Identifying Highly Correlated Stocks
Using the Last Few Principal Components

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Abstract: We show that the last few components in principal component analysis of the correlation matrix of a group of stocks may contain useful financial information by identifying highly correlated pairs or larger groups of stocks. The results of this type of analysis can easily be included in the information an investor uses to manage their portfolio.

Keywords: Principal component analysis, stock selection, diversification, stock portfolios

JEL Classifications: G11

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1 Introduction

In recent years Principal Component Analysis (PCA) has been widely applied to the study of financial markets. PCA (Jolliffe, 1986) is a standard method in statistics for extracting an ordered set of uncorrelated sources of variation in a multivariate system. Given that financial markets are typically characterised by a high degree of multicollinearity, implying that there are only a few independent sources of information in a market, PCA is an attractive method to apply.

Using random matrix theory, or, more specifically, the spectral decomposition theorem (Jolliffe, 1986, p13) many authorities have divided the eigenvectors into three distinct groups based on their eigenvalues. For example, Kim and Jeong (2005) decomposed a correlation matrix of 135 stocks which traded on the New York Stock Exchange (NYSE) into three parts:

1. The first principal component (PC1) with the largest eigenvalue which they asserted represents a market wide effect that influences all stocks.

2. A variable number of principal components (PCs) following the market component which represent synchronized fluctuations affecting groups of stocks.

3. The remaining PCs indicate randomness in the price fluctuations.

Authorities such as Driessen et al. (2003), Kim and Jeong (2005), Pérgion et al. (2007), Kritzman et al. (2011), Billio et al. (2012), and Zheng et al. (2012) all assumed without any real question that the PCs of interest were PC1 and, depending on the author’s purposes, some or all of the PCs in group (2) above.

It is widely understood in the statistical literature that there may be two more categories of PC in addition to the three listed above. The first is the detection of outlier variables (rather than outlier observations). In the analysis of our data below we did not find any PCs of this type so will not pursue this further, see Jolliffe (1986, Ch10) for details on this type of PC.

The other type of PC is near constant relationships between variables. To see how these are detected, consider two stocks which are highly correlated. Assume the eigenvalue of principal component $k$ is small and very close to zero. The
eigenvalue of each principal component is a linear combination of all variables (Jolliffe 1986), which can be written as

$$\alpha_k'x = \sum_{i=1}^{p} \alpha_{ki}x_i$$

(1)

where $\alpha_k'x$ is the eigenvalue of component $k$, and $\alpha_{ki}$ is the coefficient of variable $i$ (in our case stocks) in component $k$. If variables $x_1$ and $x_2$ are the two highly correlated variables (stocks) being detected in component $k$, each variable will have a large coefficient while the remainder of the variables will have near zero coefficients. Equation (1) then reduces to

$$0 \approx \alpha_{k1}x_1 + \alpha_{k2}x_2 + 0.$$ (2)

As a consequence, the closer $\alpha_{k1}$ and $\alpha_{k2}$ are in magnitude, the more correlated the $x_1$ and $x_2$. If $x_1$ and $x_2$ are highly positively correlated then $\alpha_{k1}$ and $\alpha_{k2}$ will have opposite signs. If they are negatively correlated they will have the same sign. In this illustration we have used two stocks but larger associations may be found.

In any given market such correlated assets may or may not exist. If they do exist, finding them is straightforward as we will show below and the implications for stock selection and portfolio management are easily understood.

The remainder of this paper is structured as follows; Section (2) describes the data and methods, Section (3) presents our results and Section (4) concludes.

2 Data and Methods

2.1 Data

Our research is based on the Australian market. The main index for the market is the ASX200, which is a market capitalization weighted index of the 200 largest shares by capitalization listed on the Australian Securities Exchange. The index, in its current form, was created on 31 March 2000. We investigated the constituents of the ASX200 index from inception to February 2014. The ASX200 index is a capitalization index and so does not adjust for dividends. In our research we calculated the returns for all constituents which included the dividends paid.

There was a high frequency of stocks that were added to or deleted from the index over time, so we identified all stocks which had been in the ASX200 for the whole study period. After adjusting for mergers, acquisitions, and name changes we obtained a final data set of 524 unique stocks. We obtained daily closing prices and dividends for each stock from the SIRCA database[1]. All the prices and

dividends were adjusted to be based on the AUD. The return was calculated in
the following steps:

1. We created a new variable associated with each stock called the Dividend
Factor. We started with a factor of 1 and every time a dividend was paid we
multiplied the Dividend Factor,

\[
\text{Daily Dividend Factor}_i(t) = \begin{cases} 
1 & \text{if no dividend} \\
1 + \frac{D_i(t)}{P_i(t)} & \text{if dividend}
\end{cases}
\]

Cumulative Dividend Factor_i(t) = \prod_{j=1}^{t}(\text{Daily Dividend Factor}_i(t))

where \(D_i(t)\) is the dividend for stock \(i\) in time \(t\), \(P_i(t)\) is price for stock \(i\) at
time \(t\) in units of one trading day.

2. We adjusted the price series with the dividend factor, the adjusted price was
calculated by

\[
P_{\text{NEW}}i(t) = P_i(t) \times \text{Cumulative Dividend Factor}_i(t).
\]

3. The return series for a given stock \(i\) was calculated as

\[
R_i(t) = \frac{P_{\text{NEW}}i(t + 1) - P_{\text{NEW}}i(t)}{P_{\text{NEW}}i(t)}.
\]

We extracted a set of stocks that had complete return information for the
whole study period, and there were 156 such stocks. The remaining 368 stocks
were either listed after April 2000 or delisted before February 2014.

2.2 Principal Component Analysis

PCA can be applied to either a correlation matrix or a covariance matrix. All
PCAs reported in this paper were carried out on correlation matrices generated
from the return series.

Correlation matrices were generated with the \texttt{cor} function in the \texttt{stats}
package, PCAs were carried out using the function \texttt{eigen} in base R (R Core Team,
2014).

We made biplots of the last few PCs using plotting functions in the \texttt{graphics}
package in base R and examined them for near constant relationships. See Jolliffe
(1986, Sec. 5.3) for details on biplots.
3 Results

In this section, we present some details on the eigenvalues of the last six PCs: biplots of their loadings, which successfully picked up groups of stocks with highly correlated returns, together with time series plots of their adjusted prices. We start with PCs 151 and 152 which picked up three pairs of near linear relationships and then discuss the 'big four’ banks and two mining firms.

These six low variance principal components detected stocks with high correlations. In some applications the eigenvalues associated with the last few principal components are very close to zero. In our case the eigenvalues the last few principal components were small but clearly different from zero. Nevertheless, they still picked up near linear relationships between some stocks, see Table (1).

In Figure (1), we present biplots of PCs 151 and 152. BHP Billiton (BHP) and CFS Retail Property Trust Group (CFX) in the real estate industry (CFX changed its name to Novion Property Group after the close of the study period and now has the ticker symbol NVN) clearly form a pair, they have high loadings of opposite signs on PC151 but low loadings on PC152. Mirvac Group (MGR), Stockland (SGP), Santos Limited (STO) and Woodside Petroleum Limited (WPL) form a group of four and have high loadings on both PC151 and PC152. This group of four can be broken into two pairs, STO with WPL and MGR with SGP based on the signs of their loadings in PC151 and PC152.

Curiously the first pair of stocks are not in the same industry. BHP is in Basic Materials and CFX in the real estate industry. They tended to move in the same direction from the start of the study period until 2011. Their price trajectories then began to move in different directions after this time, see Figure (4).

The second pair, MGR and SGP, are both large diversified real estate groups. In the beginning of 2000, they had nearly the same stock price level and have diverged since then. The similarity in their price movements can be easily seen over short time frames, but over the longer term their price level has diverged, see Figure (5).

The third pair of stocks are STO and WPL, which are in the Oil & Gas industry. Both companies explore for and produce oil and gas from onshore and offshore wells. The high correlations in their price movements over both the short and long term are clearly evident, see Figure (6).

The last four components (PC 153-156, Figures 2 and 3) all picked up the four largest banks in Australia, often referred to as the “four pillars”, their ticker symbols are ANZ, CBA, NAB and WBC. The strong relationships in price (and consequently returns) are easily seen in Figure (7). To help visualizing the price co-movement of the four big banks, we used a different scale for CBA. Its dividend-adjusted price changed from $20 in the beginning of our study period to approximately $150 at the end of study period while the other three banks had price
Table 1: Eigenvalues and variances explained by the last six principal components.

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Variance explained(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC151</td>
<td>0.40</td>
</tr>
<tr>
<td>PC152</td>
<td>0.38</td>
</tr>
<tr>
<td>PC153</td>
<td>0.32</td>
</tr>
<tr>
<td>PC154</td>
<td>0.30</td>
</tr>
<tr>
<td>PC155</td>
<td>0.29</td>
</tr>
<tr>
<td>PC156</td>
<td>0.24</td>
</tr>
</tbody>
</table>

levels that ranged from $10 to $70. Obviously, NAB was least correlated with others among the four banks. But after the 2008 financial crisis, all four banks converged to move very similarly.

In the bi-plot of PC155 and PC156, Figure (3), Australia’s two biggest mining firms were also picked up, BHP and Rio Tinto Ltd (RIO). However, they are clearly different from the four banks because they have high loadings of opposite signs in PC155 and near zero loadings in PC156. A plot of their price trajectories is presented in Figure (8). At the beginning of our study period, the price of RIO was approximately 1.4 times of BHP. Before the price collapsed in 2008, both two stocks increased significantly and RIO increased even more. At the end of 2007, the price of RIO was about 2.5 times of BHP. However, during the 2008 financial crisis, RIO also declined more than BHP. At the end of our study period, the price of RIO was about 1.5 times of BHP, which is almost the same as it was at the beginning of our study period.

4 Conclusions

Our results above differ from that of other authorities such as Kim and Jeong (2005) who reported that only the market component (PC1) and the subsequent group PCs contained useful information about the financial market analysed. Our results illustrate that further financially useful information may be contained in the last few principal components because these may identify stocks with near linear correlations. This fact seems to have been overlooked in the finance literature despite it being well known in the statistics literature.

The identification of highly correlated stocks (or other assets) can aid the task of portfolio selection and management because it identifies pairs or groups of stocks which provide little benefit for diversification; holding one of the pair or group will provide most of the benefits of diversification while freeing funds to be invested in other assets. It is not a given that such associations exist in any particular market.
Figure 1: Bi-plots of relative weights of each stock in components 151 and 152 arising from a PCA on a correlation matrix from the whole study period. The stocks are colour coded using the Industry Classification Benchmark Industry (ICB) classification. Financials are Blue (33 stocks), Health Care are Red (9 stocks), Industrials are Yellow (24 stocks), Consumer Services are Brown (19 stocks), Basic Materials are Green (31 stocks), Oil&Gas are Purple (16 stocks), Utilities are orange (5 stocks), Consumer Goods are Black (9 stocks), Telecommunications are Orchid (4 stocks), Technology are Grey (6 stocks). Stocks with a loading of at least 0.2 in one of the PCs are labelled with their ticker symbol.
Figure 2: Bi-plots of relative weights of each stock in components 153 and 154 arising from a PCA on a correlation matrix from the whole study period. The stocks are colour coded using the colour scheme described in Figure 1. Stocks with a loading of at least 0.25 in one of the PCs are labelled with their ticker symbol.
Figure 3: Bi-plots of relative weights of each stock in components 155 and 156 arising from a PCA on a correlation matrix from the whole study period. The stocks are colour coded using the colour scheme described in Figure 1. Stocks with a loading of at least 0.2 in one of the PCs are labelled with their ticker symbol.
Figure 4: Time series plot of two stocks identified in the biplot of components 151 and 152: BHP-Billiton (BHP) in basic materials and CFS Retail Property Trust Group (CFX) in the real estate industry.
Figure 5: Time series plots of near linear correlated stocks identified in a biplot of components 151 and 152: Mirvac Group (MGP) and Stockland (SGP), two stocks in the real estate sector.

Adjusted Price Series for MGR and SGP

![Graph showing time series plots of MGR and SGP prices]
Figure 6: Time series plot of two stocks in Oil & Gas industry identified in the biplot of components 151 and 152: Santos (STO) and Woodside Petroleum (WPL).
Figure 7: Time series plots of near linear correlated stocks identified in the biplots of components 153 to 156: the four big banks in Australia. ANZ (ANZ), Commonwealth Bank of Australia (CBA), National Australia Bank (NAB) and Westpac (WBC).

### Adjusted Price Series for the Four Major Banks

- **WBC**
- **NAB**
- **ANZ**
- **CBA**

Adjusted price series for the four major banks from 2000-04 to 2014-02.
Figure 8: Time series plot of two stocks in Basic Materials identified in component 155: BHP and Rio Tinto (RIO).
But if they do exist they can be easily detected and that information fed into the fund manager’s or other investor’s task of managing their portfolio well.

Even if, say, a pair of highly correlated stocks are identified there may be considerations other than diversification which mean that an investor may hold both. Figures (4), (5), and (7) show that high correlations in price movements do not necessarily indicate that the returns over the longer term will be similar. Thus depending on how much short-fall risk an investor is willing to take on, he or she may decide to hold more than one of the stocks identified in each of these groups.

We applied the PCA to the full sample period to illustrate the value of the method. In practical applications a fund manager would apply the PCA on a rolling window basis to a much shorter period of data. However, our results show that highly correlated stocks tend to remain highly correlated over long periods of times. Nevertheless, these correlations may break down as is evident in Figure (4). It is clear that since about 2011 the strong correlation between them evident in the first 11 years of the sample no longer held.

While the method presented here was applied to a stock market, it is general and can be applied to any set of assets for which a correlation matrix can be generated.

References


