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Univariate Unit Root Tests Perform Poorly When Data Are Cointegrated

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Univariate Unit Root Tests Perform Poorly When Data Are Cointegrated

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Abstract: This note demonstrates that unit root tests can suffer from inflated Type I error rates when data are cointegrated. Results from Monte Carlo simulations show that three commonly used unit root tests – the ADF, Phillips-Perron, and DF-GLS tests – frequently overreject the true null of a unit root for at least one of the cointegrated variables in reasonably sized samples. While the addition of lagged differenced (LD) terms can sometimes eliminate the size distortion, standard diagnostics such as (i) testing for serial correlation in the residuals and (ii) using information criteria to select lags are unable to identify the appropriate number of terms.

Keywords: Unit root testing, cointegration, DF-GLS test, Augmented Dickey-Fuller test, Phillips-Perron test, Monte Carlo, simulation

JEL Classifications: C32, C22, C18

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I. INTRODUCTION

When estimating relationships among time series data, it is standard practice to first test for unit roots in the individual series. If the data are integrated, one then moves to testing for cointegration. Assuming the test results are positive, one then estimates an error correction model. This note identifies practical problems with this approach. Its main finding is that unit root tests commonly suffer from size distortions when data are cointegrated. These size distortions can be substantial.

I illustrate this using a simple autoregressive, distributed lag (ARDL) system of two variables. The ARDL framework has a number of features which make it attractive for modelling dynamic relationships. It allows for interactions between variables, and incorporates both endogeneity and own- and cross-lagged effects. These features capture likely behaviors of real economic time series. The ARDL framework can be solved to identify parameter values consistent with the two variables being cointegrated. Furthermore, the ARDL framework is easily transformed to an error correction specification, which facilitates interpretation of dynamic relationships.

TABLE 1 illustrates the problem with size. X and Y are two simulated data series where the data generating process (DGP) has been chosen to ensure that they are cointegrated of order CI(1,1). I subject each series to three unit root tests: the augmented Dickey-Fuller test (ADF), the Phillips-Perron test, and the DF-GLS test. 10,000 simulations of sample sizes 100 were conducted. Significance levels were set equal to 0.05. The table reports the associated Type I error rates. All simulations were done using *Stata*, Version 14.[1]

While the ADF and DF-GLS tests produce Type I error rates for X close to 0.05, the Phillips-Perron test produces an error rate over 0.40. For Y, the results are much worse. Type

[1] All programs used to produce the results for this paper are available for download. Descriptions of the respective programs, and links for downloading, can be found in Appendix B.

I error rates are 0.206, 1.000, and 0.685 for the ADF, Phillips-Perron, and DF-GLS tests, respectively.[2] The ADF regressions show good diagnostics, with little serial correlation evident in the residuals. As I show below, unit root test results such as these are quite easy to produce with cointegrated data.

I proceed as follows. Section II presents the theory that motivates the simulation work. Section III presents additional Monte Carlo evidence of size distortions for cointegrated data. Section IV provides a possible explanation for my results. Section V explores testing for cointegration rank and finds evidence of poor performance in various rank tests. Section VI summarizes the main findings of this study and suggests directions for future research.

II. THEORY

Consider the following ARDL(1,1) model.

1)
$$y_t = \beta_{12} x_t + \gamma_{11} y_{t-1} + \gamma_{12} x_{t-1} + \varepsilon_{yt} \ \ , \varepsilon_{yt} \sim NID(0,1) \ \ ,$$
$$x_t = \beta_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} x_{t-1} + \varepsilon_{xt} \ \ , \varepsilon_{xt} \sim NID(0,1) \ \ ,$$

$t = 1,2, \dots, T$. This can be rewritten in VAR form as:

(2) $$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} a_{13} & a_{14} \\ a_{23} & a_{24} \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix}.$$

where the parameters a_{ij}, *i=1,2, j=1,2,3,4* are each functions of the β and γ terms of Equation (1).

Define $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. The determinant of the matrix $(A - \lambda I) = \begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix}$ is the characteristic equation of A, and the values of λ that set this equation equal to zero are the associated characteristic roots, or eigenvalues:

(3) $$\lambda^2 + (-a_{11} - a_{22})\lambda + (a_{11}a_{22} - a_{12}a_{21}) = 0 \ .$$

[2] Lag lengths for the Phillips-Perron and DF-GLS tests were chosen using the default options supplied by *Stata*.

A necessary condition for y_t and x_t to be cointegrated of order CI(1,1) is that the corresponding solutions to (3) be given by $\lambda_1 = 1,\ |\lambda_2| < 1$.

The following conditions on a_{11}, a_{12}, a_{21} and a_{22} are sufficient to ensure that $\lambda_1 = 1$, $|\lambda_2| < 1$.[3]

(4a) $0 < a_{22} < 1$

(4b) $0 < a_{12}a_{21} < 1 - a_{22}$

(4c) $a_{11} = 1 - \frac{a_{12}a_{21}}{1-a_{22}}$.

We can work backwards from (4a) – (4c) to obtain β and γ values consistent with $\lambda_1 = 1$, $|\lambda_2| < 1$.

Let β_{12} and β_{21} take any values such that $\beta_{12}\beta_{21} \neq 1$. Then

(5a) $\gamma_{11} = a_{11}(1 - \beta_{12}\beta_{21}) - \beta_{12}\gamma_{21}$

(5b) $\gamma_{12} = a_{12}(1 - \beta_{12}\beta_{21}) - \beta_{12}\gamma_{22}$

(5c) $\gamma_{21} = a_{21} - a_{11}\beta_{21}$

(5d) $\gamma_{22} = a_{22} - a_{12}\beta_{21}$

will produce a_{11}, a_{12}, a_{21} and a_{22} values such that $\lambda_1 = 1,\ |\lambda_2| < 1$.

Equation (2) can be arranged in vector error correction (VEC) model form as:

6)
$$\Delta y_t = a_{10} + \delta_y(y_{t-1} + \theta x_{t-1}) + \epsilon_{yt} \ ,$$
$$\Delta x_t = a_{20} + \delta_x(y_{t-1} + \theta x_{t-1}) + \epsilon_{xt} \ ,$$

where the coefficients θ, δ_y, and δ_x, as well as the error terms ϵ_{yt} and ϵ_{xt}, are functions of the a_{ij} terms, *i=1,2, j=1,2,3,4*. This allows the long-run equilibrium relationship between y_t and x_t -- represented by the parameter θ -- and the speed-of-adjustment parameters δ_y and δ_x, to all be expressed as functions of a_{11}, a_{12}, a_{21} and a_{22}.

[3] These conditions are taken from Enders (2010, page 369). I have made the conditions more restrictive to make sure that the speed of adjustment parameters have the correct sign and size.

7a) $\theta = \frac{a_{22}-1}{a_{21}}$

7b) $\delta_y = -\frac{a_{12}a_{21}}{1-a_{22}}$

7c) $\delta_x = a_{21}$.

A sufficient condition for $(y_{t-1} + \theta x_{t-1})$ to be stationary is $-1 < 1 + \delta_y + \delta_x\theta < 1$. The relationships in (4a) – (4c) imply that (i) $-1 < \delta_y < 0$, and (ii) $-1 < \delta_x\theta < 0$, so that this condition is met.

III. RESULTS OF ADDITIONAL UNIT ROOT TESTS

This section reports results from ten additional cases that highlight the problem with size distortions. The first two columns of TABLE 2 describe the model parameters and time series characteristics associated with the data generating processes (DGPs) for each case. I have chosen cases that cover a wide range of behaviours. The cases are sorted in ascending order of δ_y, the speed of adjustment coefficient for the Y series. δ_y ranges from a low of -0.16 to a high of -0.90. The last column reports the characteristic roots associated with the respective model parameters. In all cases, $\lambda_1 = 1$, $|\lambda_2| < 1$.

TABLE 3 reports more simulation findings, demonstrating that the results from TABLE 1 are not isolated outcomes. The ten panels of TABLE 3 correspond to the ten cases of TABLE 2. The top panel reports Monte Carlo results using the parameter values from Case 1:

$$\beta_{10} = 0,\ \beta_{12} = 2, \gamma_{11} = 1.24,\ \gamma_{12} = -1.70,\ \beta_{20} = 0,\ \beta_{21} = 5,\ \gamma_{21} = -4.40, \gamma_{22} = 1.75.$$

The fact that both β_{12} and β_{21} are nonzero implies that if one of the series is I(1), the other must be as well (cf. Equation 1). These values generate a VEC model with long-run equilibrium and speed of adjustment parameters $\theta = 1.25$, $\delta_y = -0.16$, and $\theta\delta_x = -0.25$. This implies that the long-run relationship between y_t and x_t is given by $y_t = -1.25x_t$. A one-unit increase in y_t from its equilibrium value causes its next period's value to decrease by

0.16 units. A one-unit increase in x_t from its equilibrium value causes its next period's value to decrease by 0.25 units (=$\delta_x \theta$). As in TABLE 1, the Monte Carlo results are based on 10,000 simulations of sample sizes 100. As a point of comparison, TABLE 3 also reports the results of unit root tests for a random walk variable, $z_t = z_{t-1} + \varepsilon_{zt}$. $\varepsilon_{zt} \sim NID(0,1)$ The *Z* column is useful as a benchmark for comparing the respective test results.

The results for the *X* variable demonstrate that the size distortions associated with each of the tests can be quite substantial. The Type I error rates for the ADF, Phillips-Perron, and DF-GLS tests are 0.536, 0.876, and 0.640, respectively. Thus, given sample data from this DGP and applying any of the three *Stata* tests, a researcher would incorrectly conclude over half the time that x_t was stationary. The results for y_t also show size distortions, but of a smaller magnitude. These results are to be compared to those reported for z_t, which is a pure random walk process. All three unit root tests produce Type I error rates for z_t that are close to 5 percent.

As is well-known, results from unit root tests can differ substantially depending on the number of lagged differenced (LD) terms included in the unit root specification. *Stata* automatically selects the number of LD terms for the Phillips-Perron and DF-GLS tests. The ADF test requires the user to supply the number of lags. For the ADF tests, I chose lag orders that were sufficient to generate white noise behaviour in the residuals.[4] The last row of the panel reports the results of a Breusch-Godfrey test where the null hypothesis is no serial correlation. The test results for x_t and y_t are close to the value of 0.05 that one would expect were there no serial correlation. These are virtually identical to those for the random walk *Z* variable which has no serial correlation by construction. Based on these results, a researcher would conclude that the ADF test was correctly specified.

[4] Lag lengths for Cases 1 to 10 are 2, 2, 4, 3, 6, 2, 3, 4, 4, and 5, respectively.

The next nine panels report more unit root test results. I have highlighted the results that show substantial size distortions. In the second panel, unit root tests for both X and Y reveal Type I error rates well above 5 percent for all three tests. For the ADF test, Type I error rates are 0.390 and 0.309, respectively. The Breusch-Godfrey test results indicate that sufficient lags have been included in the ADF specification. For the Phillips-Perron and DF-GLS tests, the Type I error rates are 0.770 and 0.655, and 0.492 and 0.394, respectively. These results for X and Y contrast with the results for the benchmark Z variable, which are approximately 5 percent across all three tests. The results from this second panel indicate that a researcher would frequently conclude that X and Y were both stationary.

The highlighted areas in the subsequent panels accumulate further evidence that unit root tests of cointegrated data frequently suffer from substantial size distortions. An egregious example is Case 6, where the Type I error rates for Y are 0.921, 1.000, and 0.931. A researcher would incorrectly classify the order of integration for this variable over 90 percent of the time using any of the three *Stata* tests.

It turns out that the size distortions for the ADF test can be eliminated by adding sufficient lagged differenced (LD) terms to the ADF specification. However, in practice, knowing the correct number of LD terms to add is impossible. Two common methods for determining the number of LD terms are (i) testing the residuals for serial correlation; and (ii) using information criteria such as the AIC and SIC (Harris, 1992).

TABLE 4 reports the results of an analysis where these methods are employed to determine the appropriate number of LD terms to add to the ADF specification. The X and Y data for TABLE 4 are generated using the DGP for Case 1. As before, the Z data are pure random walk data and are included as a benchmark. One to ten LD terms are successively added to the ADF specification. Breusch-Godfrey tests for each LD specification are reported in the top panel of TABLE 4. Average AIC and SIC values for each LD specification are reported in the subsequent two panels.

TABLE 4 is designed to address the following thought experiment: based on the results from the first three panels, how many LD terms would a researcher think is the "correct" number of terms? For example, when LAGS = 1, the null hypothesis of no serial correlation is rejected approximately 6.0 and 5.7 percent of the time for the *X* and *Y* series. For LAGS = 2, rejection rates are 5.6 and 5.5 percent.[5] The average AIC values for the *X* and *Y* series when LAGS = 1 are 168.96 and 41.51. These successively increase as additional LD terms are added. Likewise, the average SIC values for the *X* and *Y* series achieve their minimum when LAGS = 1. Using the diagnostics from these three panels, a researcher might conclude that the "correct" number of LD terms to add was 1 or 2.

The fourth panel of TABLE 4 reports the ADF Type I error rates for each LD specification. When LAGS = 1, the Type I error rates for the *X* and *Y* variables are 0.734 and 0.148. When LAGS = 2, they are 0.546 and 0.106, respectively. In other words, using commonly accepted methods for determining the appropriate number of LD terms, a researcher would likely conclude that one, or at most two, LD terms was sufficient to control for serial correlation in the ADF specification. Either strategy would lead one to conclude over half the time that the *X* variable was stationary. In fact, it would take ten or more LD terms to reduce the size of the ADF test to 5 percent. I conclude that diagnostic tests are unable to identify the appropriate number of LD terms.

Similar results are obtained for the remaining cases (Appendix A reports the results of a similar procedure applied to Case 2). In all cases, the information criteria select a single LD

[5] In practice, a researcher only has a single test for serial correlation to go on, so that it is likely that that he/she would find a single LD term to be sufficient in this case.

term. Tests for serial correlation generally indicate that more than one LD term should be included, but not so many as to eliminate the size distortion.

IV. DISCUSSION OF UNIT ROOT TESTS

The reason for the poor performance of the above unit root tests is unclear and needs further investigation. One possibility is that the bivariate cointegrated, vector auto-regression (CVAR) systems produce ARMA representations with strong negative MA components. It is well known that unit root tests do not perform well in this setting, and that the Phillips-Perron test is particularly vulnerable.[6]

An example of a CVAR system that produces a strong MA component is the following triangular system.[7]

8)
$$y_t = x_t + \varepsilon_{yt} \ , \ \varepsilon_{yt} \sim NID(0,1) \ ,$$
$$x_t = x_{t-1} + \varepsilon_{xt} \ , \ \varepsilon_{xt} \sim NID(0,1) \ .$$

Here, x_t is a random walk, and y_t and x_t are cointegrated. It follows that

9)
$$\Delta y_t = \varepsilon_{xt} + \left(\varepsilon_{yt} - \varepsilon_{y,t-1}\right) \ ,$$
$$\Delta x_t = \varepsilon_{xt} \ .$$

Results of applying the unit root tests to 10,000 simulated datasets of 100 observations each are given below.

TEST	***X***	***Y***
ADF	0.0487	0.0520
Phillips-Perron	0.0657	0.4591
DF-GLS	0.0442	0.1186
BREUSCH-GODFREY TEST:	0.0624	0.0586

[6] I thank Pierre Perron for pointing this out in private correspondence.
[7] All simulations used four lags in the augmented Dickey-Fuller specifications. I thank Peter Phillips for suggesting this triangular system.

As noted above, the Philllips-Perron test is well known to perform particularly poorly in the presence of a strong, negative MA term. The results in the table are consistent with this observation. Whether this explanation applies to other cases, and whether it is sufficient by itself to explain the poor performances of the unit root tests, is a subject for future research.

V. RESULTS OF COINTEGRATION TESTS

The standard procedure for estimating error correction models first tests for unit roots in the individual variables, then tests if the variables are cointegrated. The preceding analysis has focussed on unit root testing. This section turns to testing for the number of cointegrating relations in the VEC model.

I use three tests. The first is Johansen's trace statistic test (Johansen, 1995). Given the bivariate CVAR system from Equation (1), there are three possibilities: 0, 1, and 2 cointegrating relations. The trace test is sequential: First test if there are no more than 0 cointegrating relations. If one fails to reject, then conclude that there 0 cointegrating relations. If one rejects, then test if there are no more than 1 cointegrating relation. If one fails to reject, conclude that there is 1 cointegrating relation. If one rejects again, conclude that there are 2 cointegrating relations. A 5% significance level is used. The next two tests select the number of cointegrating relations based on minimum Schwarz Bayes and Akaike Information Criterion values.

TABLE 5 reports the results of testing the ten cases of TABLE 3. The top panel reports results when the respective samples have 100 observations, which is the same number of observations used for the simulations in the previous tables. The values in the table indicate the rate that the respective tests select the wrong number of cointegrating relations.

For example, when the DGP is characterized by the parameter values for Case 1 ($\beta_{12} = 2.00$, $\gamma_{11} = 1.24$, $\gamma_{12} = -1.70$, $\beta_{21} = 5.00$, $\gamma_{21} = -4.40$, $\gamma_{22} = 1.75$), Johansen's trace test gets it wrong 32.3 percent of the time, concluding that either 0 or 2 is the correct number of cointegrating relations. Using minimum SBIC and AIC values would lead to the wrong conclusion 23.9 and 57.9 percent of the time. The false positive rates are virtually identical across all ten cases.

As a point of comparison, I also investigate two VEC models with 0 and 2 cointegrating relations. In the first VEC model, both variables are independent random walks.

10)
$$y_t = y_{t-1} + \varepsilon_{yt} \ , \ \varepsilon_{yt} \sim NID(0,1) \ ,$$
$$x_t = x_{t-1} + \varepsilon_{xt} \ , \ \varepsilon_{xt} \sim NID(0,1) \ .$$

In the second VEC model, both variables are independent, Gaussian white noise processes.

11)
$$y_t = \varepsilon_{yt} \ , \ \varepsilon_{yt} \sim NID(0,1) \ ,$$
$$x_t = \varepsilon_{xt} \ , \ \varepsilon_{xt} \sim NID(0,1) \ .$$

For the first set of processes, the trace test selects the wrong number of cointegrating relations 13.4 percent of the time. The minimum SBIC and AIC criteria select the wrong number 10.2 and 75.5 percent of the time, respectively. The results are very different for the second set of processes. All three tests have stellar records when it comes to choosing the correct number of cointegrating relations. The bottom panel of TABLE 5 reports the results when the sample sizes are increased to 1000 observations per simulated sample.

The conclusion from these experiments is that none of the cointegration rank tests is generally reliable when it comes to choosing the correct number of cointegrating relations. The best performing test from TABLE 5 is the minimum SBIC criterion. However, for the bivariate CVAR processes examined in this study, this test incorrectly selects the true number of cointegrating relations approximately a fourth of the time. It is true that when sample size increases to 1000 observations, the Minimum SBIC criterion performs quite well. But most studies other than financial time series have far fewer than 1000 observations.

VI. CONCLUSION

This note highlights a problem with conventional testing of unit roots in applied research. While these tests perform well when variables are random walks, a problem arises when variables are part of a cointegrated system. Using simulated data from a bivariate cointegrated vector autoregression (CVAR) model, I show that unit root tests have poor size, frequently overrejecting the correct null hypothesis of a unit root. Type I error rates can be substantial, over 50 percent in some cases.

This is a problem for applied researchers. Unit root testing is often the first step in estimating an error correction model. If the respective variables are determined to be I(1), then the next step is to test for cointegration. If the variables are cointegrated, one can proceed to estimate the long-run relationship between the variables. However, if unit root testing finds that one variable is I(1), while the second is I(0), then conventional practice would conclude that the variables cannot be cointegrated, as no long run relationship between the two variables is possible. The experiments in this study demonstrate that unit root testing would frequently lead a researcher to the wrong conclusion in this application.

As noted by the comments to the discussion paper version of this study, a possible solution to this problem is to adopt an alternative approach. This alternative approach would first determine the number of cointegrating relations, and then check for the stationarity of the individual variables by testing that a unit vector is a "cointegrating vector." This requires that there be a reliable system for determining cointegration rank. Our tentative analyses are not encouraging. Using three different testing procedures (Johansen's trace statistic test and minimum SBIC and AIC values), we find that none of the testing procedures performed well, at least for samples of size 100.

The main contribution of this study is that it identifies some important problems associated with unit root testing and the estimation of cointegrated systems in applied research.

Addressing these problems is an important task for future research. It may be that there are alternative unit root tests that address these shortcomings. The Ng and Perron unit root test is one possibility (Ng and Perron, 2001; Qu, 2007).[8] Likewise, alternative tests and procedures for determining cointegration rank are also worth exploring.

The experiments in this study are almost exclusively based on simulations of sample size 100. Another topic for future research is to investigate test performance for other sample sizes. It may be that some of the problems identified here become inconsequential if sample sizes are larger. Finally, all the simulations in this study were conducted using *Stata*. It is not impossible that some of the built-in procedures in *Stata* have errors, or other problems. Investigating the performance of unit root tests in other software packages would therefore also be of interest.

Most applied researchers depend on statistical software packages such as *Stata, Eviews, RATS, R,* and others to undertake their time series analyses. They assume that these programs produce reliable results when applied to well-behaved datasets of reasonable size. This study shows that this confidence may be misplaced when testing for unit roots with cointegrated data. Gaining a better understanding of the reasons for the problems identified here, and devising reliable solutions, is an important task for future research.

[8] I thank Pierre Perron for suggesting this in private correspondence.

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TABLE 1
Example of Unit Root Test Results Using Cointegrated Data

UNIT ROOT TEST	***X***	***Y***
ADF	0.051	0.206
Phillips-Perron	0.410	1.000
DF-GLS	0.087	0.685

NOTE: Values in the table are Type I error rates associated with the null hypothesis that the data series have a unit root. The Monte Carlo experiments simulated 10,000 datasets, with each dataset having 100 observations. The underlying DGP is the ARDL framework represented by Equation 1 in the text, with the following parameter values:

$\beta_{10} = 0, \beta_{12} = 3, \gamma_{11} = 1,\ \gamma_{12} = -3,\ \beta_{20} = 0,\ \beta_{21} = -1,\ \gamma_{21} = -0.2, \gamma_{22} = 0.2$

The corresponding long-run equilibrium and speed of adjustment parameters (see Equations 7a)-7c) are given by: $\theta = -0.1, \delta_y = -0.5, \delta_x = 1, \theta\delta_x = -0.1$.

TABLE 2
DESCRIPTION OF CASES

CASE	***MODEL PARAMETERS (1)***	***TIME SERIES CHARACTERISTICS (2)***	***CHARACTERISTIC ROOTS (3)***
1	$\beta_{12} = 2.00, \gamma_{11} = 1.24,\ \gamma_{12} = -1.70,$ $\beta_{21} = 5.00,\ \gamma_{21} = -4.40, \gamma_{22} = 1.75$	$\delta_y = -0.16, \delta_x = -0.20, \theta\delta_x = -0.25, \theta = 1.25$	$\lambda_1 = 1, \lambda_2 = 0.59$
2	$\beta_{12} = 2.00, \gamma_{11} = 1.80,\ \gamma_{12} = -1.60,$ $\beta_{21} = 5.00,\ \gamma_{21} = -4.50, \gamma_{22} = 1.25$	$\delta_y = -0.20, \delta_x = -0.50, \theta\delta_x = -0.25, \theta = 0.50$	$\lambda_1 = 1, \lambda_2 = 0.55$
3	$\beta_{12} = 2.00, \gamma_{11} = 0.35,\ \gamma_{12} = 0.60,$ $\beta_{21} = 3.00,\ \gamma_{21} = -2.05, \gamma_{22} = -2.80$	$\delta_y = -0.25, \delta_x = 0.20, \theta\delta_x = -0.80, \theta = -4.00$	$\lambda_1 = 1, \lambda_2 = -0.05$
4	$\beta_{12} = 1.00, \gamma_{11} = 1.45,\ \gamma_{12} = -0.90,$ $\beta_{21} = 5.00,\ \gamma_{21} = -3.65, \gamma_{22} = 1.30$	$\delta_y = -0.45, \delta_x = -0.90, \theta\delta_x = -0.20, \theta = 0.22$	$\lambda_1 = 1, \lambda_2 = 0.35$
5	$\beta_{12} = 0.50, \gamma_{11} = 0.72,\ \gamma_{12} = -0.85,$ $\beta_{21} = 1.00,\ \gamma_{21} = -0.92, \gamma_{22} = 1.10$	$\delta_y = -0.48, \delta_x = -0.40, \theta\delta_x = -0.50, \theta = 1.25$	$\lambda_1 = 1, \lambda_2 = 0.02$
6	$\beta_{12} = 3.00, \gamma_{11} = 1.00,\ \gamma_{12} = -3.00,$ $\beta_{21} = -1.00,\ \gamma_{21} = -0.20, \gamma_{22} = 0.20$	$\delta_y = -0.50, \delta_x = -1.00, \theta\delta_x = -0.10, \theta = 0.10$	$\lambda_1 = 1, \lambda_2 = 0.40$
7	$\beta_{12} = 2.00, \gamma_{11} = 2.20,\ \gamma_{12} = -1.80,$ $\beta_{21} = 5.00,\ \gamma_{21} = -2.90, \gamma_{22} = 1.35$	$\delta_y = -0.60, \delta_x = -0.90, \theta\delta_x = -0.15, \theta = 0.17$	$\lambda_1 = 1, \lambda_2 = 0.25$
8	$\beta_{12} = 0.80, \gamma_{11} = 0.72,\ \gamma_{12} = -1.08,$ $\beta_{21} = 0.60,\ \gamma_{21} = -0.64, \gamma_{22} = 0.96$	$\delta_y = -0.60, \delta_x = -0.40, \theta\delta_x = -0.40, \theta = 1.00$	$\lambda_1 = 1, \lambda_2 = 0$

CASE	***MODEL PARAMETERS (1)***	***TIME SERIES CHARACTERISTICS (2)***	***CHARACTERISTIC ROOTS (3)***
9	$\beta_{12} = 0.80, \gamma_{11} = 0.52,\ \gamma_{12} = -1.16,$ $\beta_{21} = 0.60,\ \gamma_{21} = -0.52, \gamma_{22} = 1.06$	$\delta_y = -0.80, \delta_x = -0.40, \theta\delta_x = -0.30, \theta = 0.75$	$\lambda_1 = 1, \lambda_2 = -0.1$
10	$\beta_{12} = 2.00, \gamma_{11} = 1.90,\ \gamma_{12} = -1.90,$ $\beta_{21} = 5.00,\ \gamma_{21} = -1.40, \gamma_{22} = 1.40$	$\delta_y = -0.90, \delta_x = -0.90, \theta\delta_x = -0.10, \theta = 0.11$	$\lambda_1 = 1, \lambda_2 = 0$

NOTE: Each of the cases above is based on the ARDL framework of Equation (1), with associated parameter values given in Column (1). The values of the characteristics δ_y, δ_x, and θ reported in Column (2) are, respectively, the values of the speed of adjustment parameters and the long-run relationship parameter between *Y* and *X* in the error correction models of Equation (6) that correspond to the values of the model parameters for that case. $\theta\delta_x$ identifies the systematic change in Δx_t corresponding to a one-unit change in x_t. Sufficient conditions for the two I(1) series to be CI(1,1) are $-1 < \delta_y < 0$, and (ii) $-1 < \delta_x\theta < 0$. The last column reports the values of the characteristic roots in the VAR specification of Equation (2) for that case. A necessary condition for the series to be cointegrated is $\lambda_1 = 1$, $|\lambda_2| < 1$.

TABLE 3
More Examples of Unit Root Test Results Using Cointegrated Data

CASE	*TEST*	*X*	*Y*	*Z*
1	***ADF***	0.536	0.100	0.055
	Phillips-Perron	0.876	0.203	0.062
	DF-GLS	0.640	0.121	0.045
	BREUSCH-GODFREY TEST:	0.056	0.058	0.059
2	***ADF***	0.390	0.309	0.055
	Phillips-Perron	0.770	0.655	0.062
	DF-GLS	0.492	0.394	0.045
	BREUSCH-GODFREY TEST	0.059	0.057	0.059
3	***ADF***	0.113	0.048	0.049
	Phillips-Perron	0.970	0.029	0.062
	DF-GLS	0.429	0.066	0.045
	BREUSCH-GODFREY TEST	0.056	0.063	0.062
4	***ADF***	0.200	0.403	0.053
	Phillips-Perron	0.787	0.963	0.062
	DF-GLS	0.419	0.680	0.045
	BREUSCH-GODFREY TEST	0.063	0.069	0.057
5	***ADF***	0.159	0.106	0.045
	Phillips-Perron	1.000	1.000	0.062
	DF-GLS	0.756	0.639	0.045
	BREUSCH-GODFREY TEST	0.053	0.063	0.063
6	***ADF***	0.051	0.921	0.055
	Phillips-Perron	0.024	1.000	0.062
	DF-GLS	0.038	0.931	0.045
	BREUSCH-GODFREY TEST	0.061	0.056	0.059
7	***ADF***	0.083	0.578	0.053
	Phillips-Perron	0.452	0.998	0.062
	DF-GLS	0.170	0.808	0.045
	BREUSCH-GODFREY TEST	0.057	0.066	0.057
8	***ADF***	0.266	0.405	0.049
	Phillips-Perron	0.999	1.000	0.062
	DF-GLS	0.677	0.782	0.045
	BREUSCH-GODFREY TEST	0.062	0.062	0.062

CASE	***TEST***	***X***	***Y***	***Z***
9	***ADF***	0.087	0.311	0.049
	Phillips-Perron	0.952	1.000	0.062
	DF-GLS	0.354	0.696	0.045
	BREUSCH-GODFREY TEST	0.055	0.061	0.062
10	***ADF***	0.050	0.347	0.048
	Phillips-Perron	0.316	1.000	0.062
	DF-GLS	0.085	0.826	0.045
	BREUSCH-GODFREY TEST	0.059	0.063	0.060

NOTE: The values in the table are the rejection rates of the respective null hypothesis. The associated Monte Carlo experiments simulated 10,000 datasets, with each dataset having 100 observations. For the unit root tests (*ADF*, *Phillips-Perron,* and *DF-GLS)*, the null hypothesis is that the series has a unit root. For the Breusch-Godfrey tests, the null hypothesis is that the residuals associated with the *ADF* test are not serially correlated.

TABLE 4
The Effect of Adding Lagged Differenced Terms to the Dickey-Fuller Unit Root Regression Equation: Case 1

	X	***Y***	***Z***
BREUSCH-GODFREY TESTS:			
LAGS = 1	0.060	0.057	0.057
LAGS = 2	0.056	0.055	0.063
LAGS = 3	0.062	0.056	0.060
LAGS = 4	0.056	0.054	0.059
LAGS = 5	0.059	0.059	0.060
LAGS = 6	0.060	0.060	0.062
LAGS = 7	0.059	0.061	0.061
LAGS = 8	0.067	0.058	0.063
LAGS = 9	0.056	0.064	0.065
LAGS = 10	0.060	0.061	0.064
AIC VALUES:			
LAGS = 1	168.96	41.51	279.67
LAGS = 2	169.93	42.39	280.44
LAGS = 3	170.91	43.55	281.41
LAGS = 4	171.84	44.49	282.39
LAGS = 5	172.91	45.38	283.40
LAGS = 6	173.65	46.40	284.03
LAGS = 7	174.65	47.31	284.93
LAGS = 8	175.51	48.31	285.79
LAGS = 9	176.73	49.25	286.81
LAGS = 10	177.49	50.05	287.48
SIC VALUES:			
LAGS = 1	179.34	51.89	290.05
LAGS = 2	182.90	55.37	293.42
LAGS = 3	186.48	59.12	296.98
LAGS = 4	190.00	62.66	300.56
LAGS = 5	193.67	66.14	304.16
LAGS = 6	197.01	69.75	307.39
LAGS = 7	200.60	73.26	310.88
LAGS = 8	204.06	76.86	314.33
LAGS = 9	207.87	80.39	317.95
LAGS = 10	211.23	83.78	321.22

TABLE 4 (continued)

	X	*Y*	*Z*
ADF UNIT ROOT TESTS:			
LAGS = 1	0.734	0.148	0.054
LAGS = 2	0.546	0.106	0.050
LAGS = 3	0.405	0.085	0.055
LAGS = 4	0.291	0.068	0.054
LAGS = 5	0.222	0.063	0.047
LAGS = 6	0.166	0.054	0.048
LAGS = 7	0.143	0.054	0.043
LAGS = 8	0.116	0.046	0.042
LAGS = 9	0.092	0.051	0.045
LAGS = 10	0.082	0.046	0.044

NOTE: The Monte Carlo experiments used to produce the results above simulated 10,000 datasets, with each dataset having 100 observations. The values in the top panel ("Breusch-Godfrey Tests") are the rejection rates associated with the null hypothesis of no serial correlation for alternative specifications of lagged differenced (LD) terms in the ADF specification. The values in the next two panels ("AIC Values" and "SIC Values") are the average information criteria values associated with the respective LD specifications. The number of observations are held constant across the different specifications. The values in the bottom panel ("ADF Unit Root Tests") are the Type I error rates associated with the null hypothesis of a unit root using the ADF test with the designated number of lagged, differenced terms.

TABLE 5
Cointegration Tests

Cointegrating Relations	*Case*	*Trace Test*	*Minimum SBIC*	*Minimum AIC*
		T = 100		
1	***1***	0.323	0.239	0.579
1	***2***	0.323	0.242	0.580
1	***3***	0.323	0.234	0.581
1	***4***	0.317	0.234	0.576
1	***5***	0.323	0.232	0.579
1	***6***	0.317	0.232	0.580
1	***7***	0.319	0.232	0.575
1	***8***	0.316	0.230	0.575
1	***9***	0.317	0.230	0.576
1	***10***	0.322	0.232	0.574
0	$\gamma_{11}, \gamma_{22} = 1$ $\beta_{12}, \beta_{21}, \gamma_{21}, \gamma_{22} = 0$	0.134	0.102	0.755
2	$\beta_{12}, \beta_{12} = 0$ $\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22} = 0$	0.000	0.000	0.000
		T = 1000		
1	***1***	0.311	0.084	0.578
0	$\gamma_{11}, \gamma_{22} = 1$ $\beta_{12}, \beta_{21}, \gamma_{21}, \gamma_{22} = 0$	0.112	0.006	0.731
2	$\beta_{12}, \beta_{12} = 0$ $\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22} = 0$	0.000	0.000	0.000

NOTE: The values in the table indicate the percent of times the following conditions were not met. *For Trace Test (Rank = 0)*: Fail to reject the hypothesis that the VEC model has no more than 0 cointegrating relations. *For Trace Test (Rank = 1)*: (i) Reject the hypothesis that the VEC model has no more than 0 cointegrating relations, and (ii) Fail to reject the hypothesis that it has no more than 1 cointegrating relation. *For Trace Test (Rank = 2)*: (i) Reject the hypothesis that the VEC model has no more than 0 cointegrating relations, and (ii) Reject the hypothesis that it has no more than 1 cointegrating relation. *For Minimum SBIC (Rank = j):* The number of cointegrating relations that minimizes the Schwarz Bayes Information Criterion is *j*. *For Minimum AIC (Rank = j):* The number of cointegrating relations that minimizes the Akaike Information Criterion is *j*. The Monte Carlo experiments for the top panel simulate 10,000 datasets, with each dataset having 100 observations. The Monte Carlo experiments for the bottom panel simulate 10,000 datasets, where each dataset has 1000 observations.

APPENDIX A
The Effect of Adding Lagged Differenced Terms to the Dickey-Fuller Unit Root Regression Equation: Case 2

	X	*Y*	*Z*
BREUSCH-GODFREY TESTS:			
LAGS = 1	0.069	0.069	0.057
LAGS = 2	0.060	0.060	0.063
LAGS = 3	0.063	0.059	0.060
LAGS = 4	0.056	0.057	0.059
LAGS = 5	0.062	0.056	0.060
LAGS = 6	0.059	0.058	0.062
LAGS = 7	0.059	0.061	0.061
LAGS = 8	0.065	0.056	0.063
LAGS = 9	0.060	0.060	0.065
LAGS = 10	0.063	0.064	0.064
AIC VALUES:			
LAGS = 1	179.18	14.35	279.67
LAGS = 2	179.93	15.04	280.44
LAGS = 3	181.00	16.15	281.41
LAGS = 4	181.89	16.95	282.39
LAGS = 5	182.89	17.86	283.40
LAGS = 6	183.62	18.93	284.03
LAGS = 7	184.67	19.96	284.93
LAGS = 8	185.51	20.94	285.79
LAGS = 9	186.71	21.91	286.81
LAGS = 10	187.58	22.78	287.48
SIC VALUES:			
LAGS = 1	189.56	24.73	290.05
LAGS = 2	192.90	28.02	293.42
LAGS = 3	196.57	31.72	296.98
LAGS = 4	200.06	35.12	300.56
LAGS = 5	203.65	38.62	304.16
LAGS = 6	206.97	42.28	307.39
LAGS = 7	210.62	45.91	310.88
LAGS = 8	214.06	49.49	314.33
LAGS = 9	217.85	53.05	317.95
LAGS = 10	221.32	56.52	321.22

APPENDIX (continued)

	X	*Y*	*Z*
<u>ADF UNIT ROOT TESTS</u>:			
LAGS = 1	0.592	0.477	0.054
LAGS = 2	0.411	0.324	0.050
LAGS = 3	0.287	0.218	0.055
LAGS = 4	0.202	0.163	0.054
LAGS = 5	0.145	0.123	0.047
LAGS = 6	0.113	0.094	0.048
LAGS = 7	0.098	0.083	0.043
LAGS = 8	0.079	0.065	0.042
LAGS = 9	0.067	0.056	0.045
LAGS = 10	0.059	0.052	0.044

NOTE: The Monte Carlo experiments used to produce the results above simulated 10,000 datasets, with each dataset having 100 observations. Values in the top panel ("Breusch-Godfrey Tests") are the rejection rates associated with the null hypothesis of no serial correlation. Values in the next two panels ("AIC Values" and "SIC Values") are the average information criteria values associated with the respective lag specifications. The number of observations are held constant across the different specifications. The values in the bottom panel ("ADF Unit Root Tests") are the Type I error rates associated with the null hypothesis of a unit root, using the ADF test with the designated number of lagged, differenced terms.

APPENDIX B
Simulation Programs Used to Produce the Results of this Study

The programs used to generate the results in this study, and to confirm that they are doing what they are supposed to be doing, may be downloaded from the links below.

http://www.economics-ejournal.org/dp-2015-57-charroots.do/view

http://www.economics-ejournal.org/dp-2015-57-function_data.do/view

http://www.economics-ejournal.org/dp-2015-57-parameter-values.xls/view

http://www.economics-ejournal.org/dp-2015-57-table3a.do/view

http://www.economics-ejournal.org/dp-2015-57-table3b.do/view

http://www.economics-ejournal.org/dp-2015-57-table4a.do/view

http://www.economics-ejournal.org/dp-2015-57-table4b.do/view

http://www.economics-ejournal.org/dp-2015-57-table5a.do/view

http://www.economics-ejournal.org/dp-2015-57-table5b.do/view

http://www.economics-ejournal.org/dp-2015-57-vecrank.do/view

http://www.economics-ejournal.org/dp-2015-57-vecstable.do/view

These programs allow one to not only check all the programing in the paper, but to derive other combinations of coefficient values for the DGP to check the performance of the corresponding unit root tests. The remainder of this appendix explains the respective programs and how to use them.

The Excel spreadsheet "Parameter Values". The worksheet "ParameterValues" allows one to select parameter values that produce CI(1,1) series. The yellow-highlighted cells identify where the researcher enters values for a_{22} (STEP ONE), and a_{12} and a_{21} (STEP TWO) and gives instructions to ensure they satisfy the necessary conditions.

STEP ONE	Choose 0 < a22 < 1			
a22 =	0.9			
STEP TWO	Choose a12 and a21 such that 0 < a12a21 < 1-a22			
a12 =	-0.1			
a21 =	-0.9			
a12a21 =	0.09	1-a22=	0.1	

From here, the spreadsheet calculates the corresponding value of a_{11} that guarantees that the largest eigenvalue = 1 (STEP THREE).

STEP THREE	a11 is set equal to 1 - (a12a21/1-a22) to guarantee that the second eigenvalue is < 1										
a11 =	0.1										

The next step works backwards to calculate the values of β and γ that will produce the values of a_{11}, a_{12}, a_{21} and a_{22} that one selected in STEPS ONE through THREE. It is straightforward to show that the values of a_{11}, a_{12}, a_{21} and a_{22} in Equation (2) are related to the values of β and γ in Equations (1) and (2) by the following:

(8a) $$a_{11} = \frac{\gamma_{11}+\beta_{12}\gamma_{21}}{1-\beta_{12}\beta_{21}}$$

(8b) $$a_{12} = \frac{\gamma_{12}+\beta_{12}\gamma_{22}}{1-\beta_{12}\beta_{21}}$$

(8c) $$a_{21} = \frac{\gamma_{21}+\beta_{21}\gamma_{11}}{1-\beta_{12}\beta_{21}}$$

(8d) $$a_{22} = \frac{\gamma_{22}+\beta_{21}\gamma_{12}}{1-\beta_{12}\beta_{21}}$$

(8a) – (8d) provide 4 equations with 6 unknowns, $\beta_{12}, \beta_{21}, \gamma_{11}, \gamma_{12}, \gamma_{21}$, and γ_{22}. Accordingly, the yellow-highlighted cells in STEP FOUR allow one to set β_{12} and β_{21} to any value so long as their product does not equal 1.

STEP FOUR	Let b12 and b21 take any values such that b12*b21 is not equal to 1										
b12 =	2										
b21 =	5										
b12b21 =	10										
X =	-9										

Once β_{12} and β_{21} have been set, one can use Equations (8a) – (8d) to solve for the corresponding values of $\gamma_{11}, \gamma_{12}, \gamma_{21}$, and γ_{22} (STEP FIVE).[9]

[9] Note that g11, g12, g21, and g22 represent $\gamma_{11}, \gamma_{12}, \gamma_{21}$, and γ_{22}, respectively.

STEP FIVE		This uses the definitions of a11, a12, a21, and a22 to solve for g11, g12, g21, and g22, given b12 and b22
g21 =	-1.4000	NOTE: It is important that the last two digits be zero so that
g11 =	1.9000	these EXACT parameter values can be entered into Stata
g22 =	1.4000	
g12 =	-1.9000	

Note that these parameter values will be entered into the Stata .do files. The values need to be exact in order for the series to be cointegrated. Approximate values, such as 0.333 for 1/3, will not suffice. So this section warns that the user must be sure that the respective parameter values do not have trailing values.

The last part of the worksheet reports the corresponding values of $\theta, \delta_y,$ and δ_x.

THIS REPORTS LR EQUILIBRIUM RELATIONSHIP (theta) AND SPEED OF ADJUSTMENT VALUES (deltay/deltax)		
theta =	0.111111	
deltay =	-0.9	NOTE: Need to confirm that this is between -1 and 0.
deltax =	-0.9	
deltax*theta=	-0.1	NOTE: Need to confirm that this is between -1 and 0.

This allows one to check that the speed of adjustment parameters satisfy the conditions (i) $-1 < \delta_y < 0$, and (ii) $-1 < \delta_x\theta < 0$, which are imposed to guarantee that the series return smoothly to their equilibrium values following shocks to the system.

Also included in the Excel spreadsheet is a worksheet by the name of "PasteValues." It summarizes all the parameter values that were calculated on the "ParameterValues" worksheet (see below).

local b10 =	0	b10 =	0
local b12 =	2	b12 =	2
local g11 =	1.9	g11 =	1.9
local g12 =	-1.9	g12 =	-1.9
local b20 =	0	b20 =	0
local b21 =	5	b21 =	5
local g21 =	-1.4	g21 =	-1.4
local g22 =	1.4	g22 =	1.4

These are formatted for easy copy-and-pasting into the subsequent .do files.

The Stata .do file "CharRoots". The .do file "CharRoots" takes the parameter values from the spreadsheet and calculates the associated eigenvalues. It also provides a plot of some of the y_t and x_t values so that one can visualize how they relate. All one has to do is copy the appropriate cells from the "PasteValues" worksheet, paste them into the corresponding section of the .do file, and then run the program (see below).

```
// This section fixes the parameter values of the DGP
b10 =    0
b12 =    2
g11 =    1.9
g12 =    -1.9
b20 =    0
b21 =    5
g21 =    -1.4
g22 =    1.4
```

The associated output confirms that the two eigenvalues satisfy the conditions $\lambda_1 = 1, \ |\lambda_2| < 1$:

```
        1    2

1       1    0

abs(lambda)
        1    2

1       1    0
```

<u>The Stata .do file "VECrank"</u>. The .do file "VECrank" takes the parameters from the spreadsheet and uses the respective testing procedures in Stata to test for the rank of the matrix $(A - I) = \begin{bmatrix} a_{11} - 1 & a_{12} \\ a_{21} & a_{22} - 1 \end{bmatrix}$. A rank of 1 is consistent with the variables Y and X being cointegrated. To run this program, follow the same procedure as above and copy and paste the parameter values from the "PasteValues" worksheet into the appropriate section of the .do file, as discussed above.

This program produces output in two parts. The first part calculates the rank of $(A - I)$ directly from the population parameter values of the DGP:

```
A1I = A1-I(2)

rank(A1I)
1
```

The second part simulates 1000 observations of *X* and *Y* and performs (i) the trace test, (ii) the maximum eigenvalue test, and (iii) presents a series of information criterion values using the Schwarz Bayesian information criterion (SBIC), the Hannan and Quinn information criterion (HQIC), and the Akaike information criterion (AIC). All the results from the table below indicate that $(A - I) = \begin{bmatrix} a_{11} - 1 & a_{12} \\ a_{21} & a_{22} - 1 \end{bmatrix}$ has a rank of 1, consistent with the data being cointegrated.

```
                     Johansen tests for cointegration
Trend: constant                                      Number of obs =      998
Sample:  3 - 1000                                             Lags =        2
-------------------------------------------------------------------------------
                                                         5%
maximum                                      trace    critical
  rank    parms       LL       eigenvalue  statistic    value
    0       6     -832.09042           .    430.6955    15.41
    1       9     -616.91072     0.35029      0.3362*    3.76
    2      10     -616.74265     0.00034
-------------------------------------------------------------------------------
                                                         5%
maximum                                       max     critical
  rank    parms       LL       eigenvalue  statistic    value
    0       6     -832.09042           .    430.3594    14.07
    1       9     -616.91072     0.35029      0.3362     3.76
    2      10     -616.74265     0.00034
-------------------------------------------------------------------------------
maximum
  rank    parms       LL       eigenvalue     SBIC        HQIC        AIC
    0       6     -832.09042               1.709033    1.690751    1.67954
    1       9     -616.91072     0.35029    1.29857*   1.271146*   1.25433
    2      10     -616.74265     0.00034   1.305153    1.274682   1.255997
-------------------------------------------------------------------------------
```

<u>The Stata .do file "VECstable"</u>. The .do file "VECstable" takes the parameters from the spreadsheet, simulates 1000 observations using those parameter values, estimates the VEC model, and then reports the estimated eigenvalues of the system. This program also has output in two parts. The first part estimates of the VEC model and allows one to compare the

estimated values of θ, δ_y, and δ_x with their true values, as reported at the bottom of the "ParameterValues" worksheet.

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D_y						
_ce1 L1.	-.9075725	.0403104	-22.51	0.000	-.9865793	-.8285656
y LD.	-.0016972	.0332669	-0.05	0.959	-.0668991	.0635047
x LD.	-.0019733	.0196572	-0.10	0.920	-.0405007	.0365541
_cons	.0124837	.0079948	1.56	0.118	-.0031858	.0281533
D_x						
_ce1 L1.	-.935039	.0906559	-10.31	0.000	-1.112721	-.7573566
y LD.	.0216589	.0748156	0.29	0.772	-.1249769	.1682947
x LD.	-.0001907	.044208	-0.00	0.997	-.0868369	.0864554
_cons	-.012117	.0179799	-0.67	0.500	-.047357	.0231229

Johansen normalization restriction imposed

beta	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
_ce1						
y	1	.	.	.	.	.
x	.1106794	.0012192	90.78	0.000	.1082897	.113069
_cons	.0069962	.	.	.	.	.

From the output above, we see that the VEC estimates of δ_y, δ_x, and θ are -0.908, -0.935, and 0.111. The true values are reported on the bottom of the "Parameter Values" worksheet:

THIS REPORTS LR EQUILIBRIUM RELATIONSHIP (theta) AND SPEED OF ADJUSTMENT VALUES (deltay/deltax)		
theta =	0.111111	
deltay =	-0.9	NOTE: Need to confirm that this is between -1 and 0.
deltax =	-0.9	
deltax*theta=	-0.1	NOTE: Need to confirm that this is between -1 and 0.

The last part of the program estimates the value of the second eigenvalue, assuming that the larger of the two eigenvalues equals one:

```
Eigenvalue stability condition

  Eigenvalue                    | Modulus
  ------------------------------+---------
              1                 |       1
  .04103287                     | .041033
  -.0269914 + .01791487i        | .032396
  -.0269914 - .01791487i        | .032396

The VECM specification imposes a unit modulus.
```

$|\lambda_2|$ is estimated to be 0.041. We know from "CharRoots" program above that the true value of the second eigenvalue is zero. While standard errors are not calculated, the estimated value of $|\lambda_2|$ is well below 1, confirming again that the series are cointegrated.

The Stata .do files "function_data", "Table3A", and "Table3B". The .do files "function_data", "Table3A", and "Table3B" are a suite of three programs that are designed to be used together. These programs perform the simulation exercises that test each of the series for unit root using the (i) ADF, (ii) Phillips-Perron, and (iii) DF-GLS tests. The programs simulate 10000 data sets of 100 observations each of *X*, *Y*, and a third variable *Z*. *X* and *Y* are simulated using the parameter values pasted into the program "Table3B":

```
// This section fixes the parameter values of the DGP
local b10 = 0
local b12 = 2
local g11 = 1.9
local g12 = -1.9
local b20 = 0
local b21 = 5
local g21 = -1.4
local g22 = 1.4
```

The variable *Z* is a classic random walk variable:

(9) $z_t = z_{t-1} + \varepsilon_{zt} \; , \varepsilon_{zt} \sim NID(0,1)$.

It is included in the simulations for comparison's sake.

The programs must be run in order (first "function_data", then "Table3A", and then "Table3B"). The output consists of two parts. The first part are the Type I error rates associated with the three unit root tests:

```
RESULTS[3,3]
              X      Y      Z
DFuller    .035   .325   .053
PPerron    .291      1   .066
  DFGLS    .075   .831   .039
```

While the Phillips-Perron and DF-GLS tests automatically select lag lengths, the default option in Stata is to include no lagged differenced terms in the Dickey-Fuller specification. As serial correlation in the error term distort test results, it is advisable to add lagged differenced terms.

My simulations add 4 lags to the Dickey-Fuller test. To ensure that this is sufficient to account for serial correlation, I perform a Breusch-Godfrey test following each Dickey-Fuller test. The associated rejection rates are reported in the second part of the output:

```
BGODFREY[1,3]
              X      Y      Z
BGodfrey   .056   .056   .055

. display as text "Number of lags = `lagz'"
Number of lags = 5
```

This provides a check to ensure that I have added sufficient lagged differenced terms to ensure that the Dickey-Fuller test results are not distorted by the presence of serial correlation.

The Stata .do files "function_data", "Table4A", and "Table4B". These programs are designed to illustrate the effect of increasing the number of lagged differenced terms to the Dickey-Fuller unit root specification. As with the TABLE 3 programs, they must be run in order (first "function_data", then "Table4A", and then "Table4B").

The Stata .do files "function_data", "Table5A", and "Table5B". These programs are designed to test the performance of various cointegration rank tests. They also must be run in order (first "function_data", then "Table5A", and then "Table5B").