

# Trade Union Objectives and Economic Growth

by

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## Abstract

A trade union whose purpose is to raise wages above the competitive level may foster economic growth if it succeeds in shifting income away from the owners of capital to the workers and if the workers' marginal propensity to save exceeds the one of capitalists. We make this point in an overlapping generations framework with unionized labor. Considering a monopoly union which cares for wages and employment, we determine a range of trade union objectives and characterize the aggregate technology so that the union's policy spurs per capita income growth and increases welfare of all generations that adhere to the union.

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## 1. Introduction

A trade union is usually defined as an organization “whose purpose is to improve the material welfare of members, principally by raising wages above the competitive level.” [Booth (1995), p. 51]. This paper considers a trade union acting in this way and asks whether and how it affects economic growth.

The argument that leads us to answer these questions points to the link between the distribution of factor incomes and aggregate savings. It is well established [see, e.g., Bertola (1993, 1996)] that the growth performance of an economy is closely related to aggregate savings, i.e., to the part of aggregate output used for capital formation. In turn, aggregate savings are linked to the factor-income distribution if the propensities to save out of wage and capital income in the economy do not coincide. Therefore institutions or policies that impinge on the factor-income distribution are likely to affect economic growth.

The central idea of this paper is that a centralized trade union may qualify as such an institution. Indeed, a union that succeeds in shifting income away from the owners of capital to the workers by raising wages above the competitive level may foster growth if the economy’s propensity to save out of wage income exceeds its propensity to save out of capital income.

The study of this idea requires an analytical framework which allows for aggregate savings to be *endogenously* linked to both the factor-income distribution of the economy and the rate of economic growth. For simplicity we consider a two-period overlapping generations (OLG) economy exposed to endogenous growth à la Romer (1986). We introduce a monopoly union which sets wages at the beginning of each period so as to maximize an objective function having the real wage and the level of employment of union members as its arguments. The notion of a “trade union objective” refers to the relative weight a union attaches to either argument.

We analyze balanced growth equilibria and use the equilibrium under *laissez-faire* as a benchmark to which we relate the equilibrium with unionized labor. The comparison allows us to determine a range of trade union objectives and conditions on the aggregate technology so that the equilibrium with unionized labor exhibits faster per capita income growth. The intuition behind these findings is as follows. In an OLG economy savings are closely linked to the economy’s wage income as only (young) workers save. Therefore, a union policy that raises aggregate wage

income spurs economic growth. In turn, this is possible if the aggregate technology is such that the effect of a reduction in employment due to wages above *laissez-faire* levels induces a *pronounced* shift in the factor-income distribution in favor of wage incomes but only a *small* reduction in aggregate output.

Having identified the growth effects of unionized labor, we consider its impact on individual welfare. We find that unionized labor may lead to higher welfare of all generations that adhere to the union if it has a strong positive effect on per capita income growth. This is because the old of each generation suffer a loss in capital income which has to be offset against the increase in wage income. The negative effect on the old arises as unemployment lowers the marginal productivity of capital implying a reduced rate of return on old age savings. For the same reason, introducing a labor union cannot be Pareto-improving as the current old will only be affected by the union's policy in the form of reduced old age capital income.

There are two strands of the literature on economic growth which are related to the present paper. The first strand includes papers on endogenous growth and labor market imperfections such as Agell and Lommerud (1993), Cahuc and Michel (1996), and Hellwig and Irmen (1999). Agell and Lommerud consider a labor union that pursues an egalitarian wage policy. They show that the union may foster structural change in favor of increased productivity growth by compressing wage differentials between low-productivity and high-productivity sectors. Related arguments are employed by Cahuc and Michel and Hellwig and Irmen who consider minimum wage legislation. These authors show that minimum wages may move the economy towards more human capital respectively knowledge intensive production again stimulating per capita income growth.

The second line of research studies the growth effects of intergenerational transfers. Saint-Paul (1992), Wiedmer (1996) and Wigger (1999), among others, demonstrate that intergenerational transfers from young to old generations in the form of pay-as-you-go public pensions tend to lower per capita income growth by discouraging private savings and investment. For a similar reason, policies that imply transfers from the old to the young may foster growth as they are likely to stimulate private savings. In fact, Jones and Manuelli (1992) demonstrate that tax-financed transfers from the old to the young augment per capita income growth. A similar argument underlies Uhlig and Yanagawa (1996) who consider a policy that by shifting the tax burden away from labor to capital income moves the tax burden from the young to the old which again may have a positive impact on growth.

In light of these contributions we can state our results as follows. A union formed by the working young which succeeds in raising the aggregate wage bill effectively transfers resources from the dissaving old to the saving young which, in turn, may lead to higher aggregate savings and per capita income growth.

We establish and discuss our results in the following five sections. Section 2 sets up the basic model. Section 3 studies the competitive equilibrium which serves as a benchmark for the subsequent analysis. Our main result is presented in Section 4 where we study the equilibrium with unionized labor and highlight the link between the union's objective and economic growth. Section 5 analyzes the welfare implications of the equilibrium with unionized labor. Finally, Section 6 considers some extensions and discusses the robustness of our results.

## 2. The Model

### 2.1. The Household Sector

The household sector has a simple overlapping generations structure à la Samuelson (1956) and Diamond (1965). Each generation is represented by a single individual who lives for two periods. In the first period the individual supplies labor out of her initial labor endowment which is normalized to one, and receives wage income. This income is used to consume and to save. In the second period of life the individual retires and lives on the proceeds of her savings.

An individual born at time  $t$  draws utility from young and old age consumption. Lifetime utility  $u_t$  is determined by:

$$u_t = u(c_t^y, c_{t+1}^o),$$

where  $c_t^y$  and  $c_{t+1}^o$  denote young and old age consumption, respectively. For simplicity we assume the utility function  $u$  to take the form  $u(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$ , with  $0 < \beta < 1$  as the discount factor common to all generations. It is well known that this specification neutralizes income and substitution effects associated with changes in the interest rate.<sup>1</sup> Each generation takes the real wage  $w_t$  in  $t$  and the real interest rate  $r_{t+1}$  on savings from  $t$  to  $t + 1$  as given and

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<sup>1</sup> See Section 6 for a discussion of how a more general utility function would affect our results.

maximizes lifetime utility under the constraints:

$$\begin{aligned} c_t^y + s_t &\leq w_t L_t, \\ c_{t+1}^o &\leq (1 + r_{t+1}) s_t, \\ L_t &\leq \min\{\hat{L}_t, 1\}, \end{aligned}$$

where  $s_t$  is savings in  $t$ ,  $L_t$  is the actual labor supply, and  $\hat{L}_t$  is a quantity constraint on employment which is binding whenever  $\hat{L}_t$  is less than 1. The difference  $1 - \hat{L}_t$  can be interpreted as the rate of (involuntary) unemployment in  $t$ .

The optimal consumption plan is implicitly determined by:

$$u_{1,t} = (1 + r_{t+1}) u_{2,t},$$

where  $u_{1,t}$  and  $u_{2,t}$  are the partial derivatives of  $u$  at  $(c_t^y, c_{t+1}^o)$ . For logarithmic utility this involves the following savings function:

$$s_t = \frac{\beta}{1 + \beta} w_t L_t. \tag{1}$$

Moreover, given that an individual does not care about leisure, she always desires to supply as much labor as possible so that:

$$L_t = \min\{\hat{L}_t, 1\}.$$

## 2.2. The Production Sector

Identical firms hire the aggregate capital stock,  $K_t$ , and demand labor supplied by the young. Both factors are used to produce a homogeneous good according to a neoclassical production function  $F(K_{it}, A_t L_{it})$ , where  $K_{it}$  and  $L_{it}$  are capital and labor inputs hired by firm  $i$ , whereas  $A_t$  is an index of the economy-wide stock of knowledge in  $t$ . The function  $F$  exhibits constant returns to scale and satisfies standard concavity and differentiability conditions.

Factor markets are competitive in the sense that firms take factor prices as given. In equilibrium all firms produce with the same capital intensity so that  $K_{it}/A_t L_{it} = K_t/A_t L_t \equiv k_t$ . The respective first order conditions for profit maxi-

mization are:

$$\begin{aligned} r_t &= f'(k_t), \\ w_t &= A_t [f(k_t) - k_t f'(k_t)], \end{aligned}$$

where  $f(k_t) \equiv F(k_t, 1)$  with  $f' > 0$  and  $f'' < 0$ .

We endogenize productivity growth following Arrow (1962), Sheshinski (1967), and Romer (1986) as a “learning-by-investing” effect, i.e.,

$$A_t = K_t. \tag{2}$$

Hence, an increase in the aggregate stock of capital augments the stock of knowledge available in the economy one-to-one. An immediate implication of (2) is that the economy produces with a capital intensity  $k_t = 1/L_t$  so that factor prices become:

$$r_t = f'(1/L_t), \tag{3}$$

$$w_t = \omega(L_t) K_t, \quad \text{with } \omega(L_t) = [f(1/L_t) - f'(1/L_t)/L_t]. \tag{4}$$

Equation (4) implies that for a given level of employment the wage rate is proportional to the aggregate stock of capital. The factor of proportionality  $\omega(L_t)$  represents the external return on capital per unit of employed labor caused by the spillover from cumulated investment on labor productivity. Indeed, if productivity growth stems from (2), aggregate production is determined by  $Y_t = F(K_t, K_t L_t)$ . In addition, if factor prices are determined by (3) and (4), one finds that the social return on capital is  $dY_t/dK_t = r_t + \omega(L_t) L_t$ . Since the productivity enhancing effect of aggregate investment is not priced, the social return on capital exceeds its private counterpart where  $\omega(L_t) L_t$  is the external return that accrues to employed labor.

For further reference, observe that:

$$dr_t/dL_t = -f''(1/L_t)/L_t^2 > 0, \tag{5}$$

$$dw_t/dL_t = K_t d\omega(L_t)/dL_t = K_t f''(1/L_t)/L_t^3 < 0, \tag{6}$$

i.e., a higher employment per firm raises the marginal productivity of the existing capital and lowers the marginal productivity of labor.

### 3. The Competitive Equilibrium

Before introducing a trade union we make a short detour and quickly recall the (perfect foresight) equilibrium under full employment which serves as a benchmark for the subsequent analysis.

Given an initial level of capital  $K_0$  owned by the old generation, a competitive equilibrium determines a sequence  $\{s_t, c_t^y, c_t^o, w_t, r_t, K_{t+1}, L_t, A_t\}_{t=0}^{\infty}$  such that:

(E1) each generation  $t$  saves according to (1),

(E2) for all  $t$  aggregate savings equals aggregate investment:  $s_t = K_{t+1}$ ,

(E3) for all  $t$  there is full employment:  $L_t = 1$ ,

(E4) for all  $t$  the factor price conditions (3) and (4) hold.

The unique competitive equilibrium is a balanced growth equilibrium with a constant interest rate and a constant growth rate of wages, capital, and per capita output. Indeed, from (E3) and (E4) one finds that for all  $t$  the following holds:

$$\begin{aligned} r_t &= r^* = f'(1), \\ w_t &= w_t^* = K_t \omega(1), \end{aligned} \tag{7}$$

where the latter implies that wages and capital grow at the same pace. From (E1) and (E2) one finds that capital accumulation in all periods obeys to:

$$\frac{\beta}{1+\beta} w_t = K_{t+1}.$$

Substituting for  $w_t$  employing (7) gives:

$$\frac{\beta}{1+\beta} \omega(1) = \frac{K_{t+1}}{K_t}.$$

Hence, the laissez-faire growth rate,  $g^*$ , of capital, per capita output, and wages can be written as:

$$g^* = \frac{\beta}{1+\beta} \omega(1) - 1.$$

As  $g^*$  is independent of time, the system jumps immediately to the balanced growth equilibrium.

## 4. Introducing a Trade Union

### 4.1. The Union's Objective

The trade union is formed by the working population. The OLG structure set out above implies that only the young of each period are union members. Following Pencavel (1984) we model union preferences over pairs of wages and levels of employment in  $t$ . More precisely, the union evaluates a tuple  $(w_t, L_t)$  according to the function:

$$V_t = V(w_t, L_t) = (w_t - w_t^*)^\gamma L_t^{1-\gamma}. \quad (8)$$

The first argument in (8) is a wage mark-up defined as the difference between the actual and the competitive wage. The second argument is the rate of employment in  $t$ . The parameter  $\gamma \in (0, 1)$  determines how much weight the union attaches to wages and employment, respectively.

The union rationally anticipates aggregate labor demand which results from the firms' profit maximizing behavior. Considering equations (4) and (7) and the fact that the aggregate stock of capital  $K_t$  is predetermined at  $t$ , (8) can be written as:

$$V_t = K_t^\gamma [\omega(L_t) - \omega(1)]^\gamma L_t^{1-\gamma}.$$

As  $K_t$  is a given constant at time  $t$ , the union's maximization problem reduces to:

$$\max_{0 \leq L_t \leq 1} \tilde{V}(L_t) = [\omega(L_t) - \omega(1)]^\gamma L_t^{1-\gamma}.$$

The solution is characterized in the following proposition.

**Proposition 1.** Denote  $\rho(L_t) = -d \ln \omega(L_t) / d \ln L_t$  the elasticity of  $\omega(L_t)$ . Then, for all  $\gamma$  satisfying

$$0 < \gamma < \bar{\gamma} \equiv \lim_{L_t \rightarrow 0} \frac{1}{1 + \rho(L_t)}, \quad (9)$$

the union's maximization problem has an interior solution  $\hat{L}_t \in (0, 1)$  given

in implicit form by :

$$\frac{\omega(\hat{L}_t)}{\omega(1)} = \frac{1 - \gamma}{1 - \gamma - \gamma\rho(\hat{L}_t)}. \quad (10)$$

*Proof:* See the Appendix.

Proposition 1 states a condition for an interior solution to the union's maximization problem. Since the union attaches some weight to wages ( $\gamma > 0$ ), the chosen level of employment falls short of full employment. On the other hand, condition (9) gives an upper bound on the weight on wages so that the union never chooses to reduce employment to zero.

Equation (10) shows that the chosen level of employment  $\hat{L}_t$  relates the wage mark-up to the available technology via the elasticity  $\rho$  and the preference parameter  $\gamma$ . In view of (4),  $\rho$  is the elasticity of wages with respect to the level of employment. To get more intuition for the economics implied by (10), it is useful to link  $\rho$  to the elasticity of substitution between capital and efficient labor and to the output elasticity of efficient labor.

**Lemma 1.** Let  $\sigma(L_t)$  denote the elasticity of substitution between capital  $K_t$  and labor in efficiency units  $A_t L_t$ , and  $\varepsilon(L_t)$  the output elasticity of labor in efficiency units. Then,  $\rho(L_t)$  can be written as:

$$\rho(L_t) = \frac{1 - \varepsilon(L_t)}{\sigma(L_t)}. \quad (11)$$

*Proof:* Considering equations (4) and (6),  $\rho(L_t)$  becomes:

$$\rho(L_t) = \frac{f''}{L_t (L_t f - f')},$$

where the argument of  $f$  is  $1/L_t$ . Straightforward manipulation of the right hand side leads to (11), where:

$$\varepsilon(L_t) = 1 - \frac{f'}{L_t f} \quad \text{and} \quad \sigma(L_t) = -\frac{f'}{f} \frac{(L_t f - f')}{f''}. \quad Q.E.D.$$

In light of Lemma 1 condition (10) becomes:

$$\frac{\omega(\hat{L}_t)}{\omega(1)} = \frac{(1 - \gamma) \sigma(\hat{L}_t)}{(1 - \gamma) \sigma(\hat{L}_t) - \gamma [1 - \varepsilon(\hat{L}_t)]}.$$

This form reveals that the resulting wage mark-up is inversely related to both the elasticity of substitution between capital and efficient labor and the output elasticity of efficient labor. The economic intuition behind this result is as follows. If capital and labor in efficiency units become better substitutes, setting wages above the competitive level becomes more costly in terms of foregone employment. As a consequence, the union chooses a higher level of employment and the wage mark-up falls. Furthermore, if the output elasticity of efficient labor is high, a reduction in employment has a substantial impact on aggregate output and, henceforth, on the share of output that accrues to labor. This implies that the costs associated with a reduction in employment are high which induces the union to choose a small mark-up.

Condition (10) implicitly relates employment to the union's preference parameter  $\gamma$ .

**Lemma 2.**  $d\hat{L}_t/d\gamma < 0$  for all  $\gamma \in (0, \bar{\gamma})$  and  $\lim_{\gamma \rightarrow 0} \hat{L}_t = 1$ .

*Proof:* Applying the implicit function theorem to the first-order condition of the union's maximization problem and considering the respective second-order condition gives the result. *Q.E.D.*

As expected the chosen level of unemployment increases as the union attaches more weight to the wage mark-up. Yet, as (9) shows,  $\gamma$  must be bounded from above. In light of Lemma 1, the upper bound  $\bar{\gamma}$  becomes:

$$\bar{\gamma} \equiv \lim_{L_t \rightarrow 0} \frac{\sigma(L_t)}{\sigma(L_t) + 1 - \varepsilon(L_t)}.$$

The following CES example demonstrates how  $\bar{\gamma}$  depends on the elasticity of substitution. The bound  $\bar{\gamma}$  is equal to 1 if the elasticity of substitution is smaller than 1 and strictly smaller than 1 otherwise. Moreover, the relationship between  $\bar{\gamma}$  and the elasticity of substitution is discontinuous and non-monotonic.

**Example.** In the CES case the production function takes the form:

$$Y_t = \left[ \alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where  $\sigma$  is constant by definition. Straightforward algebra yields:

$$\varepsilon(L_t) = 1 - \frac{\alpha L_t^{\frac{1-\sigma}{\sigma}}}{\alpha L_t^{\frac{1-\sigma}{\sigma}} + 1 - \alpha},$$

implying:

$$\lim_{L_t \rightarrow 0} \varepsilon(L_t) = \begin{cases} 1 & \text{if } \sigma < 1, \\ 1 - \alpha & \text{if } \sigma = 1, \\ 0 & \text{if } \sigma > 1. \end{cases}$$

It follows that:

$$\bar{\gamma} = \begin{cases} 1 & \text{if } \sigma < 1, \\ 1/(1+\alpha) & \text{if } \sigma = 1, \\ \sigma/(1+\sigma) & \text{if } \sigma > 1. \end{cases}$$

#### 4.2. Equilibrium with Unionized Labor

A perfect foresight equilibrium with unionized labor determines a sequence  $\{s_t, c_t^y, c_t^o, w_t, r_t, K_{t+1}, L_t, A_t\}_{t=0}^{\infty}$  given  $K_0$  which satisfies (E1), (E2), and (E4) stated in Section 3 and the following labor market equilibrium condition:

$$(E3') \quad L_t = \hat{L}_t \quad \text{for all } t,$$

which states that the level of employment in the economy is determined as the quantity of labor that maximizes the union's objective. Clearly, for each firm the corresponding wage is binding so that the equilibrium wage becomes  $w_t = \hat{w}_t = \omega(\hat{L}_t) K_t$ .

From the union's problem it can be inferred that the optimal choice of employment is time-invariant, i.e.,  $\hat{L}_t = \hat{L}$ . Thus, from a similar reasoning as applied in Section 3 it follows that the equilibrium exhibits a constant interest  $\hat{r} = f'(\hat{L})$

and a constant growth rate. The latter is given by:

$$\hat{g} = \frac{\beta}{1 + \beta} \omega(\hat{L}) \hat{L} - 1. \quad (12)$$

From Lemma 2 we can infer that  $\hat{L}$  is a continuous function of  $\gamma$  in a neighborhood of  $\gamma = 0$ . Thus, as  $\hat{g}$  is a continuous function of  $\hat{L}$ , it follows that for  $\gamma$  sufficiently small, the equilibrium growth rate  $\hat{g}$  can be written as a continuous function  $\hat{g} = \hat{g}(\gamma)$  relating union preferences to the growth rate of per capita income. The following proposition states a condition on the technology and on union preferences under which the growth rate of the equilibrium with unionized labor exceeds the growth rate of the competitive equilibrium.

**Proposition 3.** Let  $\sigma(1) + \varepsilon(1) < 1$ . Then, there is some  $\tilde{\gamma} \in (0, \bar{\gamma})$  so that  $\hat{g} > g^*$  for all  $\gamma \in (0, \tilde{\gamma})$ .

*Proof:* From (12) one easily observes that the growth rate for an arbitrary level of employment  $L$  takes the form  $g = \beta/(1 + \beta) \omega(L) L - 1$ . Differentiation of  $g$  with respect to  $L$  gives  $dg/dL = \beta/(1 + \beta) \omega(L) [1 - \rho(L)]$ . Hence,  $dg/dL|_{L=1} > 0$  if and only if  $\rho(1) < 1$ . From Lemma 1 the latter is equivalent to  $\sigma(1) + \varepsilon(1) < 1$ . Then the proposition follows from Lemma 2 and the continuity of  $\hat{g}$  in the neighborhood of  $\gamma = 0$ . *Q.E.D.*

The intuitive argument behind this result is as follows. The total effect of a reduction in employment due to unionization can be decomposed in an effect on the functional distribution of income and an output effect. The former occurs as for a given output a reduction in employment increases the wage rate and reduces the interest rate. The distribution effect is measured by the elasticity of substitution between capital and efficient labor  $\sigma$ . As is well known, the labor share of aggregate income will increase if the elasticity of substitution is smaller than one. However, an increase in the labor share is not sufficient to increase total labor income (the wage bill) and, in light of (12), the growth rate. This is because a reduction in employment reduces aggregate output, i.e., causes a negative output effect. The output effect on the wage bill and the growth rate is measured by the output elasticity of efficient labor  $\varepsilon$ . Since  $\varepsilon$  is the share of aggregate output that accrues to labor, it measures to what extent labor income is reduced when aggregate production falls. In sum, for the growth rate to exceed the competitive level ( $\hat{g} > g^*$ ) the technology must be such that the effect of a

reduction in employment on the functional distribution of income in favor of labor more than outweighs the effect on aggregate output that accrues to labor. If this is the case, the aggregate wage bill rises and triggers a positive effect on aggregate savings and growth.

## 5. Unionization and Welfare

This section studies the welfare effects that materialize when the economy moves away from full employment to an equilibrium with a constant level of unemployment in all periods. We want to know how the welfare of current and future generations changes when such a switch takes place. To answer this question, we first consider a marginal reduction in employment which occurs in period  $t = 0$  and is maintained throughout all future periods and analyze its impact on the lifetime utility of all generations. Subsequently, we relate the welfare results to the findings of the previous section and provide a link between union objectives, economic growth, and individual welfare.

Suppose the economy is on an equilibrium path with  $L_t = L$ ,  $r_t = r$ , and a constant growth rate  $g$  of capital, per capita output, and wages, and consider a constant and permanent marginal reduction in employment ( $dL < 0$ ) at time  $t = 0$ . Then, the welfare of the current old, the current young, and all yet unborn generations are affected as follows.

*The Current Old.* The welfare of the old at time  $t = 0$  is given by:

$$u_{-1} = u[w_{-1}L_{-1} - s_{-1}, (1+r)s_{-1}],$$

where  $w_{-1}$ ,  $L_{-1}$  and  $s_{-1}$  are predetermined at time  $t = 0$ . Differentiating with respect to the level of employment gives:

$$\frac{du_{-1}}{dL} = u_{2,-1} \frac{dr}{dL} s_{-1},$$

so that in view of (5) one finds:

$$\frac{du_{-1}}{dL} = -u_{2,-1} \frac{f''}{L^2} s_{-1} > 0. \quad (13)$$

Thus, a reduction in employment reduces the welfare of the current old. This is due to a capital income effect. It occurs as the marginal productivity of the

existing capital stock shrinks when employment falls. This reduces the return on old age savings and, henceforth, consumption of the current old.

*The Current Young.* Welfare of the young at time  $t = 0$  is given by:

$$u_0 = u[w_0 L - s_0, (1 + r) s_0].$$

A marginal decrease in employment at time  $t = 0$  and  $t = 1$  leads to (considering the Envelope theorem and the fact that  $K_0$  is predetermined at time 0):

$$\frac{du_0}{dL} = u_{1,0} \frac{d(w_0 L)}{dL} + u_{2,0} s_0 \frac{dr}{dL}.$$

Replacing  $w_0$  by  $\omega(L) K_0$  and considering that  $u_{1,0} = (1 + r) u_{2,0}$  and  $s_0 = K_1 = (1 + g) K_0$ , this can be written as:

$$\frac{du_0}{dL} = u_{1,0} \left( \frac{d\omega}{dL} L + \omega(L) + \frac{1 + g}{1 + r} \frac{dr}{dL} \right) K_0. \quad (14)$$

Thus, what matters for the welfare of the current young is the impact of a decrease in  $L$  on the current wage bill (wage income effect) and on the discounted capital income which accrues in  $t = 1$  to the capital stock  $1 + g$  times as large as in  $t = 0$  (capital income effect). Again, the latter effect is negative. To evaluate the overall effect, substitute (4), (5), and (6) into (14) to get after some manipulations:

$$\frac{du_0}{dL} = u_{1,0} \frac{f'' f}{f' L} \left( 1 - \sigma(L) - \varepsilon(L) - \frac{1 + g}{1 + r} [1 - \varepsilon(L)] \right) K_0. \quad (15)$$

It will be seen below that this expression permits a very straightforward interpretation of the effects of a reduction in employment on the welfare of the current young.

*Future Generations.* The utility of some generation  $l \in \mathbf{N}$  is given by:

$$u_l = u[w_l L - s_l, (1 + r) s_l].$$

Considering that  $w_l = \omega(L) K_l$ ,  $K_l = (1 + g)^l K_0$ , and  $1 + g = (\beta/(1 + \beta)) \omega(L) L$ ,  $u_l$  becomes:

$$u_l = u \left[ [\omega(L) L]^{l+1} \left( \frac{\beta}{1 + \beta} \right)^l K_0 - s_l, (1 + r) s_l \right].$$

Differentiating with respect to  $L$  and considering that  $s_l = (1 + g) K_l$  and  $u_{1,l} =$

$(1+r)u_{2,l}$ , one obtains after some manipulations:

$$\frac{du_l}{dL} = u_{1,l} \left[ (l+1) \left( \frac{d\omega}{dL} L + \omega \right) + \frac{1+g}{1+r} \frac{dr}{dL} \right] K_l.$$

This expression generalizes (14) to the case of  $l > 0$ . Again, there is a capital income effect which has the same interpretation as the one in (14). However, now there is a cumulated wage income effect. This is because the reduction in employment  $dL$  does not only impinge on the wage bill in  $t = l$  but also on the wage bill in all preceding periods  $t \geq 0$ . A similar procedure as above leads to:

$$\frac{du_l}{dL} = u_{1,l} \frac{f'' f}{f' L} \left( (l+1)[1 - \varepsilon(L) - \sigma(L)] - \frac{1+g}{1+r} [1 - \varepsilon(L)] \right) K_l.$$

From this equation and equations (13) and (15) the following inferences can be drawn:

**Proposition 4.**

- i) A reduction in employment cannot be Pareto-improving.
- ii) A reduction in employment increases the welfare of generation 0 if and only if:

$$\sigma(L) + \varepsilon(L) + \frac{1+g}{1+r} [1 - \varepsilon(L)] < 1.$$

- iii) There is some  $l \in \mathbf{N}$  so that a reduction in employment increases the welfare of generation  $l$  and all subsequent generations if:

$$\sigma(L) + \varepsilon(L) < 1.$$

Part i) follows immediately from the observation that a marginal reduction in employment in period 0 hurts the old whose level of consumption solely relies on capital income. All later generations suffer similar losses in capital income when old. Yet, they may benefit from a wage income effect when young. Considering the analysis in Section 4 wage and capital income effects can be expressed in terms of distribution and output effects. In fact, part ii) of Proposition 4 states that the generation of the current young will benefit from a reduction in employment if the shift from capital income of generation  $-1$  to labor income of generation 0, i.e. the distribution effect, outweighs the output effects arising at time 0 and time 1. The distribution effect is strong if  $\sigma$  is low (see Section 4). The output effects

occur in terms of reduced labor income when young and reduced capital income when old. At time 0 the share of a reduction in aggregate output born by the wage earning generation 0 is equal to  $\varepsilon$  (the labor share). At time 1 generation 0 is interest earner and the respective share is equal to  $1 - \varepsilon$  (the capital share). Naturally, one has to consider that the output effect at  $t = 1$  is  $1 + g$  times larger than at  $t = 0$  and that it must be discounted with the rate  $r$ . Part iii) states that a reduction in employment at time  $t = 0$  increases the welfare of some generation  $l$  and all subsequent generations if it generates a rise in per capita income growth (see Proposition 3). This is because the wage income effect cumulates over time and eventually dominates the capital income effect so that generation  $l$  and all subsequent generations benefit from the permanent reduction in employment at time 0.

The link between welfare, union objectives, and growth is now easily established.

**Corollary 1.** Let

$$\sigma(1) + \varepsilon(1) + \frac{1+g}{1+r} [1 - \varepsilon(1)] < 1. \quad (16)$$

Then, there is some  $\tilde{\gamma} \in (0, \bar{\gamma})$  so that  $\hat{g} > g^*$  for all  $\gamma \in (0, \tilde{\gamma})$  and the welfare of all generations which adhere to the union is improved.

*Proof:* By part ii) of Proposition 4, condition (16) implies that  $dL < 0$  increases the welfare of each generation  $t \geq 0$  which adheres to the union. Since, condition (16) implies  $\sigma(1) + \varepsilon(1) < 1$ , the claim follows with Proposition 3. *Q.E.D.*

Corollary 1 gives the local condition under which a marginal rise in wages above the competitive level and the associated decrease in employment augments the welfare of all generations but generation  $-1$ . It emphasizes that there are union preferences of the type (8) so that a union actually chooses a level of employment which increases both per capita income growth and welfare of all generations but the current old.

## 6. Discussion

*Union Preferences.* The preceding analysis can be used to assess the implications for growth of alternative union's objective functions discussed in the

literature. Consider first the total wage bill maximization approach advocated by Dunlop (1944). Given the simple growth model with externalities from capital formation, the wage bill maximization approach implies that the union chooses a level of employment that leads to maximum growth. Consider next the case of a rents-from-unionization objective emphasized by Rosen (1969), de Menil (1971), and Calvo (1978), among others. The union objective function that we have employed in this paper converges to this case if the weight the union puts on wages,  $\gamma$ , approaches  $1/2$ . The union then chooses a level of employment which definitely exceeds the growth maximizing one. It may be the case that the growth rate of the unionized economy falls short of the competitive level, even though the local condition for unionization to stimulate economic growth stated in Proposition 3 holds true. Finally, consider a utility oriented approach, suggested, e.g., by Farber (1978) and Oswald (1982), in which the union's objective coincides with the individual objectives of its members. From equation (15) it can be inferred that in the present OLG framework such a union chooses a level of employment  $\hat{L}$  which satisfies:

$$\sigma(\hat{L}) + \varepsilon(\hat{L}) + \frac{1 + g(\hat{L})}{1 + r(\hat{L})} [1 - \varepsilon(\hat{L})] \geq 1, \quad \text{with } = \text{ if } \hat{L} < 1,$$

i.e., which maximizes lifetime utility of the union's members at each point in time. In fact, if  $\hat{L} < 1$ , the union chooses a level of employment which leads to higher growth than obtained in a competitive economy but not to maximum growth. This is because a union which maximizes lifetime utility of its members also takes into account the negative effect of unemployment on capital income born by its members when old.

*Length of Lifetimes.* The conditions for a labor union fostering per capita income growth derived in this paper are closely linked to the assumption of a two-period overlapping generations structure. In this economy only young individuals save so that aggregate savings exclusively stem from labor income. If individual lifetimes extend to more than two periods, the conditions for a monopoly union to spur economic can be expected to be more restrictive. This is because in such a framework savings generally do not only stem from labor but also from capital income. In this case the negative capital income effect of unionization discussed in Section 5 will have a dampening effect on aggregate savings and, henceforth, on growth. In the extreme case of infinite lifetimes our results would be reversed. As has been shown by Bertola (1993) in such a framework savings stem solely from

capital income implying that redistributing from capitalists to workers necessarily reduces economic growth.

*Savings Function.* By confining attention to logarithmic utility we excluded that savings depends on the interest rate. If we considered a savings function with the interest as one of its arguments, the conditions for unionization to stimulate economic growth would either be more or less restrictive, depending on whether savings would be positively or negatively related to the interest rate. If savings are increasing in the interest rate, the negative effect of unionization on the interest rate lowers aggregate savings and exerts a depressing effect on per capita income growth. If, on the other hand, savings are negatively related to the interest rate, the same reasoning points to a further positive effect of unionization on growth.

## Appendix

### Proof of Proposition 1

It is easily verified that  $\tilde{V}(L_t) > 0$  for some  $L_t \in (0, 1)$ . Thus, as  $\tilde{V}(1) = 0$  it follows that  $\hat{L}_t < 1$ . Next, consider  $V(L_t \rightarrow 0)$ . If  $\tilde{V}(L_t \rightarrow 0) = 0$ , the preceding argument reveals that  $\hat{L}_t > 0$ . Thus, assume that  $\tilde{V}(L_t \rightarrow 0) > 0$ . This implies that  $\omega(L_t \rightarrow 0) = \infty$ . We demonstrate that this leads to  $\lim_{L_t \rightarrow 0} d\tilde{V}/dL_t > 0$  for  $\gamma < \bar{\gamma}$ . Differentiation of  $\tilde{V}$  with respect to  $L_t$  yields:

$$[\omega(L_t) - \omega(1)]^{\gamma-1} L_t^{-\gamma} \left[ \gamma \frac{d\omega(L_t)}{dL_t} L_t + (1 - \gamma)[\omega(L_t) - \omega(1)] \right],$$

which can be written in the form

$$\frac{\tilde{V}(L_t)}{[\omega(L_t) - \omega(1)] L_t} \Psi,$$

where

$$\Psi \equiv \gamma \frac{d\omega(L_t)}{dL_t} L_t + (1 - \gamma)[\omega(L_t) - \omega(1)].$$

Since  $\omega(L_t) > 0$  for all  $L_t$ , it follows that  $\Psi > 0$  if:

$$\gamma \rho(L_t) + (1 - \gamma) \left( 1 - \frac{\omega(1)}{\omega(L_t)} \right) > 0.$$

From the latter and  $\omega(L_t \rightarrow 0) = \infty$  it follows that  $\Psi > 0$  if  $\gamma < \bar{\gamma}$  as given in (9). To demonstrate that  $\tilde{V}(L_t)/[(\omega(L_t) - \omega(1))L_t] > 0$  for  $L_t \rightarrow 0$ , it is sufficient to show that  $\lim_{L_t \rightarrow 0} [(\omega(L_t) - \omega(1))L_t] = 0$  since by assumption  $\tilde{V}(L_t \rightarrow 0) > 0$ . This, in turn, holds true since:

$$\begin{aligned} \lim_{L_t \rightarrow 0} [\omega(L_t) - \omega(1)] L_t &= \lim_{L_t \rightarrow 0} \omega(L_t) L_t - 0 \\ &= \lim_{L_t \rightarrow 0} \left[ f\left(\frac{1}{L_t}\right) - \frac{1}{L_t} f'\left(\frac{1}{L_t}\right) \right] L_t \\ &= \lim_{k_t \rightarrow \infty} \frac{f(k_t) - k_t f'(k_t)}{k_t} \\ &= \lim_{k_t \rightarrow \infty} \frac{f(k_t)}{k_t} - \lim_{k_t \rightarrow \infty} f'(k_t) = 0. \end{aligned}$$

Thus,  $\lim_{L_t \rightarrow 0} d\tilde{V}/dL_t > 0$ . *Q.E.D.*

## References

- Agell, J. and Lommerud K.E. (1993): "Egalitarianism and Growth"; *Scandinavian Journal of Economics*, 95, 559-579.
- Arrow, K.J. (1962): "The Economic Implications of Learning by Doing"; *Review of Economic Studies*, 29, 155-173.
- Bertola, G. (1993): "Factor Shares and Savings in Endogenous Growth"; *American Economic Review*, 83, 1184-1198.
- Bertola, G. (1996): "Factor Shares in OLG Models of Growth"; *European Economic Review*, 40, 1541-1560.
- Booth, A.L. (1995): *The Economics of the Trade Union*; Cambridge (Eng.): Cambridge University Press.
- Cahuc, P. and Michel, P. (1996): "Minimum Wages, Unemployment and Growth"; *European Economic Review*, 40, 1463-1482.
- Calvo, G.A. (1978): "Urban Unemployment and Wage Determination in LDC's: Trade Unions in the Harris Torado Model"; *International Economic Review*, 19, 65-81.
- de Menil, G. (1971): *Bargaining: Monopoly Power versus Union Power*; Cambridge (Mass.): MIT Press.
- Diamond, P. (1965): "National Debt in a Neoclassical Growth Model"; *American Economic Review*, 55, 1126-1150.
- Dunlop, J.T. (1944): *Wage Determination und Trade Unions*; London: Macmillan.
- Farber, H.S. (1978): "Individual Preferences and Union Wage Determination: The Case of the United Mine Workers"; *Journal of Political Economy*, 86, 923-942.
- Hellwig, M.F. and Irmen, A. (1999): "Wage Growth, Productivity Growth, and the Evolution of Employment"; SFB 504 Working Paper No. 99-86.
- Jones, L.E. and Manuelli, R.E. (1992): "Finite Lifetimes and Growth"; *Journal of Economic Theory*, 58, 171-197.
- Oswald, A. (1982): "The Microeconomic Theory of the Trade Union"; *Economic Journal*, 92, 576-596.

- Pencavel, J. (1984): "The tradeoff between Wages and Employment in Trade Union Objectives"; *Quarterly Journal of Economics*, 99, 215-231.
- Romer, P.M. (1986): "Increasing Returns and Long-Run Growth"; *Journal of Political Economy*, 94, 1002-1037.
- Rosen, S. (1969): "Trade Union Power, Threat Effects and the Extent of Organization"; *Review of Economic Studies*, 36, 185-196.
- Saint-Paul, G. (1992): "Fiscal Policy in an Endogenous Growth Model"; *Quarterly Journal of Economics*, 106, 1243-1259.
- Samuelson, P.A. (1958): "An Exact Consumption-Loan Model of Interest with and without the Social Contrivance of Money"; *Journal of Political Economy*, 66, 467-482.
- Sheshinski, E. (1967): "Optimal Accumulation with Learning by Doing"; in: Shell, K. (ed.): *Essays on the Theory of Optimal Economic Growth*, Cambridge (Mass.): MIT Press.
- Uhlig, H. and Yanagawa, N. (1996): "Increasing the Capital Income Tax may lead to Faster Growth"; *European Economic Review*, 40, 1521-1540.
- Wiedmer, T. (1996): "Growth and Social Security"; *Journal of Institutional and Theoretical Economics*, 152, 531-539.
- Wigger, B.U. (1999): "Public Pensions and Growth"; *Finanzarchiv*, 56, 241-263.