Optimal Monetary Policy under Inflation Targeting Based on an Instrument Rule

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Abstract:
This paper appends an instrument rule to a simple stochastic macroeconomic model and examines the optimal setting of the policy parameter under inflation targeting. It is shown that the size of the policy parameter depends on the sources of uncertainty, the policymaker's preferences, and both parameters of the model.

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Introduction

Recent contributions to the literature on monetary policy have used a forward-looking framework.[1] This paper appends an instrument rule to a simple stochastic macroeconomic model and examines the optimal setting of the policy parameter under inflation targeting.

2. The Model

The model of the economy comprises three equations:

$$y_t = -\beta r_t + E_t y_{t+1} + v_t \quad (1a)$$

$$\pi_t = E_t \pi_{t+1} + a y_t + u_t \quad (1b)$$

$$r_t = \bar{r} + \lambda(\pi_t - \pi^T) \quad (1c)$$

$$a, \beta > 0 \qquad \begin{array}{l} v_t \approx N(0, \sigma_v^2) \\ u_t \approx N(0, \sigma_u^2) \end{array}$$

Equation (1a) represents the forward-looking IS relation, according to which the current output gap moves in concert with the current expectation of the output gap next period and responds negatively to an increase in the current real rate of interest. v_t is a stochastic disturbance to the IS relation. Equation (1b) is a forward-looking Phillips Curve. The current rate of inflation depends positively on the current expectation of the rate of inflation between period *t* and period *t+1* and the current output gap. u_t is a stochastic cost-push disturbance. Equation (1c) is the instrument rule that the policymaker follows in the conduct of monetary policy. The setting of the instrument responds to deviations of the variable that monetary policy targets.[2] More specifically, the real rate of interest is equal to a constant (the *natural* rate) and

[1] See, for instance, Clarida, Gali, and Gertler (1999).
[2] As specified, the instrument rule implies that the policymaker has complete control over the setting of the real rate of interest. This assumption simplifies the analysis without compromising the results.

responds to deviations of the rate of inflation from its fixed target level. The size of the policy parameter λ indicates the speed with which the policymaker changes the tune of monetary policy in his attempt to make the observed rate of inflation equal to the target rate.

The objective function that the policymaker faces is composed of two terms, the variance of the output gap and the variance of the rate of inflation. μ gives an indication of the extent to which the policymaker cares about the variability of the rate of inflation relative to the variability of real output deviations from potential.

$$L = V(y_t) + \mu V(\pi_t) \tag{2}$$

3. Solving the Model

To obtain the variance of the output gap and the rate of inflation, respectively, we have to solve the model for y_t and π_t. We begin by substituting the Phillips Curve, equation (1b), into the policy rule, equation (1c) and proceed by substituting the resulting equation into the IS relation, equation (1a). This yields:

$$y_t(1+\beta\lambda a) = E_t y_{t+1} - \beta(\bar{r} + \lambda(E_t\pi_{t+1} + u_t - \pi^T)) + v_t \tag{3}$$

Next we pose putative solutions for the two endogenous variables:

$$y_t = \phi_{10} + \phi_{11}v_t + \phi_{12}u_t \tag{4a}$$

$$\pi_t = \phi_{20} + \phi_{21}v_t + \phi_{22}u_t \tag{4b}$$

Updating equations (4a) and (4b) by one period and taking conditional expectations yields:

$$E_t y_{t+1} = \phi_{10} \quad (5a)$$

$$E_t \pi_{t+1} = \phi_{20} \quad (5b)$$

Substituting equations (4a), (5a), and (5b) into equation (3) and matching coefficients results in the following expressions for the undetermined coefficients ϕ_{10}, φ_{11}, and ϕ_{12}:

$$\phi_{10} = 0 \quad (6a)$$

$$\phi_{11} = \frac{1}{1+\beta\lambda a} \quad (6b)$$

$$\phi_{12} = \frac{-\beta\lambda}{1+\beta\lambda a} \quad (6c)$$

In a similar vein, combine equations (3), (4b), and (5 b) to produce expressions for ϕ_{20}, φ_{21}, and ϕ_{22}:

$$\phi_{20} = \pi^T - \frac{\bar{r}}{\lambda} \quad (7a)$$

$$\phi_{21} = \frac{a}{1+\beta\lambda a} \quad (7b)$$

$$\phi_{22} = \frac{1}{1+\beta\lambda a} \quad (7c)$$

After substituting the results of equations (6) and (7) into equation (4), we are ready to compute the variances of the two endogenous variables. They are:

$$V(y_t) = \frac{1}{(1+\beta\lambda a)^2}(\sigma_v^2 + (\beta\lambda)^2 \sigma_u^2) \tag{8a}$$

$$V(\pi_t) = \frac{1}{(1+\beta\lambda a)^2}(a^2\sigma_v^2 + \sigma_u^2) \tag{8b}$$

The policymaker's objective can then be restated in the following way:

$$\underset{\lambda}{Min}\ L = \frac{1}{(1+\beta\lambda a)^2}\left[\sigma_v^2 + (\beta\lambda)^2\sigma_u^2 + \mu(a^2\sigma_v^2 + \sigma_u^2)\right] \tag{9}$$

The policymaker chooses the value for λ so as to minimize the loss function. The optimal value for the parameter in the instrument rule is:

$$\lambda^* = \frac{a}{\beta}[\mu + (1 + \mu a^2)\frac{\sigma_v^2}{\sigma_u^2}] \tag{10}$$

Thus, the optimal value for λ depends on the parameters of the model, the sources of uncertainty, and the preferences of the policymaker.[3] The size of λ varies positively with a but negatively with β. The setting of the policy parameter also depends on the ratio of the variances of the two shocks. The greater the variance of IS shocks relative to cost-push shocks, the more vigorous the response of the policy instrument to deviations of the rate of inflation from target. Finally, the greater the dislike of the policymaker for variability in the rate of inflation relative to output, the greater the response of the real rate of interest any deviations of the rate of inflation from its target level. In general, given finite values for the policymaker's preference parameter

[3] This is consistent with earlier contributions to the literature on monetary policy under uncertainty which originated with Poole (1970).

and the variances of the two disturbances, λ assumes a strictly positive and finite value.

Several special cases deserve closer scrutiny. Suppose the variance of cost-push shocks approaches zero, i.e. $\sigma_u^2 \to 0$. In this case λ will tend towards infinity as the policymaker makes whatever adjustment is necessary to the interest rate to offset any IS disturbance that would cause output to change. If instead the variance of IS shocks approaches zero, i.e. $\sigma_v^2 \to 0$, then $\lambda \to \frac{a}{\beta}\mu$.[4] In this particular case, the policy parameter assumes a finite value if μ is finite. Thus, there are two circumstances under which the policymaker would pursue a "strict" inflation target:

1. in the absence of any cost-push disturbances, and
2. if the relative weight on the variance of inflation in the loss function approaches infinity.[5]

4. Conclusion

This paper augments a simple stochastic macroeconomic model with an instrument rule. It is shown that the size of the policy parameter depends on the sources of uncertainty, the policymaker's preferences, and the parameters of the model.

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[4] With the proviso that $\sigma_u^2 > 0$.

[5] Clearly as long as $\mu > 0$, the policymaker will always pursue an active monetary policy, i.e. vary the setting of the policy instrument ($\lambda > 0$) to achieve the target for the rate of inflation. Indeed, from equation (7A) it is clear that a strictly positive value for λ is required for the coefficient ϕ_{20} to be determined. Notice that $\pi_t = \pi^T$ only if $\lambda \to \propto$. That is, the observed rate of inflation in the current period equals the target rate only if the policy parameter approaches infinity.

References:

Clarida, R., Gali, J., and M.Gertler, 1999, "The Science of Monetary Policy: A New Keynesian Perspective." Journal of Economic Literature, 27, 1661-1707.

Poole, W., 1970. "Optimal Choice of Monetary Policy Instrument in a Simple Stochastic Macro Model," Quarterly Journal of Economics, 84, no. 2, 197-216.