

Inflation Targeting in the Open Economy: Which Rate of Inflation to Target?

By

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Abstract:

This paper attempts to explain why the central banks of small open economies generally adopt an inflation target that is defined in terms of changes in the CPI instead of changes in the domestic price level. The explanation offered is a simple one. Strict CPI inflation targeting has better stabilizing features than domestic inflation targeting. A strategy of monetary policy that takes account of the degree of openness and changes in the real exchange rate leads to lower fluctuations in key economic variables such as the output gap, the real exchange rate, and the nominal rate of interest. A key aspect of the model is the role of the degree of openness in determining the size of the structural parameters in the IS relation.

The conclusions presented in this paper differ markedly from those in the existing literature. Apart from employing a different loss function, the present framework stresses the forward-looking behavior of economic agents at the expense of persistence in the rate of inflation and real output and possible lags in the transmission of monetary policy.

Key Words: CPI Inflation-Targeting, Openness, Real Exchange Rate Channel, Forward-Looking Behavior.

JEL Classification: E 52, F 41

Introduction

The case for inflation targeting is a strong one in theory and in practice. Various theoretical contributions to the literature on monetary policy evaluate the performance of inflation targeting in comparisons with other strategies of monetary policy.¹ One fairly general result that comes out of these discussions is that strict inflation targeting represents an extreme, yet optimal form of monetary policy. Over the past 10 years, a sizeable number of industrialized as well as emerging countries have adopted inflation targeting as the principal strategy guiding monetary policy. Table 1 lists a number of the industrialized countries that target inflation. Broadly speaking, these countries share two noteworthy features that concern the size of the country and the definition of the operational target of monetary policy. First, with the possible exception of the United Kingdom, all of these countries are small open economies. The degree of openness – as represented by the ratio of imports to total consumption – ranges from *0.25* to approximately *0.4*. Second, in all countries but the United Kingdom, the inflation target is expressed in terms of percentage changes in the Consumer Price Index (CPI). This index is based on the prices of final *domestic* and *foreign* consumption goods. An alternative choice for the announced inflation target would be the rate of change of prices of domestic goods. In this case, the Gross Domestic Product Deflator would be a candidate index.

This paper attempts to explain why the central banks of small open economies generally adopt an inflation target that is defined in terms of changes in the CPI instead of changes in the domestic price level. The explanation offered is a simple one. Strict CPI inflation targeting has better stabilizing features than domestic

¹ Examples are Ball (1999), Clarida, Gali, and Gertler (1999, 2001), Svensson (1997, 2000), and Guender (2001), to name but a few.

inflation targeting.² A strategy of monetary policy that takes account of the degree of openness and changes in the real exchange rate leads to lower fluctuations in key economic variables such as the output gap, the real exchange rate, and the nominal rate of interest. Underpinning this conclusion is a model of a small open economy that features three important characteristics: the degree of openness affects the structural parameters of the IS relation, the Phillips Curve has a direct real exchange rate channel, and economic agents are forward-looking. A particularly attractive feature of the present paper is that the dominance of CPI inflation over domestic inflation targeting is illustrated by means of analytically tractable results. The findings of this paper stand in marked contrast to the simulation-based results reported by Svensson (2000) who argues that strict CPI targeting leads to substantially greater fluctuations in the output gap, the real exchange rate, and the nominal rate of interest. A number of factors account for the differing conclusions. These factors range from the specification of the objective function that the policymaker minimizes to the specification of the structural relations of the model.

The model is laid out in Section II. The policymaker's preferences are discussed in Section III. This section also discusses the reaction function of the policymaker and shows how the model is solved. Section IV compares and contrasts CPI- and domestic inflation targeting by focusing on each shock in isolation. An overall assessment of the relative merits of both inflation targeting strategies is offered in Section V. Conclusions are drawn in Section VI.

2 Another simple reason for the adoption of CPI inflation targets is that the public is generally more familiar with the CPI compared to a measure of purely domestic inflation such as the GDP deflator. In addition, CPI targeting can rely on more timely information. Monthly observations of the CPI are available in most countries while the GDP deflator is generally available only on a quarterly basis. Whether monetary policy affords greater *direct* control over the GDP deflator (as it consists only of the prices of domestic goods) compared to the CPI is debatable as well. The extent to which monetary policy affects the domestic currency price of foreign goods depends on the degree of exchange rate pass-through and the lags in the transmission process of monetary policy.

II. The Model

This section lays out the building blocks of a forward-looking model for a small open economy. The model consists of three equations. All variables with the exception of the nominal interest rate are expressed in logarithms. All parameters are positive.

$$\pi_t = E_t \pi_{t+1} + a y_t + b q_t + u_t \quad (1)$$

$$y_t = E_t y_{t+1} - a_1 (R_t - E_t \pi_{t+1}^{CPI}) + a_2 (q_t - E_t q_{t+1}) + a_3 (y_t^* - E_t y_{t+1}^*) + v_t \quad (2)$$

$$a_1 = (1 - \alpha)\sigma$$

$$a_2 = \alpha[(1 - \alpha)\eta + \eta^* \alpha^*]$$

$$a_3 = \alpha\beta^*$$

$$R_t - E_t \pi_{t+1} = R_t^* - E_t \pi_{t+1}^* + E_t q_{t+1} - q_t + \varepsilon_t \quad (3)$$

where:

y_t = the real output gap.

π_t = domestic rate of inflation at time t measured as $p_t - p_{t-1}$.

$E_t \pi_{t+1}$ = the expectation of π_{t+1} formed at time t.

$E_t \pi_{t+1}^{CPI}$ = the expectation formed at time t of CPI inflation in period t+1.

$E_t \pi_{t+1}^*$ = the expectation formed at time t of the foreign rate of inflation for period t+1

R_t = the domestic nominal interest rate at time t.

R_t^* = the foreign nominal interest rate at time t.

q_t = the real exchange rate defined as $e_t + p_t^f - p_t$ where e_t is the nominal exchange

rate (domestic currency per unit of foreign currency), p_t^f is the foreign price level,

and p_t is the domestic price level.

$E_t q_{t+1}$ = the expectation dated t of the real exchange rate for period t+1.

y_t^* = the foreign output gap in period t.

$E_t y_{t+1}^*$ = the expectation in period t of the foreign output gap in period t+1.

u_t, v_t , and ε_t are stochastic, white noise disturbances.

α = the weight on the domestic currency price of the foreign consumption good in the CPI. α is interpreted as the measure of the degree of openness of the domestic economy. α^* measures the degree of openness of the foreign country.

σ = intertemporal elasticity of substitution of consumption.

η = elasticity of substitution between domestic and foreign consumption in the home country. η^* is the foreign counterpart.

β^* = elasticity of foreign consumption with respect to foreign income.

The first two relations incorporate the forward-looking behavior typical of the New Keynesian framework. Equation (1) represents the forward-looking Phillips curve relation for the open economy. In this economy, real output is produced by monopolistically competitive firms.³ These firms set the price of output in order to minimize a cost function that takes into account the existence of menu costs and the cost of charging a price different from the optimal price. The existence of price stickiness introduces the difference between the current rate of inflation and the current expectation of future inflation into the Phillips curve. There is also a direct real exchange rate effect on domestic inflation. The rationale behind this is that a depreciation causes the domestic currency price of the foreign good, $(p_t^f + e_t)$, to increase. As explained in the appendix, the increase in the exchange rate forces up the optimal price, which, ceteris paribus, induces firms to raise the price of their output so as to minimize the deviation between the optimal price and the actual price charged.

³ The reader is referred to Appendix A where a detailed derivation of the New Keynesian Phillips Curve is presented.

At the aggregate level, the increase in the domestic price level causes the rate of inflation to rise. Thus, we observe the positive link between the exchange rate and the rate of inflation.

Equation (2) defines an open economy IS relation. The domestic monetary transmission mechanism operates through the standard expected real interest rate channel. In line with conventional open-economy macroeconomic models, the present model features a real exchange rate channel and accounts for the effect of foreign real output on domestic real output. Notice, however, that the open-economy transmission mechanism is captured by expected (negative) changes in both the real exchange rate and foreign real output. The expectations of next period's domestic real output gap, the real exchange rate, and the foreign real output gap enter the IS relation because agents are forward-looking. Another important characteristic of the IS relation pertains to the structural coefficients a_1 , a_2 , and a_3 . They vary in size in accordance with the degree of openness of the economy.⁴ The sensitivity of these coefficients to the degree of openness will be explored further in Section IV of the paper.

Equation (3) is the uncovered interest rate parity condition (UIP), expressed in real terms. This equation embodies the assumption of perfect capital mobility, reflecting the high level of integration of a small open economy's financial sector with the rest of the world. The disturbance term, ε_t , can be interpreted as a time-varying risk premium.

III.A. The Preferences of the Policymaker

The policymaker's preferences extend over the variability of the domestic real output gap and the rate of inflation, the latter measured in terms of the change in the

⁴ The derivation of this forward-looking IS curve is explained in the appendix (Part B). There is no LM relation as the policymaker uses the nominal interest rate as the policy instrument.

consumer price index between period t and $t-1$.⁵ The explicit objective function that he attempts to minimize is given by:⁶

$$L = [V(y_t) + \mu V(\pi_t^{CPI})] \quad (4)$$

where the CPI rate of inflation π_t^{CPI} is defined as:

$$\pi_t^{CPI} = (1 - \alpha)\pi_t + \alpha(\pi_t^f + \Delta e_t) \quad \text{or} \quad (5)$$

$$\pi_t^{CPI} = \pi_t + \alpha\Delta q_t. \quad (6)$$

Δe_t represents the change in the nominal exchange rate, and Δq_t represents the change in the real exchange rate. While the level of the real exchange rate does not enter in its own right as a variable in the loss function, the change in the real exchange rate affects the CPI inflation rate. It follows that if the policymaker is concerned about CPI inflation, he must pay heed to

- i. domestic inflation
- ii. movements in the real exchange rate as they have a direct impact on CPI inflation. In fact, the greater the degree of openness α , the greater the effect of changes in the real exchange rate on CPI inflation.

III.B. The Reaction Function of the Policymaker.

The policymaker sets policy on the basis of the two variables that appear in his loss function: the output gap y_t and the rate of CPI inflation π_t^{CPI} :

⁵ This specification of the loss function implies that the policymaker is not worried about the volatility of the policy instrument. The choice of this particular specification is in part motivated by its analytical tractability and in part by formal analysis. On the latter point, see Woodford (1999). He provides a rationale for including only the rate of inflation and the real output gap in the loss function on choice-theoretic grounds.

The variability in the setting of the policy instrument appears in the loss functions in the contributions by Rudebusch and Svensson (1999), Svensson (2000), Dennis (2000), and others.

⁶ The target rate of inflation is assumed to be equal to zero. The output target is potential output.

$$(\theta y_t + \pi_t^{CPI}) = 0 \quad (7)$$

θ is the weight the policymaker places in the policy rule on the output objective relative to the inflation objective. The policymaker can go about choosing the size of θ in various ways. He can either follow a rigid policy rule in which case he would arbitrarily select a numerical value for θ . For instance, a choice of $\theta=1$ would indicate that he places a unitary tradeoff on deviations of real output from its potential level and deviations of the rate of inflation from its (zero) target level. Alternatively, his choice for the size of θ could be guided by optimal monetary policy considerations. Examples of policy rules that are consistent with the optimality criterion are strict inflation targeting and strict real output targeting.⁷

Using equation (6), we can restate the above relation as:

$$(\theta y_t + \pi_t + \alpha \Delta q_t) = 0 \quad (8)$$

In order to see how the policymaker responds to changes in economic conditions, we first solve equation (3) for q_t and substitute the resulting expression into equations (1) and (2). These two expressions are substituted in turn into equation (8). Solving this equation for R_t yields the policymaker's reaction function:

$$R_t = \Delta \left[\frac{(\theta + a)(E_t y_{t+1} + a_3(y_t^* - E_t y_{t+1}^*) + v_t) + ((\theta + a)(a_1 + a_2) + b + \alpha + 1)E_t \pi_{t+1} + ((\theta + a)a_2 + b + \alpha)\Omega + (b + \alpha)E_t q_{t+1} - \alpha q_{t-1} + u_t}{(\theta + a)(a_1 + a_2) + b + \alpha} \right] \quad (9)$$

$$\text{where } \Delta = \frac{1}{(\theta + a)(a_1 + a_2) + b + \alpha} \quad \Omega = (R_t^* - E_t \pi_{t+1}^* + \varepsilon_t)$$

There are several noteworthy features about this reaction function. Apart from the stochastic domestic disturbances and the lagged real exchange rate, the

7 Ball (1999) calls policies that are consistent with optimal policy "efficient". For more details on how optimal monetary policy is chosen in a simple, forward-looking open-economy framework, see Guender (2001). He shows that in case of a strict inflation target, $\theta \Rightarrow 0$ as $\mu \Rightarrow \infty$. Alternatively, in case of a strict output target, $\theta \Rightarrow \infty$ as $\mu \Rightarrow 0$.

policymaker varies the current setting of the nominal interest rate in line with his expectations for the domestic and foreign variables in the following period. It is apparent that the response of the nominal interest rate to changes in the domestic economy and abroad depends on the size of θ , the weight on the output gap in the policy rule. Notice further that the coefficient on expected domestic inflation always exceeds one. In contrast, the policymaker's response to changes in foreign inflation and the risk premium is much more muted as the coefficient on Ω is strictly less than one.

The above reaction function also sheds some light on the role of the real exchange rate and the degree of openness in formulating monetary policy. In the forward-looking framework explicit consideration must be given to the real exchange rate. Both the past and the expected future level of the real exchange appear in the reaction function.⁸ The lagged level of the real exchange rate is included because the policymaker cares about CPI inflation and not domestic inflation alone. The degree of openness is also important in determining the response of the instrument to changes in the economic environment. The coefficients on all variables and shocks depend on the size of α .

III.C. Solving the Model

To solve the model, substitute equation (9) into equation (2), the IS relation.

The resulting equation takes the following form:

$$y_t = \Delta \left[\begin{array}{l} (b + \alpha)(E_t y_{t+1} + a_3(y_t^* - E_t y_{t+1}^*) + v_t - a_1 \Omega) + \\ (a_1 + a_2)(-E_t \pi_{t+1} + \alpha q_{t-1} - (b + \alpha)E_t q_{t+1} - u_t) \end{array} \right] \quad (10)$$

⁸ Taylor (2001) argues against the inclusion of the exchange rate in the policy reaction function on the grounds that its inclusion would lead to greater instability in the variables that the policymaker cares about. Taylor arrives at this conclusion after considering the role of the exchange rate in a *backward-looking* framework where the policymaker cares about *domestic* and not CPI inflation.

The current real output gap depends on the difference between current and expected foreign real output in period $t+1$, the foreign real interest rate and the risk premium (Ω), the IS and cost-push disturbances, the *lagged* real exchange rate, and the endogenously determined expectations of the output gap, domestic inflation, and the real exchange rate in period $t+1$. These expectations are formed rationally. The putative solution for each of the three variables depends on the predetermined variable q_{t-1} and all stochastic disturbances of the model:⁹

$$y_t = \gamma_{10}v_t + \gamma_{11}u_t + \gamma_{12}\varepsilon_t + \gamma_{13}R_t^* + \gamma_{14}\pi_t^* + \gamma_{15}y_t^* + \gamma_{16}q_{t-1} \quad (11)$$

$$\pi_t = \gamma_{20}v_t + \gamma_{21}u_t + \gamma_{22}\varepsilon_t + \gamma_{23}R_t^* + \gamma_{24}\pi_t^* + \gamma_{25}y_t^* + \gamma_{26}q_{t-1}$$

$$q_t = \gamma_{30}v_t + \gamma_{31}u_t + \gamma_{32}\varepsilon_t + \gamma_{33}R_t^* + \gamma_{34}\pi_t^* + \gamma_{35}y_t^* + \gamma_{36}q_{t-1}$$

Updating and forming expectations of each equation above yields:

$$E_t y_{t+1} = \gamma_{16}q_t \quad (12)$$

$$E_t \pi_{t+1} = \gamma_{26}q_t$$

$$E_t q_{t+1} = \gamma_{36}q_t$$

Equation (12) underscores an important result. When forming expectations of real output, domestic inflation, and the real exchange rate in period t , economic agents take account of the *current* level of the real exchange rate. It bears repeating that the *current* real exchange rate matters in the formation of expectations about the *future* because the *lagged level* of the real exchange rate matters in the determination of the *current* rate of CPI inflation.

The important role of the lagged real exchange rate is further brought out in Table 2. This table provides summary information about the behavior of the real

⁹ The three foreign variables R_t^* , π_t^* , y_t^* follow stochastic processes with mean zero and constant variance.

output gap, the domestic rate of inflation, and the real exchange rate in response to the various shocks that impinge upon the economy and shows how the three variables are related to the lagged real exchange rate. Apart from the parameters of the model, the size of the coefficients on the shocks and the lagged exchange rate clearly depends on θ , the weight on the output gap in the policy rule, and γ_{26} , the coefficient on q_{t-1} in the equation for π_{t-1} .¹⁰

IV. Strict Inflation Targeting

In this section we examine two strategies of monetary policy that focus solely on the rate of inflation. At first, we discuss the implications of strict CPI inflation targeting. This is then followed by the examination of a more restrictive form of strict inflation targeting, domestic inflation targeting. The remainder of this section presents a comparison of the stabilizing properties of the two strategies. The degree of openness plays a key role in this comparison.

IV.A. Strict CPI Inflation Targeting

If the policymaker pursues a strict CPI inflation target then the weight on the output gap in the policy rule (equation 8) reduces to zero. With θ set equal to zero, monetary policy focuses exclusively on meeting the CPI inflation target and pays no heed to the real output gap.¹¹ Imposing this condition on the policy rule helps simplify the coefficients that appear in Table 2. Nevertheless, as Table 3 underscores, the fact that the past (i.e. the lagged value of the real exchange rate) is instrumental in

10 The presence of the lagged real exchange rate in the solutions makes the calculation of the variances of the real output gap and the CPI inflation rate somewhat cumbersome. Part C of the appendix provides a short description of the procedure that was adopted to compute the variances of the relevant variables.

11 This strategy constitutes an efficient form of policy because the choice of policy occurs along the output-inflation trade-off frontier. At the same time, the strategy is extreme as only the variance of the real output gap appears in the loss function under either strategy of inflation targeting.

determining the current value of the endogenous variables under CPI inflation targeting. Indeed, the presence of the lagged exchange rate makes the analysis of CPI inflation targeting a rather complicated affair relative to domestic inflation targeting.

The first column of Table 3 shows the coefficient of the variance of real output in the face of demand shocks and cost-push shocks under strict CPI inflation targeting. The complex interaction of the parameters of the model makes it difficult to interpret the coefficient on the variances of the demand-side shocks. It is apparent though that the existence of a direct exchange rate channel in the Phillips Curve ($b > 0$) and the degree of openness of the economy (α) are very important in determining the size of the coefficient on the variances of the demand shocks. A clear result emerges in the case of the coefficient on the variance of the cost-push shock. Inspection of this coefficient reveals that it bears an inverse relationship to both b and α .

IV.B. Strict Domestic Inflation Targeting

A regime of strict domestic inflation targeting requires the policymaker to impose two conditions on the policy rule. Just like under strict CPI targeting, the policymaker sets the weight on the output gap in the policy rule equal to zero under strict domestic inflation targeting. In addition, the policymaker ignores the “pass-through” effect of the real exchange rate on CPI inflation. The second condition thus requires him to set α equal to zero in the policy rule. The policy rule under strict domestic inflation targeting thus reduces to:

$$\pi_t = 0 \tag{13}$$

The second column of Table 3 contains the coefficients on the variances of the demand shocks and the variance of the cost-push shock that appear in the policymaker’s loss function under strict domestic inflation targeting. Not

surprisingly, it is far easier to explain the mechanics of strict domestic inflation targeting compared to strict CPI inflation targeting. The performance of strict inflation targeting in the wake of demand-side disturbances depends critically on the size of the coefficient on the real exchange rate in the Phillips Curve. The greater the size of b , the worse the performance of strict domestic inflation targeting becomes as the losses of real output mount. However, in the absence of a direct exchange rate channel in the Phillips Curve ($b=0$), strict domestic inflation targeting completely offsets demand-side shocks to real output. Not having to worry about any resulting exchange rate effects on the domestic inflation rate, the policymaker need merely adjust the interest rate to keep real output at its target. As a consequence, there are no output losses and the loss function reduces to zero.

The direct exchange rate channel has a role to play, too, in the performance of strict domestic inflation targeting in the wake of cost-push disturbances. The variance of real output is a decreasing function of the size of b , a result similar to that obtained under strict CPI inflation targeting.

IV.C. The Role of Openness in Assessing the Performance of CPI Inflation Targeting vis-à-vis Strict Domestic Inflation Targeting

In this section we take a closer look at the stabilizing properties of CPI inflation targeting and compare them to those under a regime of strict domestic inflation targeting. The degree of openness of the economy plays a key role in this comparison. The focus of our attention lies on tracking the performance of the two strategies of inflation targeting as the degree of openness increases from a low value of zero – where by definition the two strategies are identical - to a high value of 0.9. Varying the degree of openness of the economy affects our open economy model in two ways. The first channel works through the direct pass-through effect. As α

increases in size, the effect of changes in the real exchange rate on the rate of CPI inflation increases. Clearly, varying α has repercussions for the performance of CPI targeting but not for domestic inflation targeting as $\alpha = 0$ under the latter strategy. The other channel, however, affects the stabilizing properties of both strategies. Close inspection of equation 2 reveals that the magnitude of the structural coefficients that appear in the IS relation depend on the size of α . Indeed,

- i. Increasing openness (i.e. increasing α) decreases the effect of the expected real rate of interest (defined in terms of the rate of CPI inflation) on real output.
- ii. Increasing openness increases the effect of foreign output shocks on domestic real output.
- iii. Increasing openness initially increases the effect of the real exchange rate on real output. For high values of α , however, this effect may be reversed.¹²

The analysis of the merits of CPI- versus domestic inflation targeting continues to be based on comparing the loss functions of the two competing strategies. However, the comparisons presented in this section take explicit account of the effect of a *varying* degree of openness on CPI-inflation and on the size of the structural parameters in the IS relation. As a start, we take values that have been reported in the literature for the intertemporal elasticity of substitution (σ), the intratemporal substitution (η, η^*), the degree of openness of the foreign economy (α^*), and the income elasticity of foreign real consumption (β^*) and use them in the

12 Exactly where the maximum size of the coefficient on the real exchange rate is attained depends on the size of the other parameters. For the values suggested by Svensson (2000), the maximum is reached at 0.4225 when $\alpha = 0.65$.

calculation of the parameters that appear in the IS relation.¹³ As pointed out above, the size of these parameters is sensitive to the degree of openness of the domestic economy. As the parameters figure in the determination of the coefficient on the variances of both demand-side and cost-push shocks in the loss function under both strict CPI and strict domestic inflation targeting, we can gauge the effect of varying the degree of openness on the relative attractiveness of the two strategies. The first comparison examines the coefficient on the variance of demand-side disturbances under strict CPI inflation targeting to the coefficient on the variances of demand-side disturbances under strict inflation targeting. The second comparison repeats this exercise for the coefficients of the variances of cost-push shocks under both strategies.

Figure 1 illustrates how varying the degree of openness affects the attractiveness of strict CPI targeting relative to strict domestic inflation targeting in the face of demand-side shocks. It is evident that domestic inflation targeting is unambiguously superior to CPI targeting for $\alpha > 0$. The coefficient on the variance of demand-side disturbances is always smaller under domestic inflation targeting than under CPI targeting. In fact, the degree of openness has no material effect on the size of the coefficient on the demand-side variances under domestic inflation targeting. In stark contrast, there is a positive monotonic relationship between the degree of openness and the size of the coefficient under CPI targeting. As the degree of openness increases, the performance of strict CPI targeting worsens in absolute terms. Notice also, that over a range of fairly low but increasing values of α the performance of CPI inflation targeting *relative* to domestic inflation becomes poorer.

Inspection of Figure 2 reveals that CPI targeting is clearly superior to domestic inflation targeting in the case of cost-push (inflation) shocks. For the variance of the

13 The values that underlie the calculation of the parameters are taken from Svensson (2000). They are: $\sigma=0.5$, $\eta=1$, $\eta^*=2$, $\alpha^*=0.15$, $\beta^*=0.9$.

cost-push shock in the loss function, we find again a monotonic relationship between the size of its coefficient under CPI targeting and the degree of openness. As Figure 2 illustrates, however, an increase in α is associated with a decrease in the size of the coefficient on the variance of cost-push shocks. While the size of this coefficient decreases monotonically – especially for low values of α - under CPI targeting, there is no evidence of such behavior under domestic inflation targeting. Indeed, the size of the coefficient on the variance of cost-push shocks increases initially, reaches a maximum and then declines. For $0.2 < \alpha < 0.5$ the difference in the size of the coefficients under the two targeting strategies is considerable. Over this range, CPI targeting is far superior to domestic inflation targeting.¹⁴

Figures 3 and 4 consider the case where the direct exchange rate channel in the Phillips Curve is suppressed. According to Figure 3, with $b = 0$, domestic inflation targeting perfectly stabilizes real output in the wake of demand-side disturbances. As a consequence, the attractiveness of domestic inflation targeting relative to CPI inflation targeting increases considerably. Just the opposite is the case for the cost-push disturbance, however. With $b = 0$, the coefficient on the variance of cost-push disturbance is determined exclusively by a , the coefficient on the output gap in the Phillips Curve. Since a is independent of the degree of openness it follows that the coefficient on the variance of the cost-push shock is a horizontal line. In this special case, the performance of domestic inflation targeting worsens dramatically. As shown

14 Figure A1 in the appendix provides a rough indication of the relative performance of CPI targeting vis-à-vis domestic inflation targeting. The ratio of the coefficient on the variance of demand-side disturbances under CPI inflation targeting to the coefficient on the variance of demand-side disturbances under domestic inflation targeting is recorded along the vertical axis. Values of α are recorded along the horizontal axis; same for the coefficients on the variance of cost-push disturbances under the two inflation targeting schemes. A value of less than one indicates that CPI targeting is superior to domestic inflation targeting while a value greater than one indicates that domestic inflation targeting is superior to CPI targeting. This comparison reveals that the relative attractiveness of domestic inflation targeting is most pronounced for $\alpha \approx 0.6$ in the wake of demand-side shocks while the dominance of CPI targeting over domestic inflation targeting increases in line with increases in the degree of openness for supply shocks.

in Figure 4, the coefficient on the variance of supply shock shoots up to a value of 4 compared to a maximum value of 0.5 when the exchange rate channel is operative.

Figure 4 brings out the fact that the appeal of CPI targeting increases markedly relative to domestic inflation targeting if $b = 0$. Nevertheless, just like under domestic inflation targeting, the absence of a direct exchange rate channel in the Phillips Curve causes the coefficient on the variance of cost-push shocks to increase markedly under CPI targeting. For small values of α in particular, the size of this coefficient is much greater than when the direct exchange rate channel exists.

IV.D. The Variability of the Real Exchange Rate and the Nominal Interest Rate

Neither the real exchange rate nor the (level or first difference of the) nominal rate of interest appears in the loss function that the policymaker faces (equation (4)). While changes in the real exchange rate enter indirectly the loss function under CPI targeting on account of the pass-through effect on the rate of CPI inflation, changes in the nominal rate of interest – the policy instrument – are assumed to be costless.¹⁵

Despite the limited role for the real exchange rate in and the total absence of the nominal interest rate from the loss function, it is still worthwhile to explore the behavior of the two variables under the alternative inflation-targeting strategies for two reasons. First, for the sake of completeness, it is necessary to examine the behavior of the nominal interest rate and the real exchange rate in addition to the real output gap and both measures of the rate of inflation. The other objective is to compare the variability of the three endogenous variables and the policy instrument to those reported by Svensson (2000).

¹⁵ The real exchange rate does not enter the loss function at all under domestic inflation targeting. In the literature, it is sometimes argued that the inclusion of changes in the nominal rate of interest in the loss function is warranted for “political reasons”.

Table 4 shows the coefficients on the variances of demand-side shocks and the variance of the cost-push shock that appear the equation for the variance of the real exchange rate. There are two striking results. For both strategies of inflation targeting, the size of one parameter, a , determines whether the variance of real exchange rates will be more responsive to the variance of demand-side disturbances compared to the variance of the cost-push shock. If $a < 1$ ($a > 1$), then the variance of demand-side disturbances impacts less (*more*) on the variance of the real exchange rate than the variance of cost-push shocks.¹⁶ In addition, the ratio of the coefficients under CPI and domestic inflation targeting is identical for both types of shocks.

Figure 5 plots the size of the coefficient on the variance of demand-side disturbances in the equation for the variance of the real exchange rate against the degree of openness of the economy.¹⁷ Irrespective of the size of α , in the open economy CPI inflation targeting leads to lower fluctuations in the real exchange rate compared to domestic inflation targeting. Moreover, the performance of CPI targeting *relative* to domestic inflation targeting improves in line with greater openness. Again we notice the existence of a monotonic relationship under CPI targeting but not under domestic inflation targeting and the pronounced decline in the size of the coefficient for relatively small values of α .¹⁸ Finally, under both strategies there exists an inverse relationship between the size of b and the size of the coefficients on the variances of the demand-side and cost-push shocks.

16 Strictly speaking, this assertion is correct only for the variance of the IS disturbance as the variances of the remaining demand-side disturbances are multiplied by parameters that appear in the IS relation. However, we continue with the nomenclature adopted earlier whereby all shocks that arise on the demand-side have been lumped together and are referred to as demand-side disturbances. This is justifiable because the main attention of the paper focuses on a comparison of the merits of CPI and domestic inflation targeting.

17 As the ratios of the coefficients on the variances of both demand-side and cost-push shocks are identical under the two strategies, there is no need to consider the performance of CPI targeting vis-à-vis domestic inflation targeting for cost-push shocks.

18 For high values of α , both a_1 and a_2 decrease in size. This causes the size of the coefficients on the variance of shocks under domestic inflation targeting to increase as the magnitude of the denominator shrinks. Hence the curve bends upward as α increases under domestic inflation targeting.

The expression for the variance of the nominal rate of interest under CPI- and domestic inflation targeting appears in Table 5. In view of the earlier results, it is not surprising to see that the coefficients on the variances of the shocks are more complex under CPI- than under domestic inflation targeting. Moreover, it is apparent that the simple definition of what we have called up to now demand-side disturbances is no longer tenable. The coefficient on the variance of the shocks that appear in the UIP condition is no longer proportional to the variance of the IS shock or the shock to foreign output. Hence we need to carry out a separate comparison for the two targeting strategies in the face of shocks to the UIP condition.¹⁹

A close look at the coefficients on the variances of the shocks under CPI targeting reveals that the response of the nominal interest rate to IS shocks is very similar to cost-push shocks. Indeed, just like in the case of the real exchange rate responding in identical fashion to both IS and cost-push disturbances when $a=1$, we find the same response pattern for the nominal rate of interest. In the case of foreign output shocks, the size of a_3a matters for the size of the coefficient on the variance. Notice that the same conclusions emerge from an examination of the coefficients of the variances of the shocks under domestic inflation targeting. Another result that obtains under domestic inflation targeting deserves being commented upon. The response pattern of the real exchange rate and the nominal interest rate is the same for the IS, cost-push, and the foreign output shock, respectively.

Figure 6 provides strong evidence for the argument that in an open economy CPI targeting leads to lower variability in the nominal interest rate in the wake of IS, cost-push, and foreign output shocks than domestic inflation targeting. The more open the economy is, the better the performance of CPI targeting relative to domestic

¹⁹ We shall continue, however, to use the term “demand-side” disturbances. The amended definition now includes only IS and foreign output shocks.

inflation becomes in dampening fluctuations in nominal interest rates. For medium-size values of α around 0.5 the relevant coefficients are close to zero. Again we find the usual pattern. While there exists a monotonic relationship between the degree of openness and the size of the coefficient on the variances of the disturbances under CPI targeting, no such relationship is evident under domestic inflation targeting. Instead we observe the bow-shaped relation typical for domestic inflation targeting.

A completely different pattern is revealed by the examination of the behavior of the nominal interest rate in the wake of risk premium or foreign interest rate shocks. Figure 7 illustrates clearly that in an open economy CPI targeting leads to greater variability of the nominal interest rate in the wake of UIP shocks than domestic inflation targeting. The difference in performance is greatest for $0.2 < \alpha < 0.4$.

V. An Attempt At Completing the Picture: An Assessment of the Performance of the Inflation Targeting Strategies

Up to now we have evaluated the performance of the two targeting strategies by looking at shocks in isolation. However, it is plausible to argue that the choice of a given monetary policy strategy should depend on its stabilizing properties in the face of *all* disturbances. To get a concrete idea about the *overall* performance of the two inflation-targeting strategies, we have to compare the loss functions, i.e. the variability of the output gap under both inflation-targeting regimes.²⁰ In addition, the implications for the behavior of the real exchange rate and the nominal rate of interest are worth considering. This is done for comparative purposes.

²⁰ To calculate the regime-specific loss functions, we used the variances of the shocks that underlie the calculations reported by Svensson (2000). They are: $\sigma_v^2 = \sigma_u^2 = \sigma_{R^*}^2 = \sigma_\varepsilon^2 = \sigma_{Y^*}^2 = 0.5$.

Figures 8 - 10 show how the two targeting strategies compare in terms of the standard deviations of the output gap, the real exchange rate, and the nominal interest rate as the degree of openness varies.

According to Figure 8, for low to medium-size values of α , CPI targeting yields leads to lower fluctuations in real output relative to domestic inflation targeting. However, the opposite result holds for values of α in the neighborhood of 0.5 and above. Figure 8 illustrates that the attractive stabilizing properties of CPI targeting wane quickly at higher values of α . This is because at higher values of α the effects of shocks to the exchange rate take are weighted more heavily in the loss function under CPI targeting but not under domestic inflation targeting where real exchange rate changes have no *direct* effects on the loss function.

Figure 9 compares the variability of the real exchange rate under CPI and domestic inflation targeting. The former strategy strictly dominates the latter for $\alpha > 0$. Moreover, the variability of the real exchange rate under CPI targeting improves markedly relative to domestic inflation targeting as the degree of openness increases. Similar findings are obtained for the behavior of the nominal interest rate. As illustrated in Figure 10, for $\alpha > 0$ the standard deviation of the nominal interest rate is always lower under CPI targeting than under domestic inflation targeting; the existence of an increasing differential between the standard deviation of the nominal interest rate under domestic as opposed to CPI inflation targeting is proof that the former strategy leads to greater *relative* variability in the nominal interest rate as openness increases.

VI. Summary and Conclusion

The above findings need to be put in perspective. Svensson (2000) evaluates strict CPI and domestic inflation targeting strategies in an open-economy setting as well. Comparing strict CPI inflation targeting to strict domestic inflation targeting, he finds that the former strategy leads to greater variability in the output gap, the real exchange rate, and the nominal rate of interest. In sharp contrast, the findings presented in this paper suggest that pursuing a strict CPI inflation target leads to lower fluctuations in these variables.^{21,22} The present framework stresses the forward-looking behavior of economic agents at the expense of persistence in the rate of inflation and real output and possible lags in the transmission of monetary policy. If forward-looking behavior is important while persistence and lags are not, then the choice of CPI inflation targeting over domestic inflation targeting is justifiable on the grounds that CPI inflation targeting leads to greater stability in key economic variables. Thus, there is reason to believe that the countries that have adopted strict inflation targeting as a strategy for monetary policy have chosen to target the appropriate rate of inflation.

21 The difference in results is attributable to substantial differences in the specification and evaluation of the model, as well as in the definition of what constitutes strict inflation targeting. In the current paper, *strict* inflation targeting is defined as the case where the relative weight on the variance of inflation in the loss function approaches infinity. The policymaker thus chooses an efficient, albeit extreme strategy for monetary policy. In contrast, Svensson (2000) defines strict inflation targeting as the case where the variance of the rate of inflation is the *only* argument in the expected loss function and the weight attached to it is unity. Under Svensson's definition of *flexible* inflation targeting, the weight on the variance of real output is 0.5 times the weight on the variance of inflation in the loss function.

Another critical difference concerns the specification of the structural relations. The specification of the IS relation employed by Svensson does not contain expectations of the future domestic and foreign output gap. In addition, the Phillips Curve features a complex lag structure that rules out analytical solutions for the variables that feature prominently in his model.

22 Recall that the standard deviation of the real output gap is lower under CPI targeting only for values of $\alpha < 0.5$.

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Table 1: Inflation-Targeting Countries.

	Inflation Objective	Index Targeted	Adoption Date (dd/mm/yy)	Who Sets the IT?	Openness: Imports as a fraction of total consumption ⁵
Australia	Average of 2-3%	CPI	01/01/93	Government	.237
Canada	<ul style="list-style-type: none"> • 2-4% by end of 1992 • 1.4-3.5% by mid-1994 • 1-3% Dec 1993 to Feb 1998¹ 	CPI ²	26/02/91	Joint	.360
Finland	2% from 1995	CPI	02/02/93	Central Bank	.365
New Zealand	<ul style="list-style-type: none"> • (1) 3-5% (Dec 1990; 2.5-4% (Dec 1991) • (2) 1.5-3.5% (1992:Q1-Q4) • (3) 0-2%(1993:Q1) • (4) 0-3%(1997:Q1) • (4) 0-3%(1997:Q4)³ 	CPI ⁴	02/03/90	Joint	.366
Spain	2% ⁶	CPI	01/01/95	Central Bank	.276
Sweden	<ul style="list-style-type: none"> • 2% • 1-3% since 1995 	CPI	15/01/93	Central Bank	.391
United Kingdom	<ul style="list-style-type: none"> • 1-4% until June '97 elections • 2.5% since June 1997 	Retail Price Index <i>ex</i> mortgage interest payments	08/10/92	Government	.324

Notes:

1. 1-3% Feb 1998 to end 2001.
2. Although the target is formally specified in terms of overall CPI, the Bank focuses on the CPI excluding food, energy, and the effect of indirect taxes.
3. The term of the new PTA coincides with the current term of the Governor which expires 31 August 2003.
4. Since December 1997, the CPI excluding credit services is targeted. Prior to that date, overall CPI was targeted.
5. 1950-1998 sample means except for Spain (1954-1998), Japan (1955-1998), Finland (1950-1997), and New Zealand (1950-1997).

Sources: With the exception of the last column, the information provided in the table stems from Siklos (1999). The measures of openness are based on author's own calculations. The annual data were taken from the IFS CD-ROM.

Two other countries that have begun pursuing an inflation-targeting strategy are Poland and Israel.

Table 2: The Coefficients on the Disturbances and the Lagged Real Exchange Rate in the Reduced-Form Equations for y_t , π_t , and q_t .

	y_t	π_t	q_t
v_t	$\frac{\gamma_{26} + b + \alpha}{\Lambda + \gamma_{26}}$	$\frac{-\theta(\gamma_{26} + b) + \alpha\alpha}{\Lambda + \gamma_{26}}$	$\frac{-(\theta + a)}{\Lambda + \gamma_{26}}$
u_t	$\frac{-(a_1 + a_2)}{\Lambda + \gamma_{26}}$	$\frac{\theta(a_1 + a_2) + \alpha}{\Lambda + \gamma_{26}}$	$\frac{-1}{\Lambda + \gamma_{26}}$
$\varepsilon_t + R_t^*$	$\frac{-a_1(\gamma_{26} + b + \alpha)}{\Lambda + \gamma_{26}}$	$\frac{a_1(\theta(\gamma_{26} + b) - \alpha\alpha)}{\Lambda + \gamma_{26}}$	$\frac{a_1(\theta + a)}{\Lambda + \gamma_{26}}$
y_t^*	$\frac{a_3(\gamma_{26} + b + \alpha)}{\Lambda + \gamma_{26}}$	$\frac{a_3(-\theta(\gamma_{26} + b) + \alpha\alpha)}{\Lambda + \gamma_{26}}$	$\frac{-a_3(\theta + a)}{\Lambda + \gamma_{26}}$
q_{t-1}	$\frac{\alpha(a_1 + a_2)}{\Lambda + \gamma_{26}}$	$\gamma_{26} = \frac{-((\theta + a)(a_1 + a_2) + b) \pm \sqrt{((\theta + a)(a_1 + a_2) + b)^2 + 4\alpha(a_1 + a_2) + b}}{2}$	$\frac{\alpha}{\Lambda + \gamma_{26}}$

Note:

i. $\Lambda = (\theta + a)(a_1 + a_2) + b + \alpha$

ii. the coefficient γ_{26} has a square root term that can take on a positive or a negative value. The decision to use the *positive* square root rests on the following economic argument. The lagged exchange rate enters the model through equation (6), the definition of CPI inflation. If $\alpha=0$, the rate of CPI inflation reduces to the rate of domestic inflation. As a result, the lagged real exchange rate ought not to enter the model. For this to be the case, γ_{26} must be equal to zero. However, this necessitates in turn that the positive square root be used in the calculation of γ_{26} .

Table 3: The Variance of Real Output (y_t):

Strict CPI Inflation Target	Strict Domestic Inflation Target
<i>Demand Shocks</i>	
$\frac{4[(A + \sqrt{A(4\alpha + A)})(2\alpha^2 + 4ab + b^2) + 2\alpha(3A\alpha + b^2)]}{(A + \sqrt{A(4\alpha + A)})(2\alpha + A + \sqrt{A(4\alpha + A)})(4\alpha + A + \sqrt{A(4\alpha + A)})}$	$\frac{b^2}{A^2}$
<i>Cost-Push Shock</i>	
$\frac{4(a_1 + a_2)^2}{(A + \sqrt{A(4\alpha + A)})(4\alpha + A + \sqrt{A(4\alpha + A)})}$	$\frac{(a_1 + a_2)^2}{A^2}$

Note:

- i. The entries in the table represent the coefficients on the variances of the demand shocks and the variance of the cost-push shock that appear in the expression for the variance of real output. The variances of the demand shocks are defined as $\sigma_v^2 + a_1^2(\sigma_{Rf}^2 + \sigma_\varepsilon^2) + a_3^2\sigma_y^2$, while the variance of the cost-push shock is given by σ_u^2 .
- ii. $A = a(a_1 + a_2) + b$.
- iii. Setting α equal to zero results in the coefficient(s) of the variance(s) of demand-side and cost-push shocks under strict *CPI inflation targeting* becoming equal to the coefficient(s) of the variance(s) of demand-side and cost-push shocks under strict *domestic inflation targeting*.

Table 4: The Variance of the Real Exchange Rate (q_t).

Strict CPI Inflation Target	Strict Domestic Inflation Target
<i>Demand Shocks</i>	
$\frac{4a^2}{(A + \sqrt{A(4\alpha + A)})(4\alpha + A + \sqrt{A(4\alpha + A)})}$	$\frac{a^2}{A^2}$
<i>Cost-Push Shock</i>	
$\frac{4}{(A + \sqrt{A(4\alpha + A)})(4\alpha + A + \sqrt{A(4\alpha + A)})}$	$\frac{1}{A^2}$

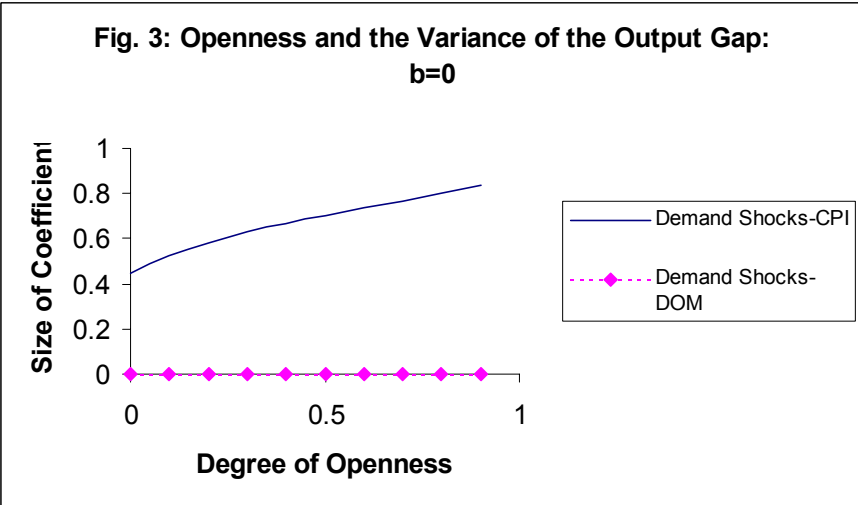
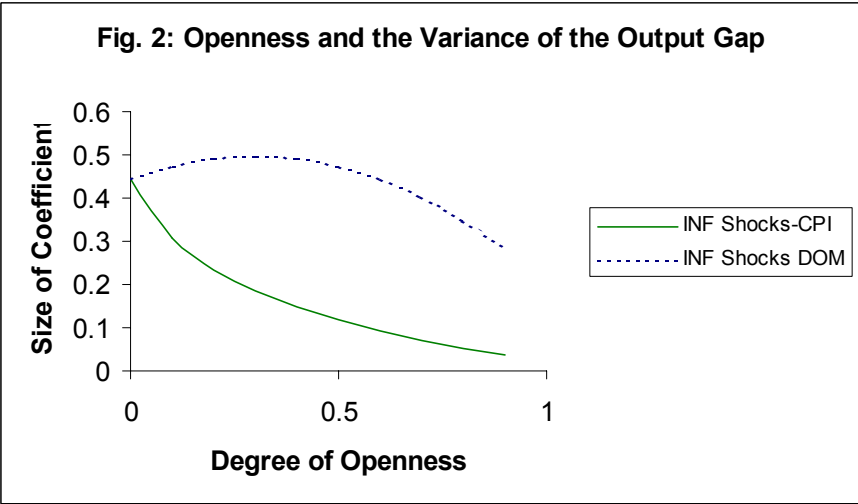
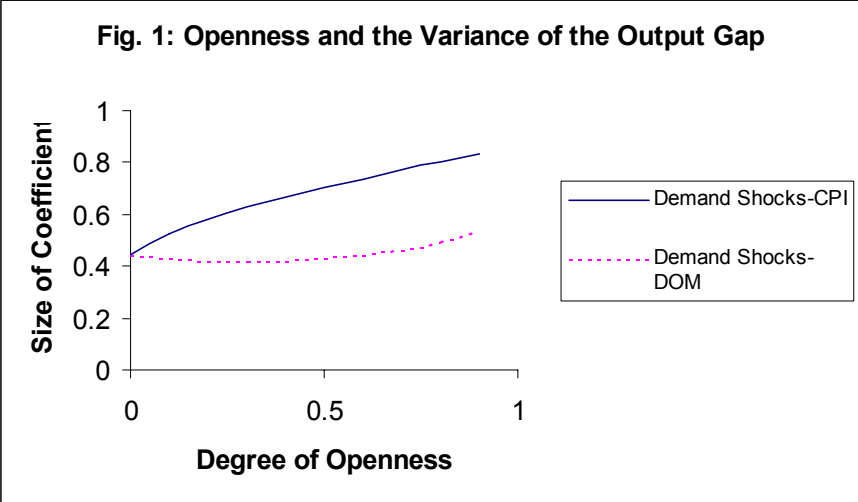
Note:

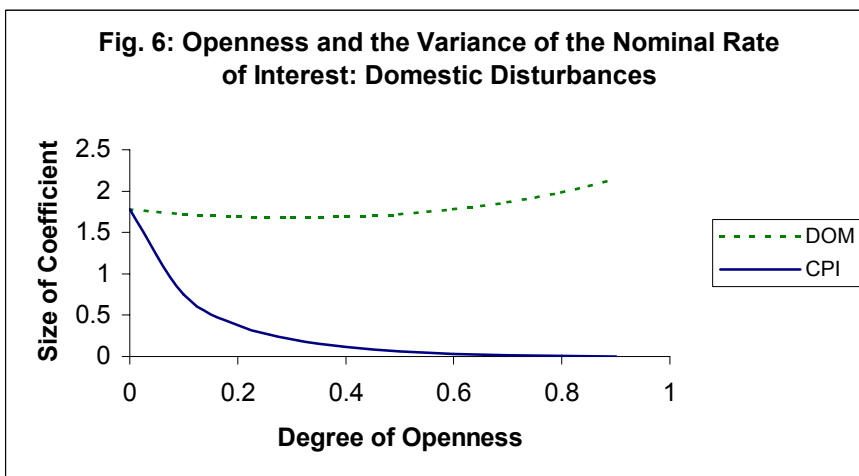
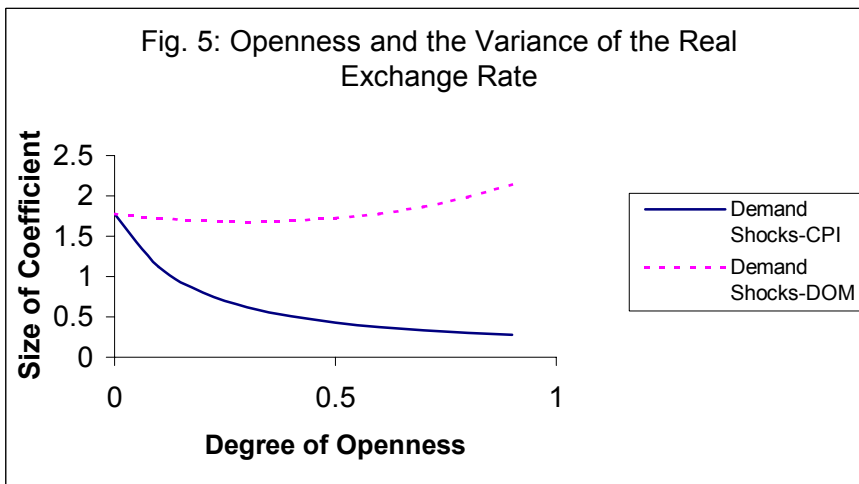
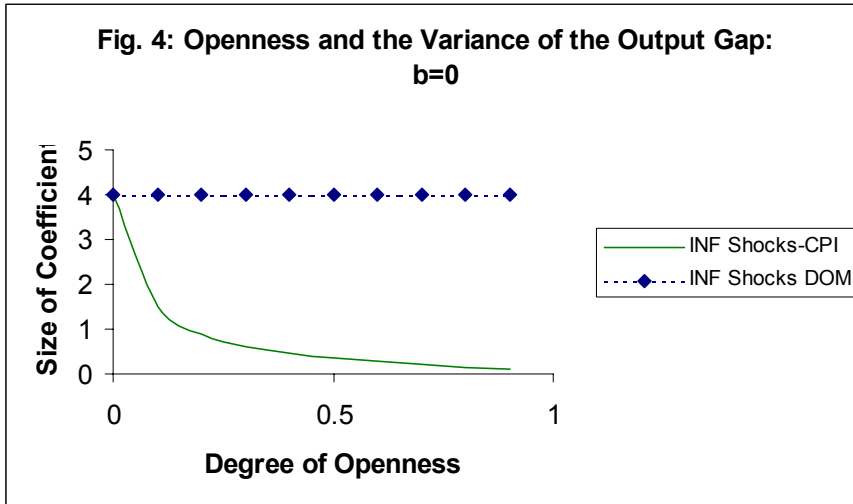
- i. The entries in the table represent the coefficients on the variances of the demand shocks and the variance of the cost-push shock that appear in the expression for the variance of the real exchange rate. The variances of the demand shocks are defined as $\sigma_v^2 + a_1^2(\sigma_{R^f}^2 + \sigma_\varepsilon^2) + a_3^2\sigma_{y^*}^2$ while the variance of the cost-push shock is given by σ_u^2 .
- ii. $A = a(a_1 + a_2) + b$
- iii. Again setting α in the first column (CPI inflation targeting) equal to zero yields the expression in the second column (domestic inflation targeting).

Table 5: The Variance of the Nominal Rate of Interest (R_t)

Variance of Shock	Strict CPI Inflation Target	Strict Domestic Inflation Target
σ_v^2	$\frac{(4a(\alpha-1)\alpha)^2 (A + \sqrt{A(4\alpha + A)})}{(2\alpha + A + \sqrt{A(4\alpha + A)})^4 (4\alpha + A + \sqrt{A(4\alpha + A)})} +$ $\frac{4a^2 \left(-1 - \frac{1}{2}(A - \sqrt{A(4\alpha + A)}) + \frac{2\alpha}{2\alpha + A + \sqrt{A(4\alpha + A)}} \right)^2}{(2\alpha + A + \sqrt{A(4\alpha + A)})^2}$	$\frac{a^2}{A^2}$
σ_u^2	$\frac{(4\alpha(\alpha-1))^2 (A + \sqrt{A(4\alpha + A)})}{(2\alpha + A + \sqrt{A(4\alpha + A)})^4 (4\alpha + A + \sqrt{A(4\alpha + A)})} +$ $\frac{4 \left(-1 - \frac{1}{2}(A - \sqrt{A(4\alpha + A)}) + \frac{2\alpha}{2\alpha + A + \sqrt{A(4\alpha + A)}} \right)^2}{(2\alpha + A + \sqrt{A(4\alpha + A)})^2}$	$\frac{1}{A^2}$
$\sigma_{R^*}^2 + \sigma_\varepsilon^2$	$\frac{(4aa_1(\alpha-1)\alpha)^2 (A + \sqrt{A(4\alpha + A)})}{(2\alpha + A + \sqrt{A(4\alpha + A)})^4 (4\alpha + A + \sqrt{A(4\alpha + A)})} +$ $\frac{4 \left(\alpha + \frac{a}{2}(-a_1(1+A) + a_2) + \frac{1}{2}(b + \sqrt{A(4\alpha + A)}(1 + aa_1)) + \frac{2a\alpha a_1}{2\alpha + A + \sqrt{A(4\alpha + A)}} \right)^2}{(2\alpha + A + \sqrt{A(4\alpha + A)})^2}$	$\frac{(-aa_2 - b)^2}{A^2}$
$\sigma_{y^*}^2$	$\frac{(4a_3a)(\alpha-1)\alpha)^2 (A + \sqrt{A(4\alpha + A)})}{(2\alpha + A + \sqrt{A(4\alpha + A)})^4 (4\alpha + A + \sqrt{A(4\alpha + A)})} +$ $\frac{4(a_3a)^2 \left(-1 - \frac{1}{2}(A - \sqrt{A(4\alpha + A)}) + \frac{2\alpha}{2\alpha + A + \sqrt{A(4\alpha + A)}} \right)^2}{(2\alpha + A + \sqrt{A(4\alpha + A)})^2}$	$\frac{(a_3a)^2}{A^2}$

Note: see remarks ii. and iii. that appear below Table 3 or 4.





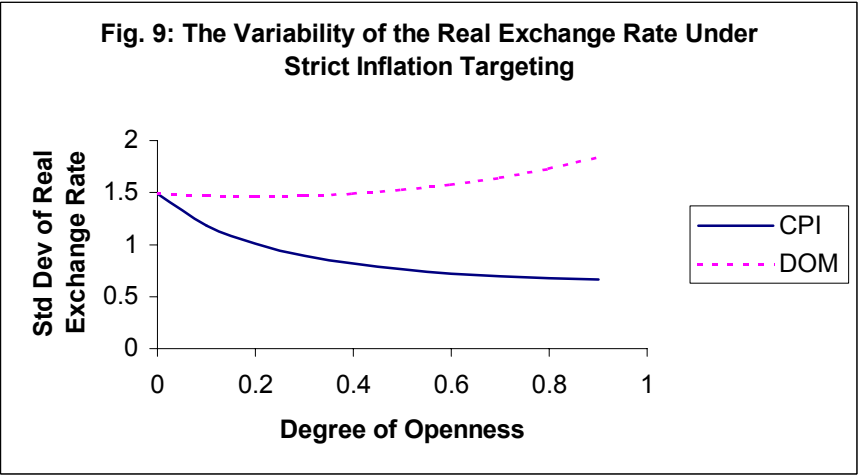
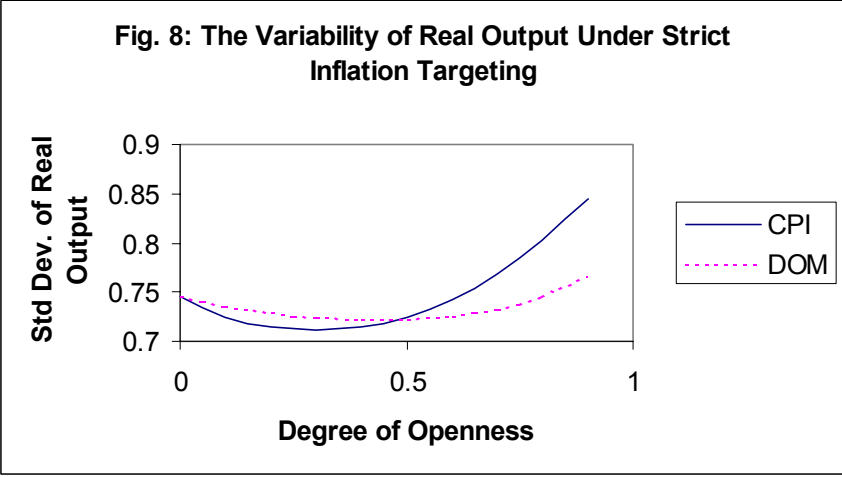
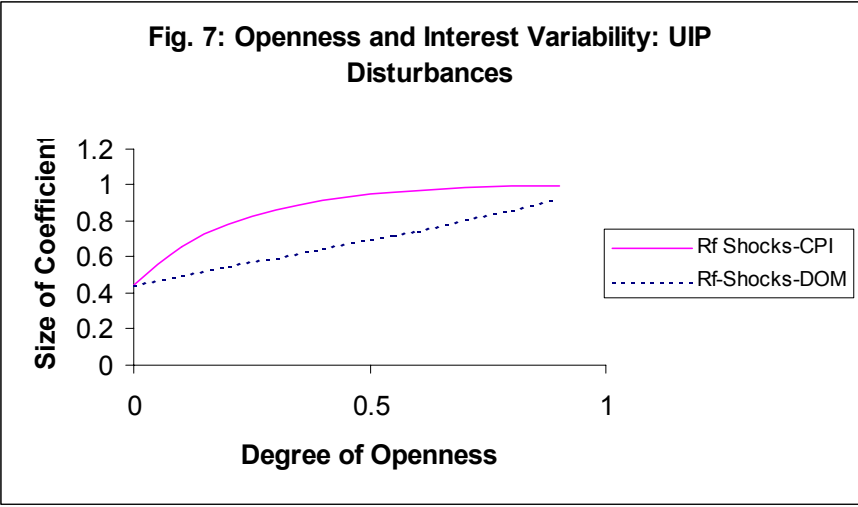
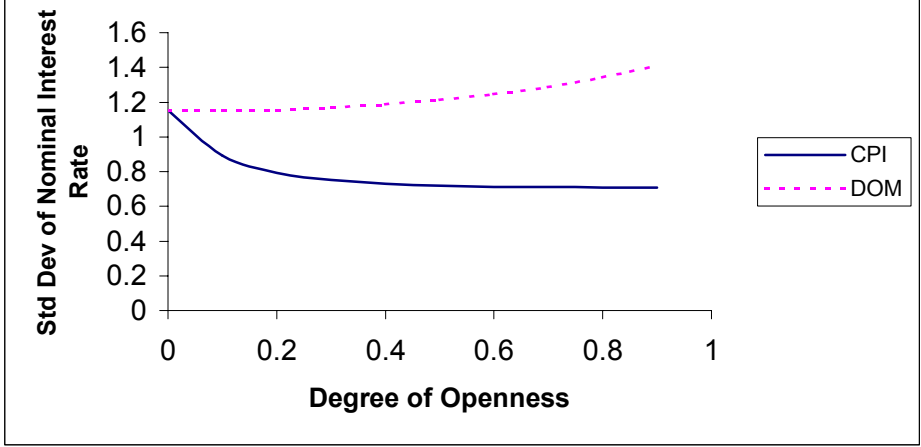


Fig. 10: The Variability of the Nominal Rate of Interest under Strict Inflation Targeting



Appendix to:
Inflation Targeting in the Open Economy:
Which Rate of Inflation to Target?

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Appendix A:

The New Keynesian approach emphasizes the price setting behavior of firms with market power. The key feature is that price changes are assumed to be costly for these firms. Thus prices set by these firms have a tendency to be sticky.¹

Firms wish to minimize menu costs weighed against the cost of being away from the optimal price they would charge in the absence of those menu costs. This optimal price is denoted p^* . Their objective function is thus:

$$\min_p \Omega_t = E_t \sum_{\tau=t}^{\infty} \theta^{\tau-t} \left[(p_{\tau} - p_{\tau}^*)^2 + c(p_{\tau} - p_{\tau-1})^2 \right] \quad (1)$$

where:

Ω_t is the total cost at time t

p_t is the natural logarithm of the price at time t

p^* is the natural logarithm of the optimal price a firm charges.

θ is the constant discount factor

c is the parameter that measures the ratio of the costs of changing prices to the costs of being away from the optimal price

E_t is the expectations operator conditional on information available at time t.

The first-order condition for the above cost-minimization problem is:

$$E_t \left\{ 2(p_t - p_t^*) + 2c(p_t - p_{t-1}) - 2c(p_{t+1} - p_t) \right\} = 0 \quad (2)$$

where we have assumed θ to equal one for simplicity.

Rearranging equation (2), we get:

$$p_t - p_{t-1} = E_t (p_{t+1} - p_t) - \frac{1}{c} (p_t - p_t^*) \quad (3)$$

Notice that expectations of the future change in the price level are present in the first order condition. It is in this sense that firms are forward-looking. Next we specify the optimal price p^* as

$$p_t^* = \hat{p}_t + \beta y_t + \varsigma_t \quad (4)$$

where:

$\beta > 0$

\hat{p}_t is the natural logarithm of the price charged by foreign firms at time t

y_t is the output gap at time t, defined as the percentage deviation of aggregate output at time t from its trend.

$\varsigma_t \sim N(0, \sigma_{\varsigma}^2)$ is a stochastic disturbance.

Equation (4) assumes an upward sloping supply curve, which is reasonable, since a firm would want to raise its price if demand (or aggregate income) is high.

¹ Roberts (1995) derives a forward-looking Phillips curve in the closed-economy context.

In a small open economy, the optimal price also responds to developments abroad. The typical firm adjusts its optimal price in line with the domestic currency price of the final goods charged by its foreign competitors. This behavior of on the part of firms implies that developments abroad have implications for the way firms go about setting prices in the domestic economy.² Thus \hat{p}_t becomes:

$$\hat{p}_t = p_t^f + e_t \quad (5)$$

where:

p_t^f is the natural logarithm of the foreign price level at time t

e_t is the natural logarithm of the spot exchange rate at time t, defined as the units of domestic currency required to buy one unit of foreign currency.

Using this specification for p_t^* , we can rewrite equation (3) as:

$$p_t - p_{t-1} - E_t(p_{t+1} - p_t) = -\frac{1}{c}(p_t - p_t^f - e_t - \zeta_t) + \frac{\beta}{c} y_t \quad (6)$$

If aggregated over all firms, equation (6) represents a Phillips Curve relation for an open economy. The same equation can also expressed as:

$$\pi_t = E_t \pi_{t+1} + a y_t + b q_t + u_t \quad (7)$$

where:

$$\pi_t = p_t - p_{t-1}$$

$$E_t \pi_{t+1} = E_t p_{t+1} - p_t$$

$$q_t = p_t^f + e_t - p_t$$

$$a = \frac{\beta}{c}$$

$$b = \frac{1}{c}$$

$$u_t = \frac{1}{c} \zeta_t$$

² That such behavior applies to firms that are based in a small open economy but trade in world markets is borne out by the following excerpt from a newspaper article that appeared in the Christchurch Press on February 17, 2001: "About 95 per cent of New Zealand's milk is used for exports, and the world price for dairy products has recently climbed, making it more costly for local dairy manufacturers to compete for milk. Milk that is used for export products sets the *benchmark for the price we pay for milk processed for the local market.*" (page 2) (italics added for emphasis).

Appendix B:

This appendix explains the derivation of the IS equation. The exposition draws on Obstfeld and Rogoff (1996), McCallum and Nelson (1997), and Svensson (2000). Consumers maximize a lifetime utility function that depends on the consumption level of the domestically produced final good and an imported final good.

The period utility function takes the following form:

$$U(C_t^h, C_t^f) = \frac{C(C_t^h, C_t^f)^{1-\sigma} - 1}{1-\sigma}$$

where $\sigma > 0$ is the intertemporal elasticity of substitution and C measures aggregate consumption while C_t^h and C_t^f measures the quantity of the domestic and foreign consumption good, respectively.

From the standard *intertemporal* utility maximization problem, the following first-order condition obtains (lower case letter denote logs):

$$c_t = E_t c_{t+1} - \sigma(R_t - E_t \pi_{t+1}^{CPI}) \quad (A1)$$

where c_t denotes aggregate consumption and $R_t - E_t \pi_{t+1}^{CPI}$ denotes the real rate of interest, defined as the difference between the nominal rate of interest and the CPI rate of inflation.

The *intratemporal* first-order condition yields the following relationship: the demand for the domestic consumption good is proportional to aggregate consumption and depends inversely on its relative price:

$$c_t^h = -\eta(p_t^h - p_t^{CPI}) + c_t \quad (A2)$$

η measures the elasticity of substitution between the domestic and the foreign consumption good. p_t^{CPI} and p_t^h are defined as the consumer price index and the price of the domestic consumption good, respectively.

The consumer price index is given by

$$p_t^{CPI} = (1-\alpha)p_t^h + \alpha(p_t^* + e_t) \quad (A3)$$

where p_t^* represents the price of the foreign consumption good and α denotes the weight of the price of the foreign good in the CPI.

Substituting (A3) into (A2) yields the following expression:

$$c_t^h = \eta\alpha q_t + c_t \quad (A4)$$

where q_t represents the real exchange rate and is defined as $q_t = p_t^* + e_t - p_t^h$.

The next step consists of substituting (A4) into (A1):

$$c_t^h - \eta \alpha q_t = E_t c_{t+1}^h - \eta \alpha E_t q_{t+1} - \sigma(R_t - E_t \pi_{t+1}^{CPI}) \quad (\text{A5})$$

Expressing the resource constraint as a log-linearized equation around the steady state levels yields:

$$y_t = (1 - \alpha)c_t^h + \alpha c_t^{h*} \quad (\text{A6})$$

where y_t is real output and c_t^{h*} is foreign consumption of domestic goods, i.e. domestic exports.

Foreign demand for the domestic consumption good evolves in accordance with equation (A7):

$$c_t^{h*} = c_t^* + \eta^* \alpha^* q_t \quad (\text{A7})$$

Foreign consumption is proportional to foreign real output, i.e. $c_t^* = \beta^* y_t^*$. Hence (A7) can be written as:

$$c_t^{h*} = \beta^* y_t^* + \eta^* \alpha^* q_t \quad (\text{A8})$$

Updating and taking expectations of the resource constraint (Equation (A6)) yields:

$$E_t y_{t+1} = (1 - \alpha)E_t c_{t+1}^h + \alpha E_t c_{t+1}^{h*}$$

After solving for $E_t c_{t+1}^h$, we can restate the above equation as follows:

$$\frac{E_t y_{t+1} - \alpha E_t c_{t+1}^{h*}}{1 - \alpha} = E_t c_{t+1}^h \quad (\text{A9})$$

Next, substitute (A9) into (A5):

$$c_t^h = \frac{E_t y_{t+1} - \alpha E_t c_{t+1}^{h*}}{1 - \alpha} + \alpha \eta (q_t - E_t q_{t+1}) - \sigma(R_t - E_t \pi_{t+1}^{CPI}) \quad (\text{A10})$$

Expression (A10) can then be substituted back into expression (A6):

$$y_t = E_t y_{t+1} + (1 - \alpha)[\alpha \eta (q_t - E_t q_{t+1}) - \sigma(R_t - E_t \pi_{t+1}^{CPI})] + \alpha (c_t^{h*} - E_t c_{t+1}^{h*}) \quad (\text{A11})$$

Making use of equation (A8), we can restate equation (A11) as written below:

$$y_t = E_t y_{t+1} + \alpha [(1 - \alpha) \eta + \eta^* \alpha^*] (q_t - E_t q_{t+1}) - (1 - \alpha) \sigma(R_t - E_t \pi_{t+1}^{CPI}) + \alpha (\beta^* (y_t^* - E_t y_{t+1}^*)) \quad (\text{A12})$$

or

$$y_t = E_t y_{t+1} + a_2(q_t - E_t q_{t+1}) - a_1(R_t - E_t \pi_{t+1}^{CPI}) + a_3(y_t^* - E_t y_{t+1}^*)$$

where $a_2 = \alpha[(1 - \alpha)\eta + \eta^* \alpha^*]$

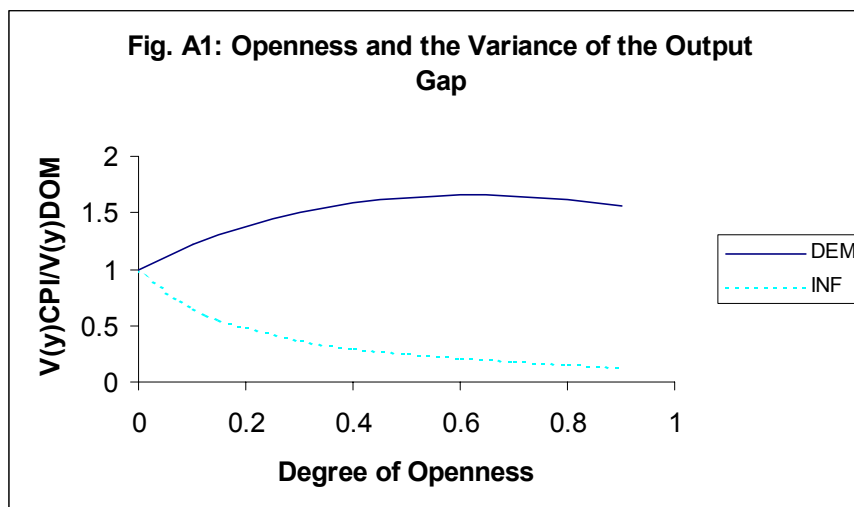
$$a_1 = (1 - \alpha)\sigma$$

$$a_3 = \alpha\beta^*$$

This is the IS equation referred to in the text.

A few remarks about this equation are in order.

- i. Because of the specification chosen for the CPI inflation rate, η is assumed to equal one.
- ii. If α is interpreted as capturing the degree of openness of the economy, then the structural coefficients of the IS equation are sensitive to the degree of openness.



Appendix C:

To obtain the variances of y_t and q_t , we take the following steps.

The equations for y_t and q_t can be written in matrix form as:

$$\begin{bmatrix} y_t \\ q_t \end{bmatrix} = \begin{bmatrix} 0 & \frac{\alpha(a_1 + a_2)}{\Lambda + \gamma_{26}} \\ \frac{\alpha}{\Lambda + \gamma_{26}} & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ q_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{\gamma_{26} + b + \alpha}{\Lambda + \gamma_{26}} & -\frac{(a_1 + a_2)}{\Lambda + \gamma_{26}} & -\frac{a_1(\gamma_{26} + b + \alpha)}{\Lambda + \gamma_{26}} & \frac{a_3(\gamma_{26} + b + \alpha)}{\Lambda + \gamma_{26}} \\ -\frac{(\theta + a)}{\Lambda + \gamma_{26}} & -\frac{1}{\Lambda + \gamma_{26}} & \frac{a_1(\theta + a)}{\Lambda + \gamma_{26}} & -\frac{a_3(\theta + a)}{\Lambda + \gamma_{26}} \end{bmatrix} \begin{bmatrix} v_t \\ u_t \\ R_t^* + \varepsilon_t \\ y_t^* \end{bmatrix}$$

or

$$\mathbf{X}_t = \mathbf{B}\mathbf{X}_{t-1} + \mathbf{C}\boldsymbol{\Omega}_t$$

Hence it follows that the variance-covariance matrix is given by:

$$E[X_t X_t'] = E[BX_{t-1} + C\boldsymbol{\Omega}_t](X_{t-1}'B' + \boldsymbol{\Omega}_t' C')$$

$$E[X_t X_t'] = BE[X_{t-1} X_{t-1}']B' + CE[\boldsymbol{\Omega}_t \boldsymbol{\Omega}_t']C'$$

$$\phi = B\phi B' + CKC'$$

Vectorizing by stacking columns yields:

$$\phi^v = (B\phi B' + CKC')^v$$

$$\phi^v = B \otimes B \phi^v + C \otimes CK^v$$

$$\phi^v = (I - B \otimes B)^{-1} (C \otimes C) K^v$$

where

ϕ is a 4x1 column vector

$(I - B \otimes B)^{-1}$ is a 4x4 matrix

$C \otimes C$ is a 4x16 matrix

K^v is a 16x1 column vector.

The first element of ϕ represents the variance of y_t , while the fourth element represents the variance of q_t .