“Leaning With the Wind”? An Open-Economy Example

By

Alfred V. Guender[*]

Department of Economics
University of Canterbury
Christchurch, New Zealand

Abstract:

This paper uses a forward-looking open-economy optimizing model to show that the existence of an exchange rate channel in the Phillips Curve dramatically alters the conduct of optimal monetary policy. The central bank’s optimal reaction function can produce a “lean with the wind” response to IS and foreign output shocks provided there are both a pronounced exchange rate channel and a high degree of persistence in the disturbances. The existence of such an exchange rate channel in the Phillips Curve generally leads to smaller fluctuations in the policy instrument and all endogenous variables except the real output gap.

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[*] Department of Economics, University of Canterbury, Private Bag 4800, Christchurch, New Zealand.
E-mail: a.guender@econ.canterbury.ac.nz
Phone: (64)-3-364-2519. Fax: (64)-3-364-2635.

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Introduction

In its simplest incarnation, a strategy of flexible inflation targeting requires a central bank to focus on two policy objectives. Apart from stabilizing the rate of inflation around its target level (or keeping it within a band), the central bank also pays heed to the state of the real economy. There is increasing evidence that flexible inflation targeting characterizes the conduct of monetary policy even in those countries where the pursuit of price stability is the overriding goal of monetary policy. That strict inflation targeting is not what central banks practice is borne out by the wording of policy targets agreements or formal contracts entered into by the executive branch of government and the head of the central bank. For instance, the policy targets agreements concluded by the Minister of Finance and the Governor of the Reserve Bank of New Zealand in 1999 and 2002 stipulate that "[i]n pursing its price stability objective, the Bank shall implement monetary policy in a sustainable, consistent and transparent manner and shall seek to avoid unnecessary instability in output, interest rates and the exchange rate."[1] Other central banks that are viewed as having adopted flexible inflation targeting are the Swedish Riksbank and the Bank of England.[2] The Reserve Bank of Australia and the Bank of Canada are also widely acknowledged as practicing flexible inflation targeting, albeit in a less formal way compared to the Swedish Riksbank or the Bank of England. Flexible inflation targeting has also been embraced by central banks in transition economies like the Czech Republic, Hungary, and Poland as well as in emerging countries like Brazil, Chile, Mexico, to name but a few.[3] The rationale for minimizing fluctuations in

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[1] Policy Targets Agreement 1999, Section 4. Implementation and Accountability, part c.
Policy Targets Agreement 2002, Section 4, Communication, Implementation, and Accountability, part b.
Notice that the economic variables are not arranged in alphabetical order. The order in which they appear is arguably indicative of their relative importance. A second observation that deserves mentioning is related to the change of emphasis in the wording of the policy targets agreement over time. The first three policy targets agreements (March 1990, December 1990, 1992) define the mandate of the Reserve Bank as the achievement of price stability without reference to the underlying state of the economy. The policy targets agreements of 1996 and 1997 mention the achievement of price stability in conjunction with sustainable economic growth and employment as the ultimate objective.

[2] According to Svensson (2001). The Bank of England Act of 1998 stipulates that "the objectives of the Bank of England shall be (a) to maintain price stability, and (b) subject to that, to support the policies of Her Majesty's government, including its objectives for growth and employment." The Swedish Riksbank Act of 1988 in contrast does not contain any references to growth and employment objectives. Indeed Article 2 of Chapter 1 merely states that "the objective of the Riksbank's operations shall be to maintain price stability. Thus, it is not clear whether the Swedish Riksbank is actually formally bound to engage in flexible inflation targeting.

[3] Bernanke (2003).

variables other than the rate of inflation is the simple realization that a narrow focus on the rate of inflation will cause excessive swings in the nominal interest rate, the exchange rate, and real output. Pronounced volatility of the nominal interest rate and the exchange rate are disruptive to the smooth working of financial markets. Huge swings in real output are not acceptable for economic as well as political reasons.[4]

The countries currently practicing flexible inflation targeting are typically small open economies.[5] In these countries the rate of inflation is determined by domestic and international factors. These factors may pull the rate of inflation in opposite directions. For instance, unexpected strong domestic demand relative to capacity will put upward pressure on the rate of inflation. With the interest rate rising, the exchange rate appreciates and reduces export demand. In addition, the appreciation of the domestic currency lowers the domestic currency price of internationally traded goods, the prices of which are determined in world markets. Thus the strengthening of the exchange rate leads to offsetting, i.e. downward pressure on domestic inflation provided that the pass-through effect on domestic prices is direct and swift. It is no surprise then that inflation-targeting central banks pay close attention to current and expected future movements in the exchange rate when setting policy.[6,7] Indeed the direction and size of any adjustment in the policy instrument depends on the strength of the effect of domestic vis-à-vis international factors on the rate of inflation.

There are two distinct ways of modeling the exchange rate effect on prices. The first way considers the *direct* effect on the CPI inflation rate of a change in the real exchange rate. This effect depends critically on the extent of exchange-rate pass-through. This paper considers a complementary exchange rate channel. The exchange rate channel emphasized in this paper traces the immediate effect of a change in the

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[4] For our purposes the mere realization that central banks are concerned about fluctuations in both the rate of inflation and the real output gap suffices. Detailing the complex interrelationships between the central bank, the executive, and the legislative branch of government goes beyond the scope of the paper.

[5] The central banks of large and rather closed countries such as the United States and Japan have not announced specific inflation targets or bands. The European Central Bank follows a strategy of monetary policy that is based on two pillars: a 2% percent target for the rate of inflation and a reference value for the growth rate of M3.

[6] This of course assumes that the law of one price holds in some shape or form.

[7] A case in point is New Zealand where the governor of the Reserve Bank frequently comments on the role of the exchange rate in determining monetary policy. “The exchange rate is *always* relevant in assessing the appropriate stance of monetary policy.” [italics in original statement] Statement by Don Brash (2000). “In setting the Official Cash Rate, we are always very conscious of exchange rate changes, and how those developments will affect the outlook for growth and inflation a little down the track.” Statement by Alan Bollard (2003). See also the Monetary Policy Statement (2003).

level of the real exchange rate on domestic inflation. The latter exchange rate effect on domestic inflation arises because domestic firms take the price of the internationally traded consumption good as a benchmark for setting the domestic price of same. In the event of a depreciation of the exchange rate, the benchmark price of the consumption good rises. This in turn induces domestic firms to raise prices, thus imparting upward pressure on the domestic rate of inflation. The importance of such a benchmark price in influencing price setting behavior has recently been highlighted by Taylor (2000) and Bergin and Feenstra (2000) in contributions that do not deal with optimal monetary policy considerations. Indeed with the exception of Walsh (1999) the literature to date has not considered formally the implications for optimal monetary policy of the existence of such an exchange rate effect in the Phillips Curve.[8]

In this paper we present a formal analysis of the behavior of a central bank that practices flexible inflation targeting in a small open economy where an exchange rate channel in the Phillips Curve is operative. The central bank acts with discretion and frames its flexible inflation targeting strategy around the domestic rate of inflation and the real output gap.[9] The key behavioral assumptions about the central bank are that it is forward-looking and optimizing in the sense of seeking to minimize its intertemporal loss function with respect to the target variables. Optimizing behavior on the part of the central bank gives rise to two first-order conditions that, when combined, characterize the targeting strategy of the central bank. With the systematic relationship between the target variables being established, it is a short step to recover the optimal reaction function of the central bank. The optimal reaction function shows how the central bank varies mechanically the setting of the policy instrument, the nominal interest rate, in response to the shocks that occur in the open-economy framework.

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8 Using the Calvo (1983) framework, Walsh (1999) introduces the real exchange rate into the Phillips Curve by assuming that wage demands are based on the CPI. In Ball (1999) lagged changes in the real exchange rate affect overall inflation. In Svensson (2000) the expectation formed in the past about the change in the real exchange rate affects the current rate of inflation. Neither contribution, however, addresses the issue of determining the optimizing condition that characterizes the relationship between the target variables of monetary policy. Gali and Monacelli (1999) and Clarida, Gali, and Gertler (2001, 2002) provide a detailed discussion of the determinants of optimal policy in the open economy but their models do not feature an exchange rate channel in the Phillips Curve.

9 In practice central banks choose a numerical value or a range of numerical values as the inflation target. In either case, the target for the rate of inflation is expressed in terms of percentage changes in the CPI. As the main emphasis of the paper is on the interpretation of *analytical* results, i.e. the derivation of the optimal reaction function and the behavior of the variances of the variables of the model, it is necessary to define the policymaker's problem in terms of the domestic rate of inflation.

The existence of an exchange rate channel in the Phillips Curve has a few noteworthy implications for the conduct of monetary policy under flexible inflation targeting. First, its existence alters dramatically the relative weight the policymaker places on the output gap in the policy rule. The optimizing condition that characterizes the systematic relationship between the target variables is shown to depend on both IS and Phillips Curve parameters. This result stands in marked contrast to the result that obtains in the closed economy framework or in a model of an open economy where an exchange rate channel in the Phillips Curve is not operative.

Second, the existence of such an exchange rate channel on the supply-side of the economy may lead to a "lean with the wind" response in the optimal reaction function in the wake of IS or foreign output disturbances. Such a response can come about because the determination of the rate of inflation is subject to domestic and international factors. The "lean with the wind" reaction is the more likely the greater the degree of persistence of the disturbances that impinge upon the economy. The analytical results also underscore the important role of the persistence parameter in propagating the effects of IS and foreign exchange market disturbances on the variances of the target variables and the setting of the policy instrument. Finally, the existence of an exchange rate channel in the Phillips Curve generally leads to smaller fluctuations in the policy instrument and all endogenous variables except the output gap.

The remainder of the paper is organized as follows. Section II lays out the model. Section III analyzes the conduct of optimal monetary policy. The optimal reaction function is described in Section IV. The behavior of the policy instrument, the real exchange rate, the output gap, and the rate of inflation is examined in Section V. The findings of the paper are summarized in Section VI.

II. The Model

The model that will serve as the foundation for the analysis of the monetary policy issue in this paper comprises the following three equations:[10]

$$y_t = E_t y_{t+1} - a_1(R_t - E_t \pi_{t+1}^{CPI}) + a_2(q_t - E_t q_{t+1}) + a_3(y_t^f - E_t y_{t+1}^f) + v_t \quad (1)$$

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[10] With the exception of the nominal rate of interest all variables are expressed as logarithms. The derivation of the IS curve and the Phillips Curve is discussed at length in the appendix.

$$\pi_t = E_t\pi_{t+1} + ay_t + bq_t + u_t \tag{2}$$

$$R_t - E_t\pi_{t+1} = R_t^f - E_t\pi_{t+1}^f + E_tq_{t+1} - q_t + \varepsilon_t \tag{3}$$

π_t = rate of domestic inflation

$E_t\pi_{t+1}^{CPI}$ = expected rate of CPI inflation

q_t = real exchange rate[11]

y_t = current real output gap

R_t = nominal rate of interest (policy instrument)

R_t^f = foreign nominal rate of interest

$E_t\pi_{t+1}^f$ = expected foreign inflation

y_t^f = foreign real output gap

Equation (1), represents the forward-looking IS equation. It has two noteworthy features. Most importantly, the forward-looking IS curve emphasizes a systematic relationship between real output and the real exchange rate as well as foreign real output that is characterized by the respective difference between the current and expected value next period. The expected rate of CPI inflation enters the definition of the expected real rate of interest. Equation (2) is an open-economy Phillips Curve. It differs from the standard closed-economy version in that the rate of domestic inflation reacts to the real exchange rate. Equation (3) represents the uncovered interest rate parity condition. Stochastic disturbances have been added to the three relations to reflect the existence of uncertainty in the economy.[12] The degree of persistence is the same for all disturbances and fixed at ρ.[13] More formally,

$$u_t = \rho u_{t-1} + \hat{u}_t \qquad \hat{u}_t \sim N(0,\sigma_{\hat{u}}^2)$$

$$v_t = \rho v_{t-1} + \hat{v}_t \qquad \hat{v}_t \sim N(0,\sigma_{\hat{v}}^2)$$

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[11] The real exchange rate is defined as the difference between the domestic currency price of the foreign good and the price of the domestic good: $q_t = s_t + p_t^f - p_t$. s_t = the nominal exchange rate, expressed in terms of domestic currency per unit of foreign currency.

[12] R_t^f, y_t^f, *and* π_t^f are considered to be exogenous stochastic variables. The home country is too small to affect prices, interest rates, and real output abroad. For simplicity, we also assume that all foreign shocks are independent of each other.

[13] Allowing for different degrees of persistence in the shocks would complicate the derivation of analytical results immensely.

$$\begin{aligned} \varepsilon_t &= \rho\varepsilon_{t-1} + \hat{\varepsilon}_t & \hat{\varepsilon}_t &\sim N(0,\sigma^2_{\hat{\varepsilon}}) \\ R^f_t &= \rho R^f_{t-1} + \hat{R}^f_t & \hat{R}^f_t &\sim N(0,\sigma^2_{\hat{R}^f}) \\ \pi^f_t &= \rho\pi^f_{t-1} + \hat{\pi}^f_t & \hat{\pi}^f_t &\sim N(0,\sigma^2_{\hat{\pi}^f}) \\ y^f_t &= \rho y^f_{t-1} + \hat{y}^f_t & \hat{y}^f_t &\sim N(0,\sigma^2_{\hat{y}^f}) \end{aligned} \tag{4}$$

III. Discretionary Monetary Policy

The Policymaker's Objective Function

Under flexible inflation targeting the policymaker has an intertemporal objective function that consists of squared deviations of the target variables, the real output gap and the rate of inflation. The rate of inflation is defined in terms of changes in the log level of domestic prices. The explicit objective function that he attempts to minimize is given by

$$\frac{1}{2}E_t[\sum_{i=0}^{\infty}\beta^i[y^2_{t+i} + \mu\pi^2_{t+i}]]. \tag{5}$$

All variables are as previously defined. β is the discount rate and μ represents the relative weight the policymaker attaches to the squared deviations of the rate of domestic inflation.[14,15]

The Optimal Policy Problem

Discretionary monetary policy is predicated upon two assumptions. First, in setting optimal monetary policy, the policymaker treats the expectations formed by economic agents as given. Second, the policymaker sets policy anew every period, i.e. he re-optimizes every period.

To set the stage for illustrating how discretionary policymaking in the open economy is carried out, it is helpful at the outset to reduce the dimension of the optimization problem to one involving only one constraint. A few simple steps need

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[14] Including only real output and the rate of inflation in the objective function is rather typical in the literature and thus facilitates comparing the results of this paper to earlier contributions (e.g. Clarida, Gali, and Gertler (1999, 2001) or Svensson (2000)).

[15] The target level for real output is the potential level of output. The target for the rate of inflation is assumed to be zero.

to be taken. First, we solve the UIP condition for the real exchange rate and substitute it into both the IS equation and the Phillips curve relation. Next, after substituting for the rate of CPI inflation in Equation (1), we solve the IS relation for the expected real rate of interest ($R_t - E_t\pi_{t+1}$).[16] Following this, we insert the expression for the expected real rate of interest into the Phillips curve relation. The following expression results:

$$\begin{aligned}\pi_t &= (a + \frac{b}{a_1 + a_2^*})y_t + E_t\pi_{t+1} \\ &+ \frac{b}{a_1 + a_2^*}\left[a_1(R_t^f - E_t\pi_{t+1}^f + \varepsilon_t) - (E_t y_{t+1} + v_t) - a_3(y_t^f - E_t y_{t+1}^f)\right] + bE_t q_{t+1} + u_t\end{aligned} \tag{6}$$

where $a_2^* = a_2 - \gamma a_1$.

As pointed out above, when setting policy with discretion, the policymaker takes the expectations of the endogenous variables y_t, π_t, q_t and the remaining terms as given.[17] Hence we can rewrite the above as

$$\pi_t = (a + \frac{b}{a_1 + a_2^*})y_t + f_t \tag{7}$$

where

$$f_t = E_t\pi_{t+1} + \frac{b}{a_1 + a_2^*}\left[a_1(R_t^f - E_t\pi_{t+1}^f + \varepsilon_t) - (E_t y_{t+1} + v_t) - a_3(y_t^f - E_t y_{t+1}^f)\right] + bE_t q_{t+1} + u_t$$

(7')

Notice further that the objective function can be neatly broken up into two separate components as future values of the endogenous variables are independent of today's policy action:[18]

$$\frac{1}{2}[y_t^2 + \mu\pi_t^2] + F_t \tag{8}$$

where $F_t = \frac{1}{2}E_t[\sum_{i=1}^{\infty}\beta^i(y_{t+i}^2 + \mu\pi_{t+i}^2)$

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[16] The rate of CPI inflation can be expressed in terms of the rate of domestic inflation and the change in the real exchange rate: $\pi_t^{CPI} = \pi_t + \gamma\Delta q_t$. The parameter γ represents the weight of the domestic currency price of the foreign good in the CPI.

[17] Here we adopt the convention of describing the conduct of discretionary policy along the lines of Clarida, Gali, and Gertler (1999).

[18] Future values of y_t and π_t are not affected by policy today as the effect of policy is contemporaneous and because of the absence of lagged endogenous variables.

The problem of setting policy under discretion thus reduces to the following simple one-period optimization problem:

$$\underset{y_t,\pi_t}{Min}\ \frac{1}{2}[y_t^2+\mu\pi_t^2]+F_t \qquad (9)$$

subject to

$$\pi_t=(a+\frac{b}{a_1+a_2^*})y_t+f_t \qquad (7)$$

Replacing a_2^* with $a_2-\gamma a_1$ and combining the first-order conditions produces a systematic negative relationship between real output and the rate of inflation:

$$y_t=-\mu(a+\frac{b}{a_1(1-\gamma)+a_2})\pi_t \qquad (10)$$

The coefficient on the rate of inflation indicates the loss of output that the policymaker is prepared to sustain if the rate of inflation exceeds its zero target level.

Equation (10) contains an important result. The optimality condition depends on *all* parameters – except a_3 - of the model. Moreover, the degree of openness γ enters explicitly into the determination of the optimizing condition.[19] Both findings are in stark contrast to earlier contributions to the literature such as Clarida, Gali, and Gertler (2001, 2002). They argue that the optimal policy problem in the open economy is isomorphic to the closed economy case and does not depend explicitly on the degree of openness.[20]

What is striking about the above optimizing condition is the relationship between the degree of openness (γ) and the sensitivity of domestic inflation to the real exchange rate in the Phillips Curve (b). The degree of openness matters only to the extent that an exchange rate channel is operative in the Phillips Curve. In case this channel is absent from the Phillips Curve, i.e. if $b=0$, then the optimal relationship between real output and the rate of inflation is independent of γ and the same for

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[19] As shown in the appendix both a_1 and a_2 depend on deep structural parameters such as the degree of openness of the economy.

[20] The reader is referred to Clarida, Gali, and Gertler (2002) pp. 892-893.

both the open and the closed economy framework: $-\mu a$. There is a straightforward explanation for this result. In the absence of an exchange rate channel in the Phillips Curve, the policymaker can offset any disturbances arising on the demand side of the economy by simply adjusting the setting of the policy instrument. Thus, demand-side factors should not have any role to play in the determination of the optimal relationship between real output and the rate of inflation. The degree of openness is, however, a characteristic of the demand-side of the economy as it denotes the share of the imported foreign consumption good in total consumption. Thus, if the policymaker is in a position to offset any demand-side disturbance, then γ should not matter in the determination of the optimizing condition.

Taken altogether, the presence of a contemporaneous exchange rate effect on inflation in the Phillips Curve permits demand-side parameters to affect the optimizing condition in the forward-looking open economy framework. Only if this distinct exchange rate channel is not operative in the Phillips Curve is the optimizing condition consistent with the one reported by Clarida, Gali, and Gertler (1999, 2001, 2002).

IV. The Optimal Reaction Function

In this section, our objective is to derive the optimal feedback rule. This rule instructs the policymaker to vary mechanically the setting of the policy instrument in response to the shocks of the model. By following the rule systematically, the policymaker minimizes his objective function.

Equation (10) forms the backbone of the optimal reaction function. This equation can be thought of as the optimal policy rule set by the policymaker. It embodies the systematic relationship between the output gap and the rate of inflation ground out by the policymaker's optimizing behavior. Combining Equation (10) with the Phillips Curve, the IS, and the UIP relation allows us to back out the optimal reaction function of the policymaker:

$$R_t = \left[\frac{ab\mu(1-\rho) + A[(1-\rho)^2 - b\rho + a^2\mu(1-\rho)]}{(1-\rho)D}\right]v_t + \tag{11}$$

$$\left[\frac{\mu(1-\rho)[aA+b] + A^2\rho}{D}\right]u_t +$$

$$a_3\left[\frac{ab\mu(1-\rho)+A[(1-\rho)^2-b\rho+a^2\mu(1-\rho)]}{D}\right]y_t^f+$$

$$\left[1-a_1\left[\frac{ab\mu(1-\rho)+A[(1-\rho)^2-b\rho+a^2\mu(1-\rho)]}{(1-\rho)D}\right]\right]\left[R_t^f+\varepsilon_t-\rho\pi_t^f\right]$$

where $A=(1-\gamma)a_1+a_2$ $\qquad D=\mu(Aa+b)^2+A^2(1-\rho)$

Closer inspection of the coefficients on the shocks reveals that the response of the monetary policy instrument to two of the five shocks is ambiguous. The ambiguity arises in case of an IS shock and a foreign output shock and is due to two factors: the existence of an exchange rate channel in the Phillips Curve $(b>0)$ and the persistence property of the stochastic disturbances $(1>\rho>0)$. The size of *b* matters because it measures the beneficial effect on the rate of inflation of an appreciating exchange rate brought about, for instance, by a positive IS shock. Indeed this effect may be so strong so as to swamp the effect of the upward pressure on inflation associated with an increase in the demand for real output, which is measured by the parameter *a*. The size of ρ matters as it captures the persistence of expected future inflation. The greater ρ, the more the current expectation of future inflation depends on the current rate of inflation.[21]

The above open-economy reaction function thus bears a striking result. It is conceivable that the policymaker responds to a positive domestic demand disturbance (v_t) and a positive foreign demand disturbance (y_t^f) by lowering the domestic nominal interest rate. Such a response is the more likely the greater the size of both b and ρ relative to a and μ.[22] Notice in particular that greater persistence of the stochastic disturbances makes it more likely that the interest rate decreases in the wake of positive realizations of the two shocks. Consider the numerator of the coefficient of both shocks: an increase in ρ causes the product $b\rho$ to increase in size

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21 Naturally, the expectations of the other endogenous variables also depend on ρ.

22 Inspection of the coefficients on v_t and y_t^f in the reaction function reveals that $b\rho>(1-\rho)^2+a^2\mu(1-\rho)$ is a necessary but not sufficient condition for the sign of the respective coefficient to be negative.

but causes the remaining terms, all of which are multiplied by $1-\rho$, to decrease in size.

The denominators of the coefficients on the shocks deserve closer scrutiny, too. D represents or appears in the denominator of the coefficients on all disturbances. While the persistence parameter ρ affects D and thus affects the denominator of every coefficient, the denominators of the coefficient on the IS shock and the foreign exchange market disturbances ($R_t^f, \varepsilon_t, \pi_t^f$) are particularly sensitive to changes in the degree of persistence. The denominator of these coefficients consists of $D(1-\rho)$. Thus, greater persistence and greater sensitivity of the rate of inflation to the real exchange rate in the Phillips Curve may elicit both a "lean with the wind" response and a sizeable change in the setting of the policy instrument in the wake of an IS or foreign exchange market shocks.

There is a further noteworthy result. Notice that the coefficient on the UIP and foreign interest rate shocks is one minus the coefficient on the domestic IS disturbance. Thus, provided that the coefficient on the IS shock is negative, the response of the domestic interest rate to an UIP shock and foreign interest rate shock exceeds unity.[23]

A clear-cut response of the policy instrument is associated with a cost-push shock: the policymaker raises the setting of the policy instrument in the wake of a positive cost-push shock.

Having established that the size of b and ρ is critical in determining the response of the policy instrument to domestic and foreign shocks that impinge upon the demand for real output, we next address the issue of the likelihood of a "lean-with-the-wind" policy response. To get some concrete idea about the conditions necessary for a negative policy response to both shocks, we carry out a simple experiment. While fixing the values of the remaining parameters of the model, we allow both ρ and b to vary over a range of plausible values.[24] More specifically, we pick values for b that fall within the [0,1] range. For the persistence parameter we choose values for ρ that satisfy $0 \leq \rho < 1$.

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[23] The response of the policy instrument to a foreign inflation shock is more muted as it depends on the degree of persistence ρ.

[24] The following parameter values were chosen: $\mu = 1; a = .25; \gamma = .3; a_1 = .35; a_2 = .3; a_3 = .27$; the appendix provides further details on the determination of the values of a_1, a_2 and a_3.

The results of this exercise are illustrated in Figures *1* and *2* and Table *1*. Figure *1* illustrates how the size of the coefficient on the IS shock in the reaction function varies in response to changes in the effect of the exchange rate channel in the Phillips Curve (b) and the degree of persistence of the shocks (ρ). The coefficient reaches its maximum if the degree of persistence and the size of b are close to zero. The size of the coefficient remains positive as long as at least either one of the parameters hovers around zero while the other parameter is allowed to vary over the whole range. Notice though that there exists a sizeable number of combinations of values of b and ρ which produce a negative coefficient on the IS shock.

The sensitivity of the response of the policy instrument to both parameters in the wake of all shocks is brought out in detail in Table *1*. This table matches four different degrees of persistence (ρ) with four different sizes of the exchange rate effect on inflation (b) and tracks the resulting changes in the size of the coefficients on the shocks. There are four panels. The top panel examines the case where persistence is low and fixed at $\rho = 0.1$ while the size of the exchange rate channel varies from a low of $b = 0$ to a high of $b = 1.$ The remaining three panels redo this exercise for steadily increasing degrees of persistence of the shocks. Consider the third column in each of the four panels. It reports the coefficient on the IS shock $(\hat{v}_t)$ in the reaction function. It is evident that low to moderate degrees of persistence $(\rho = 0.1 \textit{ and } \rho = 0.4)$ always produce a positive coefficient irrespective of the strength of the exchange rate channel. In case the degree of persistence of the shocks rises to moderately high or high levels $(\rho = 0.6 \textit{ and } \rho = 0.8)$, we observe a dramatic change in the response of the policy instrument to a positive IS shock. For $\rho = 0.6$ a moderately strong or very pronounced exchange rate effect $(b = 0.5; b = 1)$ results in a negative response of the policy instrument to a positive IS shock. In addition, the greater the sensitivity of the rate of inflation to the real exchange rate in the Phillips Curve, i.e. the greater the size of b, the greater the easing of the stance of monetary policy. Should the degree of persistence rise to fairly high levels $(\rho = 0.8)$, then the fall in the interest rate materializes even for a moderately strong exchange rate channel in the Phillips Curve $(b = 0.25)$.

Of some import is the observation that the consistent pattern in the size of the coefficient on the IS disturbance that exists at low or moderate levels of persistence does not carry over to moderately high or very high levels of persistence. For both $\rho = 0.1$ and $\rho = 0.4$, the size of the coefficient on $\hat{v}_t$ decreases as *b* increases. In contrast, at moderately high or high levels of persistence $(\rho = 0.6; \rho = 0.8)$ the size of the coefficient does not conform to a set pattern. For instance, at a high level of persistence $(\rho = 0.8)$ the (absolute) value of the coefficient rises as *b* increases from 0.25 to 0.5 but then decreases as *b* rises further from 0.5 to *1*.

The effect of changes in *b* and ρ on the size of the coefficient of the foreign output shock is illustrated in Figure *2*. The rightmost column of Table *1* provides additional quantitative information about the sensitivity of the size of the response coefficient on $\hat{y}_t^f$ in the reaction function. A similar pattern as in the case of the IS shock coefficient emerges although the size of the coefficient of the foreign output shock is smaller in absolute size.

Table *1* also provides precise information about the relative size of the coefficients on all the other shocks in the optimal reaction function. Consider first the coefficient on the cost-push shock $(\hat{u}_t)$. Of all disturbances of the model, the cost-push shock prompts the largest response of the policy instrument. It occurs when persistence is high $(\rho = 0.8)$ and an exchange rate channel does not exist $(b = 0)$. In the absence of an exchange rate channel in the Phillips Curve, the policymaker can reduce steady upward pressure on the rate of inflation only by creating a negative output gap. But this requires a pronounced increase in the setting of the policy instrument. There is a discernible pattern in the size of the coefficient on the cost-push shock for moderately high and high levels of persistence of the disturbances $(\rho = 0.6 \ \ and \ \ \rho = 0.8)$: as the magnitude of *b* increases the size of the coefficient decreases. This steady pattern in the size of the coefficient on $\hat{u}_t$ is absent at lower degrees of persistence.

For the shocks that impact on the UIP relation there is a consistent pattern except for the case of high persistence $(\rho = 0.8)$ and a potent exchange rate channel $(b = 1)$. As both *b* and ρ increase in size, the absolute size of the coefficient on $\hat{R}_t^f, \hat{\varepsilon}_t, and\ \hat{\pi}_t^f$ increases.

Finally, we take note of the fact that in the absence of an exchange rate channel (*b=0*), the response of the policymaker to IS, UIP, and foreign interest rate shocks becomes independent of the degree of persistence of the disturbances. In this case the coefficients on the shocks in question reduce to the following expressions:

$$v_t: \qquad \frac{1}{A}$$

$$R_t^f + \varepsilon_t: \qquad 1 - \frac{a_1}{A} \qquad \text{where } A = (1-\gamma)a_1 + a_2 \; A = (1-\gamma)a_1 + a_2$$

This result is also evident in the top row of each panel in Table *1*. The respective coefficient on the three shocks remains constant even though the degree of persistence varies from a low of *0.1* to a high of *.0.8*. If $b = 0$, the policymaker merely needs to adjust the setting of the policy instrument to offset the effect of the IS, UIP, and foreign interest rate shock on real output without having to worry about the resulting change in the real exchange rate affecting the current rate of inflation or its expectation. Without a contemporaneous real exchange rate effect on inflation in the Phillips Curve, stabilizing the output gap also ensures stabilizing the rate of inflation.[25] The movements required in the interest rate to offset these shocks vary depending on source of the shocks. The response (in absolute terms) of the interest rate to IS shocks is always smaller for $b > 0$ than $b = 0$. In contrast, the response of the interest rate to shocks in the UIP relation becomes greater for $b > 0$ than $b = 0$. Thus, shocks that affect conditions in the market for foreign exchange elicit a greater response of the policy instrument if an exchange rate channel in the Phillips Curve is operative. The greater policy response occurs because both target variables are hit directly by $\hat{R}_t^f$, $\hat{\varepsilon}_t$, and $\hat{\pi}_t^f$ if $b > 0$.

V. The Variability of the Endogenous Variables and the Policy Instrument

The final section of this paper assesses the implications of the existence of an exchange rate channel in the Phillips Curve <u>and</u> the presence of autocorrelated disturbances for the variability of the policy instrument and the endogenous variables of the model.

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[25] In contrast, the persistence parameter ρ affects the size of the coefficient on the other demand-side shocks. Consider the foreign output shock in the reaction function. From Equation (4) it follows that $E_t y_{t+1}^f = \rho y_t^f$. Hence the degree of persistence of the shocks matters as $y_t^f - E_t y_{t+1}^f = (1-\rho) y_t^f$ affects the output gap in the IS relation. The degree of persistence also impacts on the response coefficient on foreign inflation in the reaction function.

The model is simple enough that we can derive analytical solutions for the variances of the endogenous variables and the policy instrument. Equations (12) and (13) give the variances of the two variables that appear in the policymaker's objective function:[26]

$$V(y_t) = \frac{Z^2}{1-\rho^2}\left[\frac{\sigma_{\hat{v}}^2 + a_1^2(\sigma_{\hat{R}^f}^2 + \sigma_{\hat{\varepsilon}}^2 + \rho^2\sigma_{\hat{\pi}^f}^2)}{(1-\rho)^2} + \frac{A^2}{b^2}\sigma_{\hat{u}}^2 + a_3^2\sigma_{\hat{y}^f}^2\right] \quad (12)$$

$$V(\pi_t) = \frac{(\theta Z)^2}{1-\rho^2}\left[\frac{\sigma_{\hat{v}}^2 + a_1^2(\sigma_{\hat{R}^f}^2 + \sigma_{\hat{\varepsilon}}^2 + \rho^2\sigma_{\hat{\pi}^f}^2)}{(1-\rho)^2} + \frac{A^2}{b^2}\sigma_{\hat{u}}^2 + a_3^2\sigma_{\hat{y}^f}^2\right] \quad (13)$$

where $Z = \frac{b}{b + A(\theta(1-\rho) + a)}$ $\qquad \theta = \frac{1}{\mu[a + \frac{b}{A}]}$ $\qquad A = (1-\gamma)a_1 + a_2$

Again the importance of *b* in the propagation of demand-side disturbances is apparent. The variances of real output and inflation depend only on the variance of cost-push shocks if $b=0$.

To generate numerical values for the variances, we use the same values for the parameters of the model as in the previous section. In addition, we assume that each shock that impinges upon the economy has a variance of unity.

The standard deviations of the domestic rate of inflation, real output, the real exchange rate, the rate of CPI inflation, and the nominal rate of interest appear in Table *2*. The table consists of three panels, each of which is based on a particular assumption about the strength of the exchange rate channel in the Phillips Curve. Each row gives the value of the standard deviation for a particular degree of persistence of the disturbances. The degree of persistence varies from a low of 0.1 to a high of 0.9.

The top panel lists the standard deviations of the variables of the model for the case where an exchange rate channel in the Phillips Curve does not exist $(b = 0)$. The panel in the center represents the case where the effect of the exchange rate on domestic inflation is moderately strong $(b = 0.25)$ while the bottom panel presents the

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[26] The variance of the policy instrument follows directly from Equation (11). The variance of the real exchange rate can be obtained by combining the optimal reaction function with the UIP relation. Due to computational complexities it is very difficult to derive an analytical solution for the variance of the CPI. The variances of the variables were calculated with the help of a numerical solution algorithm developed by Dennis (2001).

standard deviations of the variables for the case where the exchange rate effect is fairly potent $(b = 1)$.

Figures *3 -7* illustrate in graphical form the behavior of the endogenous variables of the model and the policy instrument in the face of increasing persistence of the shocks and varying strengths of an operative exchange rate channel in the Phillips Curve.

Leaving aside a few minor exceptions to be commented on shortly, we find that the information contained in Table *2* and the graphs suggest a recognizable pattern in the behavior of the variables. As the potency of the exchange rate channel in the Phillips Curve increases:

- the variability of the rate of domestic inflation decreases
- the variability of real output increases
- the variability of the real exchange rate decreases
- the variability of CPI inflation decreases
- the variability of the nominal interest rate decreases.

There is a straightforward explanation for why the variability of inflation and real output move in opposite directions as the magnitude of *b* increases. Consider again the optimizing condition (Equation (10)). As the exchange rate effect in the Phillips Curve becomes larger, the policymaker increases the relative weight on the domestic rate of inflation in setting policy. The more sensitive the response of the rate of domestic inflation is to a shift in the real exchange rate, the smaller the necessary change in the setting of the policy instrument. The more pronounced this exchange rate effect in the Phillips Curve is, the less the policymaker has to rely on a change in the output gap to affect the rate of inflation. A necessary consequence of the shift in emphasis is that the policymaker is less willing to tolerate swings in the rate of inflation but more willing to accept larger fluctuations in real output.

The presence of the real exchange rate in the Phillips Curve thus expands the role of the real exchange rate in the transmission mechanism of monetary policy. Any policy change now results in a change in the real exchange rate which in turn affects domestic inflation contemporaneously. Given this, one should expect fluctuations in the real exchange rate and the policy instrument to be lower the greater the sensitivity of domestic inflation to the real exchange rate in the Phillips Curve. Inspection of columns *4* and *6* of Table *2* tends to confirm this conjecture. As the size of b increases the standard deviation of the real exchange rate decreases markedly, in particular as b rises from 0.25 to 1. The standard deviation of the nominal interest rate also decreases for a given degree of persistence as b increases except in the case where $b = 0.25$, $\rho = 0.9$.

Indeed, if an exchange rate channel in the Phillips Curve is moderately effective, i.e. $b = 0.25$, and the degree of persistence of the stochastic disturbances is high, i.e. $\rho \geq 0.8$, the standard deviation of the rate of inflation (both domestic inflation and CPI inflation) is greater than if an exchange rate channel is either absent $(b = 0)$ or fairly potent $(b = 1)$.

This puzzling result requires an explanation. Three different factors are instrumental in determining the standard deviation of inflation. They are: b, θ, and $(1 - \rho)$. Shocks arising in the goods market or in the foreign exchange market affect the rate of inflation only if $b > 0$. At the same time, however, the existence of an exchange rate channel in the Phillips Curve more than offsets the effect on inflation due to demand-side disturbances because it mitigates considerably the effect of a cost-push shock on the rate of inflation. Moreover, with $b > 0$ the weight on the rate of inflation in setting policy increases. This reduces the variability of the rate of inflation. The third factor $(1 - \rho)$ appears in the denominator of the coefficients on $\hat{v}_t$, $\hat{R}_t^f$, $\hat{\varepsilon}_t$, and $\hat{\pi}_t^f$ and causes the effect of these shocks on the rate of inflation to snowball at high levels of persistence in the disturbances. As b rises from 0 to 0.25 this snowballing effect dominates at high levels of persistence the simultaneous increase of the weight on inflation in the policy rule and the improved stabilizing property of policy in the wake of a cost-push shock. The dominance of the snowballing effect at $\rho = 0.9$ when $b = 0.25$ also accounts for the rise in the variability of the nominal interest rate relative to the case where $b = 0$ or $b = 1$.

VI. Conclusion

This paper underscores the important role of the real exchange rate in the conduct of optimal monetary policy in a small open economy. The importance of the real exchange rate in the policy process arises from its key role in the setting of domestic prices. In small open economies, domestic price setters take the world price of the internationally traded consumption good as a benchmark price. A depreciation of the domestic currency causes domestic firms to raise their prices, thereby forcing up the domestic rate of inflation. The existence of an exchange rate channel in the Phillips Curve dramatically alters the conduct of optimal policy in the open economy framework. The optimizing condition that characterizes the systematic relationship between the target variables depends on parameters of both the IS and the Phillips Curve as well as the policymaker's preferences. The optimizing condition derived in this paper is thus very different from the one that defines optimal policy in a closed economy or a standard open economy where an exchange rate channel is not operative in the Phillips Curve. Only the policymaker's preferences and the sensitivity of the domestic rate of inflation to the output gap in the Phillips Curve are involved in the determination of the optimal policy parameter in the closed economy or standard open economy model.

This paper also highlights the importance of the degree of persistence of the stochastic disturbances. If paired with a potent exchange rate channel in the Phillips Curve, a high degree of persistence may explain why in the open economy framework the setting of the policy instrument responds less forcefully to an IS disturbance or a foreign output disturbance than in the absence of this exchange rate channel. Indeed, the degree of persistence and the size of the parameter on the real exchange rate in the Phillips Curve determine whether the policymaker raises or lowers the nominal interest rate in response to a positive IS or foreign output shock. The possibility of a "lean with the wind" policy response is inconceivable in the closed economy or standard open economy model where the Phillips Curve does not feature an exchange rate channel.

The examination of the sensitivity of the policy instrument and the other variables of the model to varying degrees of persistence and varying strengths of a distinct exchange rate channel reveals that the exchange rate effect on inflation in the Phillips Curve acts generally as a stabilizing force for all variables except the real output gap. For a given degree of persistence, a more potent exchange rate channel in the Phillips Curve helps mitigate fluctuations in the domestic rate of inflation, the rate of CPI inflation, the real exchange rate, and the nominal interest rate but tends to increase the variability of the real output gap.

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Appendix:

This appendix provides a step-by-step derivation of the open economy IS relation and the open economy Phillips Curve.[27]

The IS Relation

Consumers maximize a lifetime utility function that depends on the consumption level of the domestically produced final good and an imported final good.
The period utility function takes the following form:

$$U(C_t^h, C_t^f) = \frac{C(C_t^h, C_t^f)^{1-\frac{1}{\sigma}} - 1}{1-\sigma} \qquad (1)$$

where $\sigma > 0$ is the intertemporal elasticity of substitution and C measures aggregate consumption while C_t^h and C_t^f measure the quantity of the domestic and foreign consumption good, respectively.

From the standard *intertemporal* utility maximization problem, the following first-order condition obtains (lower case letter denotes deviation from steady state value):

$$c_t = E_t c_{t+1} - \sigma(R_t - E_t \pi_{t+1}^{CPI}) \qquad (2)$$

where c_t denotes aggregate consumption and $R_t - E_t \pi_{t+1}^{CPI}$ denotes the real rate of interest, defined as the difference between the nominal rate of interest and the CPI rate of inflation.

The *intratemporal* first-order condition yields the following relationship: the demand for the domestic consumption good is proportional to aggregate consumption and depends inversely on its relative price:

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[27] The contributions by McCallum and Nelson (1997), Gali and Monnacelli (1999), Svensson (2000), and Clarida, Gali, and Gertler (2001, 2002) represent earlier attempts to model optimizing behavior by economic agents in the open economy in ways that are directly comparable to the microfoundations established in this section. The derivation of the IS curve in the current model is in parts similar to that of Svensson (2000).

$$c_t^h = -\eta(p_t^h - p_t^{CPI}) + c_t \tag{3}$$

η measures the elasticity of substitution between the domestic and the foreign consumption good. p_t^{CPI} and p_t^h are defined as the consumer price index and the price of the domestic consumption good, respectively.

With η taken to equal unity, the consumer price index can be written as a weighted average of the price of the domestic and the imported foreign consumption good, respectively:

$$p_t^{CPI} = (1-\gamma)p_t^h + \gamma(p_t^f + s_t)\,. \tag{4}$$

p_t^f represents the price of the foreign consumption good, s_t is the spot exchange rate at time *t*, defined as the units of domestic currency required to buy one unit of foreign currency, and γ denotes the weight of the price of the foreign good in the CPI.

Substituting (4) into (3) yields the following expression:

$$c_t^h = \eta\gamma q_t + c_t \tag{5}$$

where q_t represents the real exchange rate and is defined as $q_t = p_t^f + s_t - p_t^h$.

The next step consists of substituting (5) into (2):

$$c_t^h - \eta\gamma q_t = E_t c_{t+1}^h - \eta\gamma E_t q_{t+1} - \sigma(R_t - E_t \pi_{t+1}^{CPI}) \tag{6}$$

Expressing the resource constraint as a log-linearized equation around the steady state levels yields:

$$y_t = (1-\gamma)c_t^h + \gamma c_t^{hf} \tag{7}$$

where y_t is the real output gap and c_t^{hf} is foreign consumption of domestic goods, i.e. domestic exports.

Foreign demand for the domestic consumption good evolves in accordance with equation (8):

$$c_t^{hf} = c_t^f + \eta^f \gamma^f q_t \tag{8}$$

Foreign consumption is proportional to foreign real output, i.e. $c_t^f = \beta^f y_t^f$. Hence (8) can be written as:

$$c_t^{hf} = \beta^f y_t^f + \eta^f \gamma^f q_t \tag{9}$$

Updating and taking expectations of the resource constraint (Equation (7)) yields:

$$E_t y_{t+1} = (1-\gamma) E_t c_{t+1}^h + \gamma E_t c_{t+1}^{hf} \tag{10}$$

After solving for $E_t c_{t+1}^h$, we can restate the above equation as follows:

$$\frac{E_t y_{t+1} - \gamma E_t c_{t+1}^{hf}}{1-\gamma} = E_t c_{t+1}^h \tag{10'}$$

Next, substitute (10') into (6):

$$c_t^h = \frac{E_t\, y_{t+1} - \gamma E_t c_{t+1}^{hf}}{1-\gamma} + \gamma\eta(q_t - E_t q_{t+1}) - \sigma(R_t - E_t \pi_{t+1}^{CPI}) \tag{11}$$

Expression (11) can then be substituted back into expression (7):

$$y_t = E_t y_{t+1} + (1-\gamma)\left[\gamma\eta(q_t - E_t q_{t+1}) - \sigma(R_t - E_t \pi_{t+1}^{CPI})\right] + \gamma(c_t^{hf} - E_t c_{t+1}^{hf}) \tag{12}$$

Making use of equation (9), we can restate equation (12) as:

$$y_t = E_t y_{t+1} - (1-\gamma)\sigma(R_t - E_t \pi_{t+1}^{CPI}) + \gamma[(1-\gamma)\eta + \eta^f \gamma^f](q_t - E_t q_{t+1}) + \gamma\beta^f (y_t^f - E_t y_{t+1}^f)$$
(13)

or

$$y_t = E_t y_{t+1} - a_1(R_t - E_t \pi_{t+1}^{CPI}) + a_2(q_t - E_t q_{t+1}) + a_3(y_t^f - E_t y_{t+1}^f) \tag{14}$$

where $a_1 = (1-\gamma)\sigma > 0$

$a_2 = \gamma[(1-\gamma)\eta + \eta^f \gamma^f] > 0$

$a_3 = \gamma\beta^f > 0$

The numerical values reported in footnote 24 for the three parameters above are based on the following values for the deep parameters (taken from Svensson (2000)):

$$\sigma = 0.5 \qquad \eta = 1 \quad \eta^f = 2 \qquad \gamma^f = 0.15 \qquad \beta^f = 0.9$$

Equation (14) represents the open economy IS relation. The forward-looking characteristic of aggregate demand is self-evident: current real output depends not only on the current real exchange rate and current foreign real output but also on their expected values next period. More specifically, the difference between real output in the current period and expected real output in the next period depends on the difference between the real exchange rate in the current period and the expected real exchange rate in the next period. Exactly the same pattern governs the response of real output to variations in foreign real output. The standard real interest rate channel is defined in terms of expected CPI inflation.

It is common to interpret γ as reflecting the degree of openness of the economy. All structural coefficients of the IS equation are thus very sensitive to the degree of openness of the economy.

Phillips Curve

Monopolistically competitive firms aim to minimize menu costs weighed against the cost of being away from the optimal price they would charge in the absence of those menu costs. This optimal price is denoted p^{OPT}. The objective function faced by the typical firm is:

$$\min_P \Omega_t = E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[\left(p_\tau - p_\tau^{OPT}\right)^2 + c\left(p_\tau - p_{\tau-1}\right)^2 \right] \tag{15}$$

where:[28]

Ω_t = the total cost at time t

p_t = the natural logarithm of the price of the domestic good at time t

p^{OPT} = the logarithm of the optimal price a firm charges

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[28] For convenience we drop the superscript ("*h*") on domestic prices.

δ = the constant discount factor

c = the parameter that measures the ratio of the costs of changing prices to the costs of deviating from the optimal price

E_t = the expectations operator conditional on information available at time *t*.

After taking and rearranging the first-order condition for the above cost-minimization problem (where we have assumed δ to equal one for simplicity), we can characterize the relationship between past, current, and future price levels as:

$$p_t - p_{t-1} = E_t\left(p_{t+1} - p_t\right) - \frac{1}{c}\left(p_t - p_t^{OPT}\right) \tag{16}$$

The optimal price p^{OPT} is:

$$p_t^{OPT} = \hat{p}_t + \kappa y_t + \varsigma_t \qquad \kappa > 0 \tag{17}$$

where all variables are as previously defined. In addition:

$\hat{p}_t$ = the logarithm of the price charged by foreign firms at time *t*

ς_t = a stochastic disturbance.

The optimal price responds to changes in marginal cost. But marginal cost and real output are positively related.[29] Hence it is innocuous to replace marginal cost with the output gap in (17).

So far our analysis of price-setting behavior has been very much in the spirit of the closed economy "New Keynesian Framework". In a small open economy, however, the price-setting behavior of domestic firms also takes into consideration developments abroad. Being a small player in world markets, the typical firm is guided in its pricing decision by the prevailing conditions in world markets.[30] More specifically, there exists a benchmark price $\hat{p}_t$ that the firm faces in world markets.

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[29] Within a general equilibrium framework, the comovement between marginal cost and economic activity can be established by combining the labor supply and demand relations with the market clearing condition in the goods market. On this point see Clarida, Gali, and Gertler (2001, 2002) or Gali and Monacelli (1999) who derive a similar relation that stresses the positive relation between real marginal cost and domestic consumption. The positive link between output and marginal cost is also characteristic of earlier models of monopolistic competition such as Blanchard and Kiyotaki (1987). The link features also prominently in Mankiw and Reis (2002) who propose an alternative Phillips curve that is based on slow dissemination of information.

[30] In a general equilibrium setting, for the pricing decision of domestic firms to be sensitive to the prevailing price charged by foreign competitors, it is necessary to drop the assumption of constant elasticity of substitution in the utility function. Bergin and Feenstra (2000) and Taylor (2000) show how a *translog* specification for preferences or a *linear* demand relation yields an optimal pricing rule that responds to competitors' prices *in addition* to marginal cost. Equation (17) embodies this idea.

This benchmark price affects the optimal price charged by the firm. Indeed, the firm adjusts its optimal price in line with the domestic currency price of the final goods charged by its foreign competitors. Thus $\hat{p}_t$ becomes:

$$\hat{p}_t = p_t^f + s_t \tag{18}$$

where

p_t^f = the logarithm of the price of the foreign good in foreign currency at time *t*

Using this specification for p_t^{OPT}, we can rewrite equation (16) as:

$$p_t - p_{t-1} - E_t(p_{t+1} - p_t) = -\frac{1}{c}(p_t - p_t^f - s_t - \varsigma_t) + \frac{\kappa}{c} y_t \tag{19a}$$

Equation (19a) illustrates why the *level* of the real exchange rate (defined below) affects the *rate* of inflation. The benchmark price set in world markets affects only the optimal price but not the menu cost component of total cost.

If aggregated over all firms, equation (19a) represents a Phillips Curve relation for an open economy. The same equation can also be expressed as:

$$\pi_t = E_t \pi_{t+1} + a y_t + b q_t + u_t \tag{19b}$$

where

$\pi_t = p_t - p_{t-1}$

$E_t \pi_{t+1} = E_t p_{t+1} - p_t$

$q_t = p_t^f + s_t - p_t$

$a = \frac{\kappa}{c}$

$b = \frac{1}{c}$

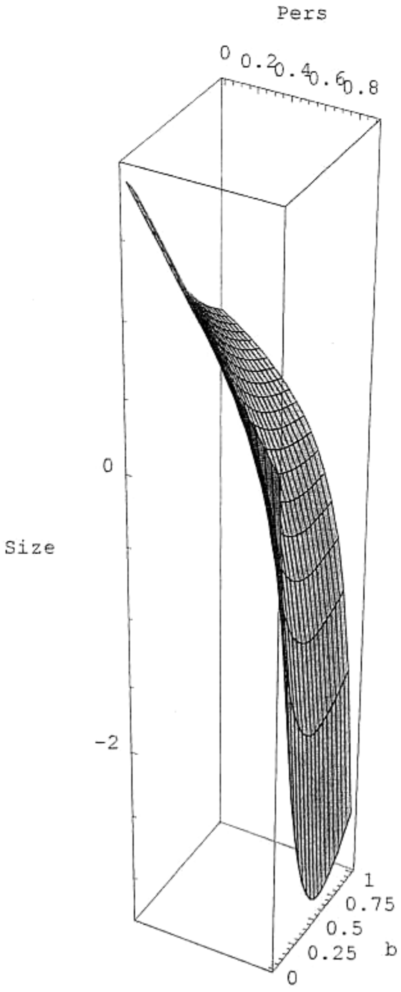

Figure 1 : The Coefficient on the IS Disturbance in the Reaction Function.
gamma = 0.3

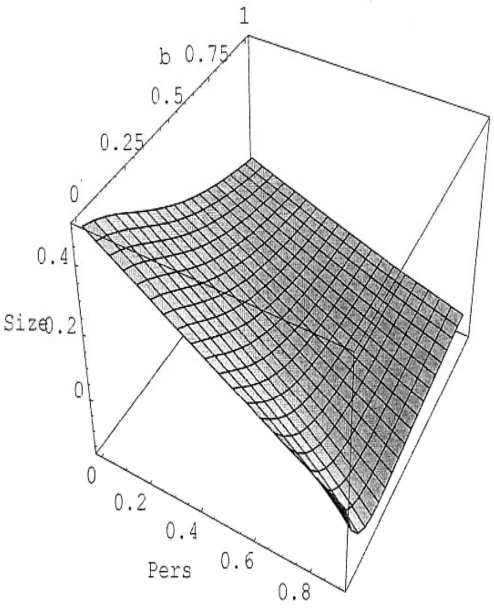

Figure 2 : The Coefficient on the Foreign Output Shock in the Reaction Function.
gamma = 0.3

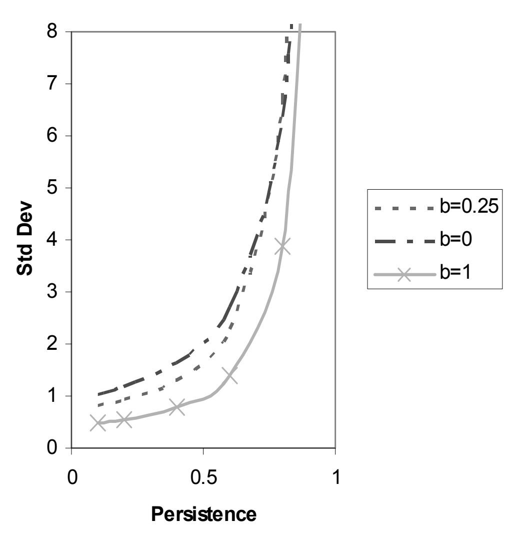

Figure 3: Inflation

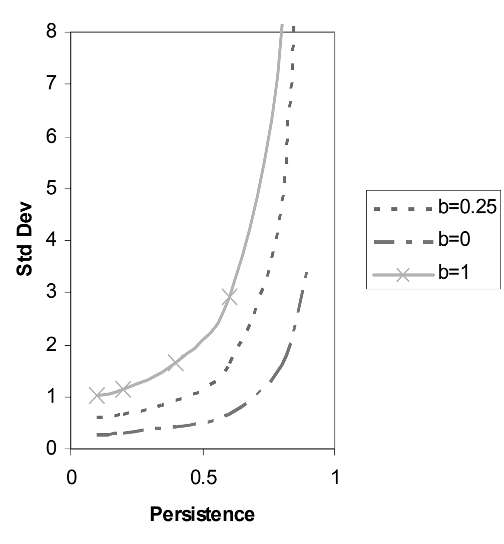

Figure 4: Real Output

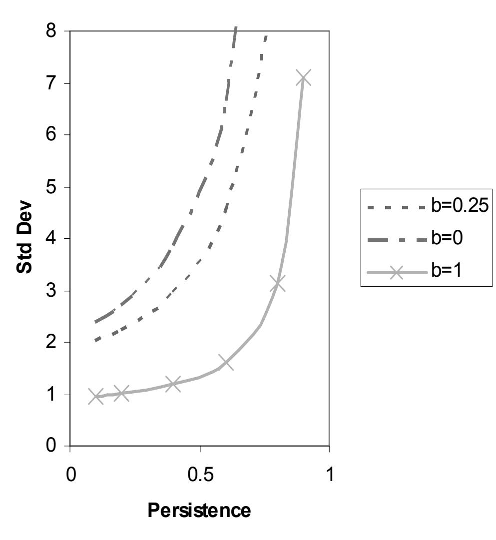

Figure 5: Real Exchange Rate

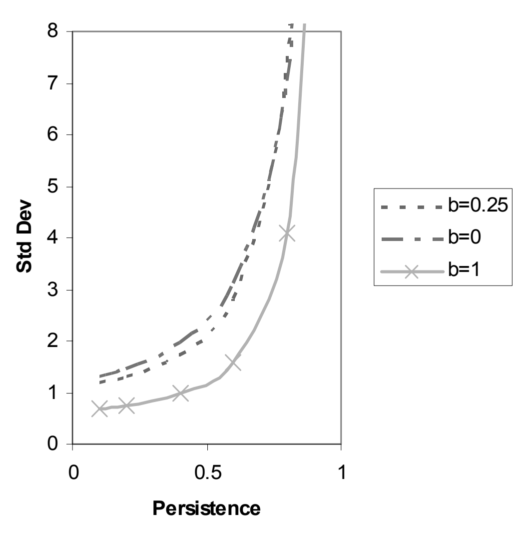

Figure 6: CPI Inflation

Figure 7: Nominal Interest Rate

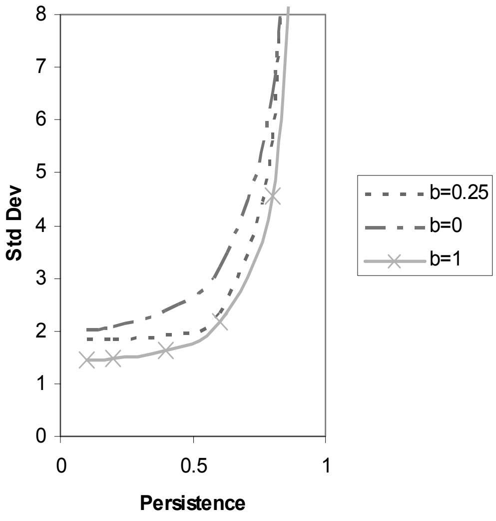

Table 1:

$\rho = 0.1$					
	$\hat{u}_t$	$\hat{v}_t$	$\hat{R}_t^f + \hat{\varepsilon}_t$	$\hat{\pi}_t^f$	$\hat{y}_t^f$
b=0	0.532	1.834	0.357	-0.035	0.445
b=0.25	0.905	1.373	0.519	-0.051	0.333
b=0.5	0.896	0.921	0.677	-0.067	0.223
b=1	0.675	0.458	0.839	-0.083	0.111
	$\rho = 0.4$				
	$\hat{u}_t$	$\hat{v}_t$	$\hat{R}_t^f + \hat{\varepsilon}_t$	$\hat{\pi}_t^f$	$\hat{y}_t^f$
b=0	1.019	1.834	0.357	-0.143	0.297
b=0.25	1.070	1.016	0.644	-0.257	0.164
b=0.5	0.858	0.522	0.817	-0.326	0.084
b=1	0.544	0.168	0.940	-0.376	0.027
	$\rho = 0.6$				
	$\hat{u}_t$	$\hat{v}_t$	$\hat{R}_t^f + \hat{\varepsilon}_t$	$\hat{\pi}_t^f$	$\hat{y}_t^f$
b=0	1.694	1.834	0.357	-0.214	0.198
b=0.25	1.241	0.411	0.856	-0.513	0.044
b=0.5	0.826	-0.060	1.021	-0.612	-0.006
b=1	0.448	-0.223	1.078	-0.646	-0.024
	$\rho = 0.8$				
	$\hat{u}_t$	$\hat{v}_t$	$\hat{R}_t^f + \hat{\varepsilon}_t$	$\hat{\pi}_t^f$	$\hat{y}_t^f$
b=0	3.397	1.834	0.357	-0.286	0.099
b=0.25	1.509	-1.627	1.569	-1.255	-0.087
b=0.5	0.785	-1.770	1.619	-1.295	-0.095
b=1	0.344	-1.323	1.463	-1.170	-0.071

Table 2:

b=0					
ρ	Standard Deviations				
	Inflation	Real Output	Real Exch. Rate	CPI Inflation	Nominal Interest Rate
0.1	1.044030	0.260768	2.389351	1.320984	2.036663
0.2	1.183215	0.294957	2.720110	1.468672	2.095709
0.4	1.646815	0.411096	3.865488	1.969263	2.383065
0.6	2.702591	0.675277	6.656049	3.112876	3.205776
0.8	6.349173	1.587135	17.84866	7.040028	6.509531
0.9	14.11789	3.529447	49.24571	15.41045	14.06936
b=0.25					
ρ	Standard Deviations				
	Inflation	Real Output	Real Exch. Rate	CPI Inflation	Nominal Interest Rate
0.1	0.828854	0.587367	2.025339	1.181524	1.842009
0.2	0.935948	0.663324	2.244548	1.307287	1.842823
0.4	1.311106	0.928977	2.966647	1.742985	1.921978
0.6	2.280789	1.616477	4.554667	2.813538	2.319051
0.8	6.702835	4.750368	9.948416	7.397769	5.636576
0.9	20.61882	14.61287	23.00649	21.46737	18.65778
b=1					
ρ	Standard Deviation				
	Inflation	Real Output	Real Exch. Rate	CPI Inflation	Nominal Interest Rate
0.1	0.485489	1.011929	0.969020	0.681175	1.454648
0.2	0.554076	1.155855	1.021274	0.744983	1.473431
0.4	0.796869	1.662528	1.204159	0.980816	1.632483
0.6	1.402498	2.924927	1.620185	1.584929	2.163562
0.8	3.890373	8.110882	3.125540	4.089401	4.572199
0.9	10.99072	22.91414	7.099923	11.23098	11.64633