

# Money Demand Functions: Cointegration and Long-Run Stability

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This paper applies conventional tests (Johansen, 1995) and new tests (Chao and Phillips, 1999) for cointegration to long-run money demand functions using Canadian data from 1872 to 1997. If cointegration is found, recently proposed tests by Quintos (1997) for stability of the cointegration rank are carried out. The paper focuses on two spans of data: one span starting in 1872, the other in 1957 or 1968. Annual data are used for the former span, and annual and quarterly data for the latter. The results show that the cointegration rank is not stable.

Keywords: Testing for cointegration; unknown change points; long spans of data.

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# 1. Introduction

The long-run money demand function has been playing an important, though different, role in macroeconomic models of the various schools of thought. A well specified money demand function is central to models of price level determination, including recent models of targeting desirable paths for inflation or interest rates.<sup>1</sup> Friedman's (1956) goal has been to demonstrate that a stable function for money demand exists and that it depends on only a very limited number of variables. Meltzer (1963), Laidler (1966), Lucas (1988), and many others have followed the same line of research. In the most simple form, all variables are in natural logarithms, except for possibly the interest rate. Money demand is set equal to money supply and real money balances are a linear function of a short or long term nominal interest rate and a measure of real income or wealth.

The stability of money demand functions has been a concern for some time.<sup>2</sup> For the last decade, empirical researchers have applied mostly cointegration techniques to uncover a stable money demand relation in the long-run.<sup>3</sup> However, the finding of cointegration does not imply that the relation is stable over time.

Stock and Watson (1993) have applied several methods of estimating cointegrating vectors to U.S. money demand functions over the period 1900 to 1988 and tested for parameter stability. They have considered a semi-logarithmic M1 money demand function with real GNP as the scale variable and various short and long term interest rates in turn to measure opportunity costs of holding money. They have concluded that a long span of data is necessary in order to estimate long-run money

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<sup>1</sup>See, for example, McCallum and Nelson (1999) for an optimizing IS-LM type model with a traditional LM function.

<sup>2</sup>See the reviews by Judd and Scadding (1982), and Goldfeld and Sichel (1990) of studies focussing on short-run stability.

<sup>3</sup>A few examples of early applications are Johansen and Juselius (1990), Hafer and Jensen (1991), Hendry and Ericsson (1991) with a reply by Friedman and Schwartz (1991), and Hoffman and Rasche (1991). See also Laidler (1993, Chapter 4) and Ericsson (1998) for surveys of other empirical research and Woodford (1998) who recently has suggested to abandon money demand functions altogether.

demand functions precisely. For the post WWII period, a lack of low frequency variation in the U.S. data does not give precise estimates and results are sensitive to sub-sample specifications. Ball (1998) has revisited Stock and Watson's study with a data set extended to 1996. He has used the same methods of estimation and testing and has found precise estimates and a stable cointegrating relation for M1 for the U.S. post war period.<sup>4</sup>

In contrast to Stock and Watson (1993) and to Ball (1998), Hoffman et al. (1995) and Bordo et al. (1997) have not assumed a known break data when testing stability of the money demand function.<sup>5</sup> Hoffman et al. and Bordo et al. have tested for the stability of the number of cointegrating vectors, which is the cointegration rank, and in addition for stability of the cointegrating parameters.<sup>6</sup> However, they held constant the short run dynamics of the money demand function for the rank stability tests, despite the well established instability of the short run money demand function in the literature.<sup>7</sup>

This paper applies the stability tests with unknown change point of Quintos (1997) to the money demand function in the framework of the vector error-correction model. The short run dynamics are not held constant for these tests. Quintos (1997, 1998) has developed a complete framework for testing for cointegration rank stability and for cointegrating parameter stability.<sup>8</sup> Quintos (1997) has derived the asymptotic

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<sup>4</sup>On the other hand, Miyao (1996) who has questioned a stable cointegrating relationship for M2. See also Friedman and Kuttner (1992) and Estrella and Mishkin (1997). In contrast, Mulligan (1997) has used instead U.S. firm-level longitudinal data for an analysis of money demand at the micro level.

<sup>5</sup>Hoffman et al. and Bordo et al. have used the vector error-correction model of Johansen (1995). Furthermore, Bordo et al. have included additional variables for the long-run money demand function to capture institutional change. I will use these variables in some of my specifications too.

<sup>6</sup>Haug and Lucas (1996) have also tested for parameter stability of money demand when the change point is unknown but did not test rank stability.

<sup>7</sup>See Ball (1998, p. 7).

<sup>8</sup>Hoffman et al. and Bordo et al. have not applied the Quintos framework and used a somewhat ad hoc approach instead. In addition, Hoffman et al. included break dummies in the error-correction model so that standard asymptotic critical values that they used are not quite appropriate.

properties of the tests and has shown that a necessary first step is to test for rank stability before proceeding to parameter stability testing in order to avoid biased results. Quintos (1997) also has demonstrated with a Monte Carlo experiment that her tests have good size and power properties in finite samples.

In addition, this paper compares the performance of Johansen's (1995) tests for cointegration to a recently proposed alternative method. Johansen's sequential tests for cointegration rank lead to overestimation of the number of cointegrating vectors even in the limit. They also require to first specify the number of lags in the vector error-correction model. I therefore apply as an alternative a new information criterion proposed by Chao and Phillips (1999). This criterion avoids the problem of overestimating the rank and allows to determine the number of lags in the model and the cointegration rank at the same time. Chao and Phillips have demonstrated in a Monte Carlo study that their criterion performs well in small samples.

For the empirical analysis in this paper, I employ a new data set with a long span back to 1872. The new data are from Metcalf et al. (1998). They constructed measures of money that take the current Bank of Canada definitions of the monetary base, M1, and M2 back in time to mid 1871. The basic semi-logarithmic money demand specification used linearly relates the natural logarithm of real money balances to the natural logarithm of real GNP and to the level of a long term interest rate. Various alternative specifications are explored, including some specifications with recently developed new measures of money.

Section 2 outlines the econometric methods used. Section 3 describes the data and Section 4 reports the empirical results. Section 5 concludes.

## **2. Econometric Methodology**

### **2.1 Unit Roots and Cointegration**

Every time-series that enters the money demand function is tested for one and two unit roots. The augmented Dickey-Fuller and Phillips-Perron tests are applied.

As a first step, Johansen's (1995) maximum likelihood based method is used to test for cointegration and to estimate the cointegrating vectors. Following Hoffman et al. (1995), among others, I allow for linear deterministic time trends in the levels vector moving-average representation of the model, which in turn implies an unrestricted constant and no deterministic time trends in the vector error-correction model (VECM) specification. Allowing for deterministic trends in the data in levels is appropriate given growth and technological change.

Chao and Phillips (1999) have drawn attention to a problem with Johansen's method of performing sequential tests to determine the cointegration rank. Johansen's (1992) Theorem 2 shows that the probability of overestimating the rank remains positive in the limit and therefore the cointegration rank is not estimated consistently with the sequential procedure. Furthermore, the VECM of Johansen requires in general to choose an appropriate lag order and results can be sensitive to lag misspecification.<sup>9</sup>

Chao and Phillips have proposed to apply the Posterior Information Criterion (PIC) of Phillips and Ploberger (1996) to VECMs as an alternative to Johansen's method. This criterion allows to determine the VECM lag order and the cointegration rank jointly and leads to consistent estimation of both. The cointegrating vectors can then be consistently estimated by Johansen's method after imposing the lag order and cointegration rank obtained with the PIC. I briefly outline the framework in which the PIC is applied, following Chao and Phillips.

The VECM is given by

$$\Delta Y_t = J^*(L)\Delta Y_{t-1} + J_*Y_{t-1} + \varepsilon_t.$$

$Y_t$  is a vector of dimension  $m$  and

$$J(L) = \sum_{i=1}^{p+1} J_i L^{i-1}$$

so that

$$J_* = J(1) - I_m$$

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<sup>9</sup>See Toda and Phillips (1994).

and

$$J^*(L) = \sum_{i=1}^p J_i^* L^{i-1}$$

with

$$J_i^* = - \sum_{l=i+1}^{p+1} J_l, \quad i = 1, \dots, p.$$

Further,  $J_* = \Gamma_r A_r'$  with loading vectors  $\Gamma_r$  and the cointegrating vectors  $A_r$ , each a matrix of dimension  $m \times r$  with full column rank  $r$  and  $0 \leq r \leq m$ . The cointegration rank is given by  $r$  and the columns of  $A_r$  contain the  $r$  cointegrating vectors. When  $r = 0$ , then  $\Gamma_0 = A_0 = 0$ . When  $r = m$ , then  $\Gamma_m = J_*$  and  $A_m = I_m$ . These special cases deliver a vector autoregression in first differences and in levels, respectively. The interest here is with the cases in between. A normalization to identify the cointegrating vectors in  $A_r$  is defined by  $A_r' = [I_r, \bar{A}_r']$ .

The lag order  $p$  and the cointegration rank  $r$  are selected by  $(\hat{p}, \hat{r}) = \arg \min \text{PIC}(p, r)$  and

$$\text{PIC}(p, r) = \ln |\hat{\Omega}_{p,r}| + [m^2 p + 2r(m-r) + mr] T^{-1} \ln T,$$

where  $T$  is sample size. The PIC attaches to the parameters  $r(m-r)$  of the cointegrating matrix twice the penalty than it does to parameters of stationary regressors. In contrast to Johansen's method, the PIC imposes a penalty on overparameterization in order to correct for upward bias of  $r$ . The residual covariance matrix

$$\hat{\Omega}_{p,r} = \left[ (\Delta Y - Y_{-1} \hat{J}_*(p, r)')' M_{W(p)} (\Delta Y - Y_{-1} \hat{J}_*(p, r)') \right] T^{-1}$$

with

$$\hat{J}_*(p, r) = \left( \hat{\Gamma}(p, r), \hat{\Gamma}(p, r) \hat{\bar{A}}(p, r)' \right),$$

where  $\hat{\Gamma}(p, r)$  and  $\hat{\bar{A}}(p, r)$  are the maximum likelihood estimators of  $\Gamma$  and  $\bar{A}$  when the cointegration rank is assumed to be  $r$  and the lag order is assumed to be  $p$ , assuming that  $\varepsilon_t$  is iid  $N(0, \Sigma)$ . Further,

$$M_{W(p)} = I_T - W(p)(W(p)'W(p))^{-1}W(p)'$$

with

$$W(p) = [W_1(p), \dots, W_T(p)]'$$

and

$$W_t(p) = [\Delta Y'_{t-1}, \dots, \Delta Y'_{t-p}]'$$

Chao and Phillips have shown how the *PIC* is based on Bayesian as well as classical principles. They have also proved weak consistency of the *PIC*.

## 2.2 Structural Change: Rank Stability Tests

I first test for the stability of the cointegration rank  $r$  of  $J_* = \Gamma_r A'_r$ , following Quintos (1997). The null hypothesis is that the rank  $r$  stays constant over the full sample:

$$H_0^r : \text{rank}(J_*^{(t)}) = r.$$

That means that the number of cointegrating vectors does not increase or decrease from some point in time on.

The test statistic depends on the form that the alternative hypothesis takes. I first consider the case of more cointegrating vectors:

$$H_1^r : \text{rank}(J_*^{(t)}) > r.$$

Quintos has suggested a likelihood ratio test based on the fully-modified vector autoregressive estimation procedure of Phillips (1995). However, she has shown in a subsequent paper (Quintos, 1998) that the likelihood ratio test for this estimator is degenerate when there are no cointegrating vectors in the system. I therefore apply instead Johansen's method to estimate the eigenvalues for the rank stability test. The test is defined by the sup of the following likelihood ratio statistic:

$$\sup_{\kappa \in \Phi} Q_T^+(\kappa) = \sup_{\kappa \in \Phi} [\kappa T] \sum_{i=r+1}^m \hat{\lambda}_i^{([\kappa T])},$$

with  $\Phi = [.15, .85]$  as suggested by Andrews (1993).<sup>10</sup> The asymptotic distribution

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<sup>10</sup>For a few small samples in the post WWII period, I used  $\Phi = [.2, .8]$  and  $[\.25, .75]$  instead, in order to achieve convergence.

depends on whether or not a constant is included in the VECM and on what restrictions, if any, are placed on that constant. The critical values from MacKinnon et al. (1999) apply to the above test statistic.

The  $\widehat{\lambda}_i^{(\cdot)}$  are the roots of the matrix

$$S_{\Delta Z \Delta Z}^{(\cdot)\frac{1}{2}} S_{\Delta Z Z}^{(\cdot)} S_{ZZ}^{(\cdot)-1} S_{Z \Delta Z}^{(\cdot)} S_{\Delta Z \Delta Z}^{(\cdot)\frac{1}{2}}. \quad (1)$$

The  $r$  largest eigenvectors of this reduced rank regression are the estimators of  $\widehat{A}^{(\cdot)}$ . Reduced rank estimation is equivalent to maximum likelihood. Further, for example for  $T$ ,

$$\begin{aligned} S_{\Delta Z \Delta Z}^{(T)} &= T^{-1} \Delta Z^{(T)'} \Delta Z^{(T)} \\ S_{\Delta Z Z}^{(T)} &= T^{-1} \Delta Z^{(T)'} Z_{-1}^{(T)} \\ S_{ZZ}^{(T)} &= T^{-1} Z_{-1}^{(T)'} Z_{-1}^{(T)}. \end{aligned}$$

$Z_{-1}^{(t)} = (z_0, \dots, z_t)'$  and  $\Delta Z^{(t)}$  are similarly defined.  $\Delta z_t$  and  $z_{t-1}$  are the residuals of a regression of  $\Delta Y_t$  on  $(\Delta Y_{t-1}, \dots, \Delta Y_{t-p})$  and of  $Y_{t-1}$  on the same set of regressors.<sup>11</sup>

Next, I consider the other alternative hypothesis of less cointegrating vectors than  $r$ :

$$\widetilde{H}_1^r : \text{rank}(J_*^{(t)}) < r.$$

At time  $t$  the estimated residuals are

$$\widehat{U}^{(t)'} = \Delta Z^{(t)'} - J_* Z_{-1}^{(t)}$$

and they are standardized to

$$\widehat{E}^{(t)'} = \widehat{\Omega}_{00}^{-\frac{1}{2}} \widehat{U}^{(t)'},$$

where  $\Omega_{00}$  is a sub-matrix of the long-run covariance matrix  $\Omega$ . I use a quadratic kernel along with an automatic data-based bandwidth selection method as suggested by Andrews (1991). Also, I pre-whiten following Andrews and Monahan (1992).<sup>12</sup>

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<sup>11</sup>For ease of exposition, the VECM is here specified without any constant terms.

<sup>12</sup>Using instead a Parzen window with four lags does not affect the results significantly. See also Phillips (1995).

Define

$$G_t = \widehat{A}_r^{(T)'} \widehat{E}^{(t)'},$$

where  $\widehat{A}_r^{(T)}$  are the  $r$  largest eigenvectors of equation (1). The Lagrange multiplier test statistic is given by

$$\overline{Q}_T = T^{-2} tr \left\{ \sum_{t=1}^T G_t G_t' \right\}.$$

The asymptotic distribution is non-standard and depends on  $r$ . Critical values were simulated by Quintos (1997) and results are given in her Table 1.

After testing for cointegration rank stability, the next step would be to test for parameter stability. Quintos (1997) has derived the asymptotic distribution of the fluctuations test of Ploberger et al. (1989) for partially nonstationary vector autoregressions. However, I do not find empirical support for rank stability in Section 4 so that tests for parameter stability are not warranted.

### 3. The Data

The annual data cover the period from 1872 to 1997. This was the largest data span available at the time this research was started. The money measures are M0 (the monetary base), M1, and M2. M1 includes currency in the hands of the public and demand deposits held by the public and provincial governments, net of float. M2 includes M1 net plus personal savings deposits and non-personal (chequing and non-chequing) notice deposits. These three measures of money have been constructed by Metcalf et al. (1998) and are historical extensions of the current Bank of Canada definitions. Details on approximations and other problems in data construction are discussed in Metcalf et al. Their monthly data cover the years 1872 to 1967. I averaged the monthly data to arrive at annual figures. Data on these money measures from 1968 to 1997 are from Statistics Canada's (March 1999) CANSIM data base, with B2055, B2033, and B2031 corresponding to M0, M1, and M2, which are the current Bank of Canada definitions.

The annual data for gross national product (GNP) and the GNP deflator for the years 1872 to 1925 are from Urquhart (1986). These series are from Statistics Canada (1975) for the years 1926 to 1960, Catalogue 13-531. For the years 1961 to 1997, the GNP series is from CANSIM, D16441, and the GDP deflator series is also from CANSIM, D205566 up to 1985 with base year 1981, and D19296 from 1986 to 1997 with base year 1992. These GDP-deflators are linked to the GNP deflator constructed by Urquhart.

The long term interest rate was kindly supplied by Pierre Siklos to cover the period 1872 to 1985. Data for the period 1986 to 1997 are from CANSIM, B14013 and are consistent with Siklos' series. The series is the annual average of Government of Canada long term bond yields of over ten years. In addition, I use data from Siklos (1993) to measure institutional change over the period 1900 to 1986 that will be explained in Section 4.

The corresponding quarterly data are from the same sources. The quarterly data cover the period from 1957 to 1997. In addition to the long term interest rate, the 3 month T-bill rate, CANSIM series B14007, is also considered, following Stock and Watson (1993) and others in considering a short and a long term interest rate.

The Bank of Canada has recently calculated new measures of the money stock to include similar deposits outside chartered banks and other relatively liquid funds not captured by M1 and M2: M1++ and M2++. These are more comprehensive measures of money from the 1970s on than M1 and M2 and are therefore considered in the empirical tests because results might be sensitive to the choice of the money measure.<sup>13</sup> These series are available from 1968 on (quarterly and seasonally adjusted) and are from CANSIM (June 1999), series B1652 and B1650, respectively. They are analyzed below in addition to the other money measures. M1++ consists of M1 plus all notice deposits. M2++ is M2 plus the sum of deposits at Trust and Mortgage Loan Companies, at Credit Unions, at Caisses Populaires, Canada Savings Bonds,

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<sup>13</sup>See, for example, Haug and Lucas (1996).

and all mutual funds.<sup>14</sup>

Seasonal adjustment, when necessary, is carried out with the weighted average (multiplicative) method in EViews 3.1. All unit root and cointegration tests, including PIC, are performed with EViews. The Quintos procedures are performed in GAUSS for Windows.

## 4. Empirical Results

First, I test the annual (1872-1997) and quarterly (1957-1997 and 1968-1997) time-series for one and possibly two unit roots with the augmented Dickey Fuller test using Akaike's criterion to select the appropriate lag lengths. I also apply the Phillips Perron unit root test with the Newey West correction as implemented in EViews 3.1. I allow in turn for a constant and also for a deterministic time trend in the test regressions. Results are available from the author on request. All money measures and GNP are in real terms and natural logarithms:  $\ln(rM0)$ ,  $\ln(rM1)$ ,  $\ln(rM2)$ ,  $\ln(rM1++)$ ,  $\ln(rM2++)$ , and  $\ln(rGNP)$ . The nominal long term interest rate is specified in natural logarithms,  $\ln(ltir)$ , and alternatively also in levels,  $ltir$ . The same applies for the short term interest rate, the 3 month T-bill rate. For all variables, the null hypothesis of a unit root cannot be rejected, whereas the null hypothesis of two unit roots is rejected, using a 5% level of significance.

The basic money demand relation, for example for M0, takes the following form:

$$\ln(rM0) - \alpha - \beta \ln(rGNP) - \gamma ltir = u_t.$$

I explore various money demand specifications that have been used by previous researchers.<sup>15</sup> I replace  $\ln(rM0)$  in turn by the other money measures. Similarly, I replace the interest rate by  $\ln(ltir)$  and also by the T-bill rate and by  $\ln(\text{T-bill rate})$  and include only one interest rate at a time.

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<sup>14</sup>See the Bank of Canada Review, January 1998, *Notes to the Tables*.

<sup>15</sup>See, for example, Hoffman and Rasche (1991), Stock and Watson (1993), and Miyao (1996).

Next, I set up the VECM using the Schwarz criterion for lag order selection, considering up to six lags in the annual data and up to eight lags in the quarterly post WWII data. The VECM is specified with an unrestricted constant. Johansen's (1995) maximum likelihood based method is used as a first step for testing for the number of cointegrating vectors with the trace test. The P values are calculated with a program available from MacKinnon et al. (1999).

The real money measures  $\ln(rM0)$ ,  $\ln(rM1)$ , and  $\ln(rM2)$  are considered, in turn, with the annual data for the period 1872 to 1997. The interest rate is specified in logarithms, however, all test results remain unchanged when the long term interest rate in levels,  $ltir$ , is used instead of  $\ln(ltir)$ . The only other variable included in the VECM is  $\ln(rGNP)$ . The VECM models with M0 and alternatively with M2 as the measure of money do not lead to a rejection of the null hypothesis of no cointegration. I therefore find no evidence for cointegration when M0 or M2 are measuring the money stock and the Johansen method is applied.

In contrast, the VECM model with M1 as the measure of money leads to the finding that there is one cointegrating vector in the system, even at a quite low level of significance. Results are reported in Table 1. The cointegrating vector estimate gives an income (GNP) elasticity of 1.04 and an interest elasticity of -.96. I tested the hypothesis that the income elasticity is not significantly different from 1 and could not reject it, using a Wald test.<sup>16</sup> This finding has important implication because it allows for a specification in terms of velocity,  $\ln(GNP/M1)$ . Of course, such a specification may not be valid if the M1 money demand relationship turns out to be unstable.

The Johansen method might possibly overestimate the cointegration rank and there could be no cointegration at all for M1.<sup>17</sup> This point was discussed in the section on econometric methodology. To explore this possibility, I apply the PIC of Chao

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<sup>16</sup>See Johansen (1995).

<sup>17</sup>Although, it is useful as a first step because it gives some idea in which neighborhood  $p$  and  $r$  may be.

and Phillips in order to choose the lag order of the VECM and the cointegration rank jointly, and to assure consistent estimation of both. The PIC rejects cointegration for the models with M0 and M2 as measures of money and therefore confirms the above Johansen results. For the model with M1, the PIC takes on a value of -16.6 for  $p = 0$  and  $r = 1$  and the same value for  $p = 0$  and  $r = 0$ . According to this criterion, there may or may not be cointegration. I will count in the evidence from the Johansen test and side with  $r = 1$ , i.e., that there is one cointegrating vector in the VECM.

The next step is to test for stability of the rank  $r = 1$  of the cointegrating relationship for M1 over the period 1872 to 1997. I first apply the  $\sup_{\kappa \in \Phi} Q_T^+(\kappa)$  test with the null hypothesis that the rank  $r$  is constant at 1 over the whole sample period. The alternative hypothesis is that the rank is greater than 1 for some  $\kappa \in \Phi$ . The test statistic takes on a value of 54.3. The 1% critical value from MacKinnon et al. is 19.93 and the P value for this statistic is less than .0001. The null hypothesis of a constant cointegration rank over the full sample is therefore decisively rejected in favor of the alternative hypothesis that there is more than one cointegrating vector for some  $\kappa$ .

The test for stability of the rank  $r = 1$  against the alternative hypothesis that  $r < 1$  leads to a test statistic  $\overline{Q}_T$  of .82. According to the critical values presented in Quintos (1997, p.302) in her Table 1, the null hypothesis of rank constancy against the alternative of a rank less than 1 cannot be rejected even at a 10% level of significance.

The long span of data does not lead to a relationship with constant rank for the models as specified so far. I therefore carry out a sensitivity analysis. First, I restrict the coefficient of income to one and reapply the above tests for M1. Qualitative results are unchanged. Second, I start the sample in 1914 instead of in 1872. Again, I find with the PIC (and the Johansen trace test) one cointegrating vector for M1. However, the rank is not stable. The  $\sup_{\kappa \in \Phi} Q_T^+(\kappa)$  equals 38.3. On the other hand,  $\overline{Q}_T$  equals 1.16 and therefore indicates no instability at usual significance levels.

Third, I include in the money demand specification a variable to measure the difference between the own yield of money and the market interest rate, as sug-

gested by Friedman and Schwartz (1982, pp. 270–271). This variable is defined as  $\ln[\text{tir} \times (\text{high powered money}/\text{total money})]$ . Tests detect for M1 one cointegrating vector but the rank is again unstable ( $\sup_{\kappa \in \Phi} Q_T^+(\kappa) = 57.9$ , whereas  $\overline{Q}_T = .34$ ).

Fourth, Siklos (1993) has argued for a money demand specification that includes variables to capture institutional and technological change in order to account for instabilities in the money demand function.<sup>18</sup> In particular, he has found two variables useful in addition to income and interest rates. One is the ratio of nonbank financial assets to total financial assets. It is supposed to capture financial sophistication. The other variable is the ratio of currency to money which is supposed to mirror the spread of commercial banking. I use M2 velocity and real per capita permanent income in addition to these two variables so that the specification is identical to that of Siklos. All these data are from Siklos and cover the period 1900 to 1986. The results with the Johansen tests and the PIC support the finding of one cointegrating vector in this model, as found by Siklos. However, the Quintos test rejects stability for both rank stability tests ( $\sup_{\kappa \in \Phi} Q_T^+(\kappa) = 183.4$  and  $\overline{Q}_T = 398.8$ ).

The overall finding with the long span of data is that the cointegration rank is not stable. I therefore analyze the post WWII data for stability. I take first annual data from 1957 to 1997 and repeat all the above test for this shorter span. M0 and M2 lead to cointegrating relations ( $r = 2$  and  $r = 1$ , respectively), using the PIC. However, the rank is not stable either in either direction ( $\sup_{\kappa \in \Phi} Q_T^+(\kappa)$  is equal to 66.5 and 42.4, and  $\overline{Q}_T$  is equal to 7.0 and 41.1). M1 leads to a relation with two cointegrating vectors. The Schwarz criterion picks one lag, however the PIC chooses no lags instead. The PIC is minimized for  $r = 2$  and  $p = 0$ . The  $\sup_{\kappa \in \Phi} Q_T^+(\kappa)$  is equal to 51.5 and  $\overline{Q}_T$  to 15.7 in this case and the null hypotheses are rejected for both tests. In summary, the annual data from 1957 to 1997 do not support rank constancy no matter which of the three measures of money is used.

Quarterly data are available for the period 1957 to 1997. I repeat the above analysis with quarterly data using the long term interest rate, and additionally the

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<sup>18</sup>See also Mulligan (1997).

T-bill rate. For M0 and M2, the PIC rejects cointegration when the long term interest rate is used, even though the Johansen trace test detects one cointegrating vector in each model. M1 with the long term interest rate leads to one cointegrating vector, using the PIC. The Johansen results are given in Table 2. But, the rank is not stable ( $\sup_{\kappa \in \Phi} Q_T^+(\kappa) = 76.5$ ; on the other hand,  $\bar{Q}_T = 1.0$  ).

The results with respect to stability do not change once the long term interest rate is replaced by the short term T-bill rate. The PIC detects one cointegrating vector for M0, M1, and M2 each. Though, rank stability is not achieved either. The  $\sup_{\kappa \in \Phi} Q_T^+(\kappa)$  statistic takes on values of 79.1, 110.1, and 64.1, and  $\bar{Q}_T = .81, 11.8,$  and 17.8, respectively.

It is of interest to study whether the recently published M1++ and M2++ measures of money improve results. These series are available from 1968 on. The quarterly models with these money measures, using the long term interest rate, do not lead to cointegration according to the PIC. Johansen's trace test would have detected one cointegrating vector in each model. Using instead the T-bill rate leads to no cointegration with the PIC for M1++. Johansen's trace test would have suggested  $r = 1$  and produced a wrong sign for the coefficient on the T-bill rate. The PIC supports one cointegrating vector for M2++ using the T-bill rate. However, the  $\sup_{\kappa \in \Phi} Q_T^+(\kappa)$  and  $\bar{Q}_T$  statistics produce highly significant values of 62.6 and 217.0. Therefore, the new measures of money do not lead to rank stability either.

Several empirical studies of money demand have included inflation as an additional explanatory variable.<sup>19</sup> I therefore include inflation in addition to income and the long term interest rate. For quarterly data, I try again the short term T-bill rate as an alternative to the long term rate. Inflation is supposed to measure the yield on non-financial assets (goods) as an alternative to holding money. I test for a unit root in the inflation rate. The augmented Dickey Fuller and the Phillips Perron tests reject a unit root at the 5% level for the long span of data from 1872 or 1914 to 1997. The same holds true for annual and quarterly post WWII data as far as

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<sup>19</sup>See, e.g., Ericsson (1998).

the Phillips Perron test is concerned. The augmented Dickey Fuller test does not reject a unit root for post WWII data. Even though the unit root evidence is not conclusive, I carry out the other tests for the post WWII period. The PIC detects cointegration in the annual data from 1957 to 1997 only for M2. Johansen's sequential tests instead detect cointegration in addition for M0 and M1. The cointegrating relationship for M2 is not stable (the  $\sup_{\kappa \in \Phi} Q_T^+(\kappa) = 93.5$  and  $\overline{Q}_T = 62.5$ ). The PIC detects cointegration for M0, M1, and M2 in the quarterly data, regardless of whether the long ( $r=1, 3,$  and  $1,$  respectively) or short term interest rate ( $r=2, 3,$  and  $3,$  respectively) is used. However, rank stability is again rejected in all cases.<sup>20</sup> Using M1++ or M2++ instead for the period of availability (1968:1-1997:4) does not lead to stability either. The PIC detects one cointegrating vector in each case, regardless of the interest rate used. Rank stability is rejected in all cases in both directions. Furthermore, all quarterly data produce an incorrect coefficient sign for either the interest rate or the (annualized) inflation rate, with the exception of M2++.

The empirical findings in this Section show that the PIC of Chao and Phillips gives results that are in line with theoretical predictions that Johansen's sequential tests possibly overestimate the cointegration rank. This finding is particularly prevalent in shorter spans of data like the post WWII period, for annual and for quarterly data. Longer spans of data do not produce differences between the PIC and Johansen's sequential method as far as the cointegration rank is concerned. In addition, the PIC chooses in most cases a lag order for the vector error-correction model below that of the Schwarz criterion for many short as well as for some long spans.

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<sup>20</sup> $\sup_{\kappa \in \Phi} Q_T^+(\kappa)$  is equal to 137.5, 158.0, and 95.5 for the long rate, and to 121.1, 46.3, and 76.1 for the short rate.  $\overline{Q}_T$  is equal to 5.6, 221.8, and 10.3 for the long rate, and to 12.1, 46.3, and 76.1 for the short rate.

## 5. Conclusion

This paper applies new econometric techniques to various alternative money demand specifications commonly used in the cointegration literature. A recently constructed data set of Metcalf et al. (1998) for three Canadian money measures is used, going back to 1872. The evidence suggests that various tests detect cointegrating relationships within a vector error–correction money demand model, however, these cointegrating relationships are not stable over time. The tests show that the problem is with the stability of the cointegration rank, independent of the data span, be it 1872 to 1997, 1957 to 1997, or 1968 to 1997. Furthermore, whether annual or quarterly post WWII observations are used is immaterial to the stability results, as is the measure of money used.

The concepts of weak, strong, and super exogeneity in the context of vector error–correction models play an important role for policy analysis. Money demand functions could possibly be inverted to obtain prices or interest rates as a function of money. However, all these interpretations hinge on a stable relationship in the first place. If a stable cointegration rank exists, testing for the various forms of exogeneity can proceed and policy implications be derived. Also, if the rank is stable, cointegrating parameters can be tested for stability and other restrictions.

This paper questions previous empirical studies on money demand within the vector error–correction framework. A stable cointegration rank is necessary before other inference is carried out. The standard cointegration framework might be too restrictive of a concept to capture the true time series behavior of money demand. Fisher and Seater (1993) have used an ARIMA approach instead to test for long–run neutrality of money. Another promising route could be to allow for nonlinear smooth transition (STR) vector error–correction processes of Granger and Teräsvirta (1993) that possibly would capture the instability found in the linear vector error–correction models. Lütkepohl et al. (1999) have recently applied single equation STR models to German money demand. It would be necessary to find a parsimonious approach

to make nonlinear multivariate models practical.

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TABLE 1: Johansen Trace Test for Cointegration and Cointegrating Vector Estimate

Sample: 1872 1997

Included observations: 125

Series: Ln(rM1) Ln(rGNP) Ln(Itir)

Lags interval: No lags

| Eigenvalue | Likelihood Ratio | 5 Percent Critical Value | P Value | Hypothesized No. of Vectors |
|------------|------------------|--------------------------|---------|-----------------------------|
| 0.21       | 37.18            | 29.80                    | 0.0059  | None                        |
| 0.06       | 7.39             | 15.49                    | 0.53    | At most 1                   |
| 0.001      | 0.16             | 3.84                     | 0.69    | At most 2                   |

Normalized:

| Ln(rM1)        | Ln(rGNP) | Ln(Itir) | Constant |
|----------------|----------|----------|----------|
| 1              | -1.04    | 0.96     | 5.40     |
| Log likelihood | 519.48   |          |          |

TABLE 2: Johansen Trace Test and Cointegrating Vector Estimate

Sample: 1957:1 1997:4

Included observations: 162

Series: Ln(rM1) Ln(rGNP) Ln(Itir)

Lags interval: 1 to 1

| Eigenvalue | Likelihood Ratio | 5 Percent Critical Value | P Value | Hypothesized No. of Vectors |
|------------|------------------|--------------------------|---------|-----------------------------|
| 0.11       | 32.12            | 29.80                    | 0.03    | None                        |
| 0.07       | 12.44            | 15.49                    | 0.14    | At most 1                   |
| 0.0009     | 0.14             | 3.84                     | 0.71    | At most 2                   |

Normalized:

| Ln(rM1)        | Ln(rGNP) | Ln(Itir) | Constant |
|----------------|----------|----------|----------|
| 1              | -1.24    | 1.25     | 2.90     |
| Log likelihood | 1205.50  |          |          |