PAYBACK AND THE VALUE OF WAITING TO INVEST

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2 July 1997

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Despite being rejected by finance theory, payback continues to be widely used as a method for evaluating capital investment projects. In situations where investment can be delayed, we show that the value of waiting to invest is an increasing function of payback period. Consequently, the optimal investment policy is equivalent to requiring that a project with positive net-present-value be launched immediately if and only if its payback period is less than a critical value P*.

1. Introduction

Surveys of corporate capital budgeting practice indicate that payback is a widely-used method of project evaluation. For example, Gilbert and Reichert (1995), Gitman and Forrester (1977), Oblak and Helm (1980) and Stanley and Block (1984) find that between 40% and 75% of U.S. firms use payback as a capital budgeting technique. Jog and Srivastava (1995), McMahon (1981), Patterson (1989), and Shao and Shao (1993) report similar findings for non-U.S. firms.

Such continued popularity is puzzling insofar as payback has long been soundly rejected by finance theory.[1] The notion that a project's acceptability can be determined by its time to payback has been criticised for ignoring the time value of money and for neglecting project cashflows subsequent to payback. By contrast, discounted cashflow methods such as net-present-value and internal-rate-of-return have been shown to provide decision rules that are consistent with the maximization of shareholder value and these methods have therefore received greater acceptance by theorists.

More recently however, the standard discounted cashflow rules have themselves been shown to be deficient if investment can be delayed. Consider, for example, the standard net-

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* For helpful comments, we are grateful to Jim Peterson and to seminar participants at Otago and Canterbury. Any remaining errors are our responsibility.

1 Authors such as Chaney (1989), Narayana (1985), and Weingartner (1969) have argued that the use of payback can be explained by various aspects of the shareholder-manager agency conflict. However, such factors seem unlikely to be sufficiently ubiquitous to account for the widespread use of payback.

present-value rule which states that a project should be launched if and only if net-present-value V is greater than zero. It is now widely recognised that such a rule implicitly assumes that the project is either fully reversible or a now-or-never proposition. If neither assumption holds, then the optimal investment policy is given by a modified net-present-value rule: A project should be launched if and only if $V \geq V^* \geq 0$. The critical point V^* represents the opportunity cost of installing the project and thereby forgoing the option to wait and invest at a later date. For this reason, V^* is known as the value of the project's delay option.[2]

In this paper, we reconsider the merits of payback in the context of projects that have irreversible, but delayable, installation costs. Why might payback be of value in this situation? **First, when a project is delayed, all expected cashflows occur later and thus are discounted more heavily. However, this timing cost of delay is lower for projects with high expected cashflow growth. For given net-present-value and discount rate, high growth projects are also long-payback projects, so the net timing costs of delay are lower for projects with long payback. Second, when investment is irreversible and cashflows are stochastic, delaying a project in order to obtain more information helps managers take advantage of favourable movements in market conditions and avoid costly mistakes. However, for a given standard deviation of future cashflows, the dispersion, and therefore the upside potential, of future cashflows is greater for high growth projects, i.e., long-payback projects. The uncertainty benefit of delay is therefore higher for long-payback projects. Thus, all else equal, projects with long payback period have lower costs and higher benefits of delay and therefore are less likely to satisfy the conditions for immediate launching.**

In subsequent sections, we provide a concrete illustration of this intuition within the framework developed by McDonald and Siegel (1986). In section 2, we obtain the optimal investment rule and derive the exact form of the delay option value V^*. In section 3, we first show that V^* is a monotonically increasing function of payback, holding all else constant, and then demonstrate that this implies the existence of a critical payback value P^* such that a project with positive net-present-value should be launched if and only if payback does not exceed P^*. In

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[2] For an excellent non-technical summary of this literature, see Dixit and Pindyck (1995). A more detailed treatment appears in Dixit and Pindyck (1994).

section 4, we obtain an exact solution for P* and derive some simple bounds. Section 5 contains some concluding remarks.

2. The Optimal Investment Policy

As in Capozza and Li (1994, 1996), we consider a project with time t cashflow x_t that evolves according to the geometric Brownian motion:[3]

$$dx_t = \mu x_t dt + \sigma x_t dz_t \qquad (1)$$

where μ is the expected cashflow growth rate, σ is the standard deviation of this growth rate, and dz_t is the increment of a Wiener process.[4] At each time t, the project can either be delayed, or it can be installed in return for the payment of a known sunk cost (which, without loss of generality, we normalize to unity). The investment decision is thus an optimal stopping problem: At what point is it optimal to pay $1 in order to install the project whose cashflows evolve according to (1)?

Standard methods (see Appendix for details) yield the optimal investment policy: Invest immediately if and only if the current cashflow x satisfies

$$x \quad x^* = \frac{\delta(\rho - \mu)}{\delta - 1}, \qquad (2)$$

where:

$$\delta = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) + \sqrt{\frac{2\rho}{\sigma^2} + \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2} \qquad (3)$$

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[3] By contrast, McDonald and Siegel (1986) assume that the *present value* of project cashflow follows a geometric Brownian motion. However, in an infinite horizon framework, a geometric Brownian motion process for cashflow is sufficient for the present value of cashflow to also follow such a process, so all the McDonald and Siegel results also apply in our model.

[4] Our principal results are not dependent on (1). For example, it is straightforward to show that either an arithmetic or a square root Brownian motion process leaves Propositions 1 and 2 unaffected. The same is true if projects have finite lives and a binomial cashflow process. Details are available from the authors.

This rule can be compared with the standard net-present-value rule which states that a project should be launched immediately if and only if net-present-value V is greater than zero. For a project with cashflows that evolve according to (1), the net-present-value if launched immediately is given by:

$$V = \frac{x}{\rho - \mu} - 1$$

Hence, the optimal investment policy (2) is equivalent to the "modified" net-present-value rule: Invest immediately if

$$V \geq V^* = \frac{1}{\delta - 1} \qquad (4)$$

otherwise wait. Since $\delta > 1$, $V^* > 0$, and the optimal investment policy therefore requires not just that the net-present-value V be positive, but also that it be sufficiently positive to exceed V^*.[5]

The investment rule contained in (4) is the well-known result of McDonald and Siegel (1986). In general, delay means that all cashflows occur later and thus are discounted more heavily, thereby reducing any positive net-present-value. However, growth ($\mu > 0$) in expected project cashflows reduces this timing cost of waiting. Moreover, uncertainty about future cashflows ($\sigma > 0$) means that there are benefits from waiting for further information. The quantitative impact of these effects on the investment decision is given by the value V^* of the option to delay. As we shall see, the interaction between these effects and their impact on $V-V^*$ can also be inferred from the length of payback period.

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5 To see that $\delta > 1$, note that $[\frac{2\mu}{\sigma^2} + (\frac{1}{2} - \frac{\mu}{\sigma^2})^2] = (\frac{1}{2} + \frac{\mu}{\sigma^2})^2$. Hence, since $\rho > \mu$,

$$\delta > (\frac{1}{2} - \frac{\mu}{\sigma^2}) + ((\frac{1}{2} + \frac{\mu}{\sigma^2})^2)^{0.5} = 1.$$

3. Net Present Value, Payback, and the Optimal Investment Policy

Corporate managers may frequently be unaware of either the existence of the modified net-present-value rule, or its the appropriate form, despite having an intuitive appreciation of the value provided by being able to delay investment projects. In this section we demonstrate that the modified net-present-value rule (4) is equivalent to a rule of the following form: Install a project with positive net-present-value V if and only if payback period P is less than or equal to a critical value P*.

If project cashflow at the time of installation is x, then the expected cumulative cashflow by time T is

$$E[\int_0^T x_t dt] = x(e^{\mu T} - 1)/\mu$$

The project's payback period P is defined as the T at which the expected cumulative cashflow equals the $1 installation cost. Therefore:

$$P = \frac{1}{\mu} \log(1 + \frac{\mu}{x}) \qquad (5)$$

Development of the relationship between payback and the optimal investment policy is facilitated by the following lemma.

Lemma 1: $\frac{\partial \delta}{\partial \mu} < 0.$

Proof: Differentiating (3) with respect to μ yields:

$$\frac{\partial \delta}{\partial \mu} = \left(\frac{-1}{\sigma^2}\right)\left(1 + \frac{(\frac{1}{2} - \frac{\mu}{\sigma^2})}{\sqrt{\frac{2\rho}{\sigma^2} + (\frac{1}{2} - \frac{\mu}{\sigma^2})^2}}\right)$$

$$= \left(\frac{-1}{\sigma^2}\right)\left(\frac{\left(\frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\frac{2\rho}{\sigma^2} + \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2}\right)}{\sqrt{\frac{2\rho}{\sigma^2} + \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2}}\right)$$

$$= \left(\frac{-\delta}{\sigma^2\sqrt{\frac{2\rho}{\sigma^2} + \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2}}\right)$$

$$< 0 \qquad \text{since } \delta > 0 \qquad \|$$

Our first result clarifies the underlying relationship between payback, the option to delay, and the optimal investment policy.

Proposition 1: Consider projects A and B with the same net-present-value $V > 0$, the same discount rate ρ, and the same cashflow volatility σ. If $P_A < P_B$, then $V_A^* < V_B^*$. Thus, if B should be launched immediately, then A should also be launched immediately.

Proof: Projects with net-present-value V and discount rate ρ are described by parameters (μ,x) satisfying $x = (V + 1)(\rho - \mu)$. Such projects have payback period:

$$P(\mu) = \frac{1}{\mu}\log\left(1 + \frac{\mu}{(V + 1)(\rho - \mu)}\right)$$

Therefore:

$$\frac{\partial P}{\partial \mu} = \frac{-1}{\mu^2}\log\left(1 + \frac{\mu}{(V + 1)(\rho - \mu)}\right) + \frac{\rho}{\mu(\rho - \mu)\{(V + 1)(\rho - \mu) + \mu\}}$$

$$\geq \frac{V}{(V + 1)(\rho - \mu)\{(V + 1)(\rho - \mu) + \mu\}}$$

since $\log(1+y) \leq y$. Hence, $\frac{\partial P}{\partial \mu} > 0$. It follows that if $P_A < P_B$, then $\mu_A < \mu_B$. Therefore, by Lemma 1:

$$\delta_A > \delta_B$$

and, since $V^* = \frac{1}{\delta - 1}$,

$$V_A^* < V_B^*.$$

Therefore, if $V \geq V_B^*$, then $V \geq V_A^*$. Since the optimal investment policy specifies installation if and only if $V\text{-}V^* \geq 0$, it follows that if high-payback project B should be installed, then so should low-payback project A. ||

Proposition 1 indicates that, all else equal, the value V* of the option to delay project installation is an increasing function of payback. This can be understood as follows. In general, a project with short payback generates more of its cashflows "early" (i.e., in the "near" future) than does a project with long payback. In particular, if two infinitely lived projects have the same net-present-value and discount rate, then any difference in payback periods must reflect differences in the time profile of their respective expected cashflows, i.e., the project with the longer payback period must have a lower initial cashflow and higher expected cashflow growth. Delay of a long-payback project therefore entails the sacrifice of low early cashflows in return for high later cashflows, while the reverse is true for a short-payback project. Consequently, the net timing costs of delay are lower for a project with long payback than they are for a project with short payback, all other project characteristics held constant.

This situation is depicted in Figure 1. Projects A and B have the same net-present-value (V = 1) and discount rate (ρ = 0.1), but project A has a shorter payback period than B (5 years and 6.17 years respectively). Project A has higher expected cashflows up to 11.5

years, B thereafter. Delay of these projects effectively moves the vertical axis rightwards. Since A has higher early cashflows than B, the cashflows sacrificed by delay are greater for A. Moreover, since B has higher later cashflows than A, the additional cashflows gained by delay are greater for B. Thus, delay is more beneficial for the long-payback project B than for the short-payback project A.

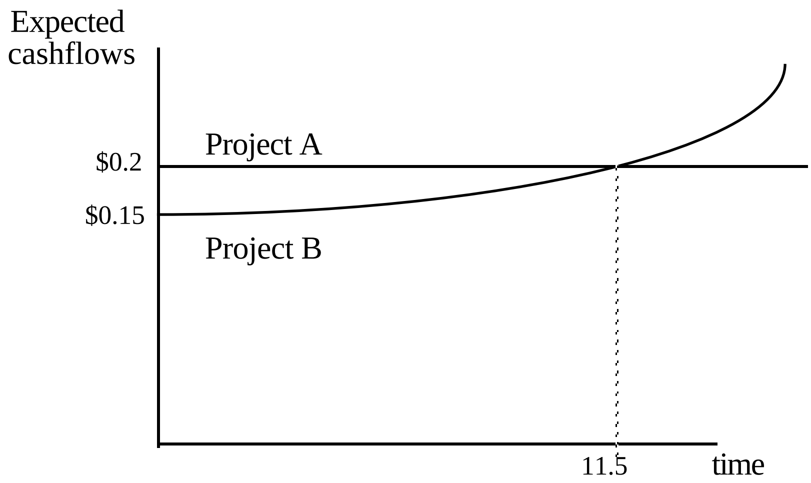

Figure 1
Project A has initial cashflow x = $0.2, expected cashflow growth μ = 0, and payback P = 5. Project B has initial cashflow x = $0.15, expected cashflow growth μ = 0.025, and payback P = 6.17. Both projects have net-present-value V = 1 and discount rate ρ = 0.1.

The length of payback period also influences V* via the uncertainty benefit to waiting. For given instantaneous volatility σ, higher expected cashflow growth increases the dispersion of future cashflow realizations. In particular, the higher the expected cashflow growth, the greater the upside potential for future cashflows and therefore the greater the incentive to delay installation. Since expected cashflow growth is increasing in payback, the uncertainty benefit of waiting is higher for a project with long payback than it is for an otherwise-identical project with short payback.[6]

[6] Another way of thinking about this is to recognise that distant cashflows are more uncertain than near cashflows, so long-payback projects, whose cashlows are more

To summarize, the net timing costs of delay are lower for long-payback projects than for otherwise-identical short-payback projects, and the uncertainty benefits are higher. Thus, the value of the option to delay is greater for long-payback projects than it is for short-payback projects. This implies that, for given net-present-value V, discount rate ρ and cashflow volatility σ, the modified net-present-value V-V* is a decreasing function of payback period. It follows that the optimal investment rule for a given project can be described in terms of payback. This observation is formalized by the following result.

Proposition 2: There exists a function P* such that a project with positive net-present-value V, discount rate ρ and cashflow volatility σ should be launched if and only if its payback period is less than or equal to P*(V, ρ, σ).

Proof: Let P*(V, ρ, σ) be the maximum payback period of all projects which (a) should be launched immediately and (b) have net-present-value V > 0, discount rate ρ and cashflow volatility σ. Consider a project with project characteristics satisfying (b) and payback period P ≤ P*(V, ρ, σ). Since P*(V, ρ, σ) is the payback period of a project that should be launched immediately, Proposition 1 implies that the project with payback period P should also be launched immediately. Now consider a project with project characteristics satisfying (b) and payback period P > P*(V, ρ, σ). Assume that this project should be launched immediately. Proposition 1 then tells us that any project with characteristics satisfying (b) and with payback period less than or equal to P should also be launched immediately. In particular, there exist projects with (a) payback period greater than P*(V, ρ, σ) and (b) net-present-value V > 0, discount rate ρ and cashflow volatility σ, which should be launched, contradicting the definition of P*. Therefore, any project with payback period greater than P*(V, ρ, σ) should be delayed.

The central lesson of Proposition 2 is as follows. If a project has net-present-value V > 0 *and* payback period P ≤ P*, then V exceeds the value V* of the option to delay and the project should be launched immediately. However, if P > P*, then the value of the option to delay

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concentrated in the distant future, have more to gain by waiting in order to obtain more information.

exceeds the net-present-value from installation and the project should be delayed. Thus, a project's suitability for immediate investment can be determined in two steps. First, calculate the standard net-present-value. If this is negative, then the project is rejected for immediate investment. Second, if the standard net-present-value is positive, calculate the payback period. If this is less than a critical value P*, then the project is accepted for immediate investment; otherwise it is rejected.

Our finding that payback can be used to evaluate projects with positive net-present-value corresponds to observed corporate practice. For example, Gitman and Forrester (1977), Jog and Srivastava (1995), Oblak and Helm (1980), Shao and Shao (1993), and Stanley and Block (1984) all report that by far the greatest use of payback is as a secondary or backup criterion to discounted cash flow methods. Our analysis indicates that such behavior is consistent with value-maximizing objectives.

4. Critical payback values

To operationalise the investment procedure identified by Proposition 2, the exact form of the function P* must be specified. In general, this will depend on the assumed project cashflow process. For projects with cashflows evolving according to (1), the solution can be obtained as follows. Projects with net-present-value V and discount rate ρ are described by parameters (μ,x) satisfying $x = (V + 1)(\rho - \mu)$. Such projects have payback period:

$$P(\mu) = \frac{1}{\mu}\log\left(1 + \frac{\mu}{(V + 1)(\rho - \mu)}\right)$$

and, by (4), should be launched if and only if $V \geq \frac{1}{\delta - 1}$, which occurs if and only if $\delta \geq (1 + \frac{1}{V})$. This occurs if and only if

$$\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) + \sqrt{\frac{2\rho}{\sigma^2} + \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2} \;\; \geq \; 1 + \frac{1}{V}$$

in which case μ satisfies

$$\frac{2\rho}{\sigma^2} + \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 \geq \left(\frac{1}{V} + \frac{1}{2} + \frac{\mu}{\sigma^2}\right)^2.$$

This occurs if and only if

$$\mu \leq \frac{\rho V}{V+1} - \frac{\sigma^2}{2V}.$$

Since $P(\mu)$ is increasing in μ (see proof of Proposition 1), it follows that

$$\begin{aligned} P^*(V, \rho, \sigma) &= \max \left\{P(\mu): \mu \leq \frac{\rho V}{V+1} - \frac{\sigma^2}{2V}\right\} \\ &= P\left(\frac{\rho V}{V+1} - \frac{\sigma^2}{2V}\right) \\ &= \left(\frac{2V(V+1)}{2\rho V^2 - \sigma^2(V+1)}\right) \log\left(\frac{2\rho V(2V+1) + \sigma^2 V(V+1)}{2\rho V(V+1) + \sigma^2(V+1)^2}\right) \end{aligned}$$

The solution for P^* is a relatively complex function of V, ρ, and σ. Further insight can be obtained by defining

$$\theta = \frac{2\rho V^2 - \sigma^2(V+1)}{2\rho V(V+1) + \sigma^2(V+1)^2}$$

so that the critical payback period P^* is

$$P^* = \left(\frac{2V}{2\rho V + \sigma^2(V+1)}\right) \frac{\log(1+\theta)}{\theta}. \qquad (6)$$

Suppose that $\theta \geq 0$ Then (see Appendix for proof)

$$\frac{1}{\rho(V+1)} \le P^* \le \frac{2}{\sigma^2}\left(\frac{V}{V+1}\right)^2 \tag{7}$$

Similarly, if $\theta \le 0$, then

$$\frac{1}{\rho(V+1)} \ge P^* \ge \frac{2}{\sigma^2}\left(\frac{V}{V+1}\right)^2 \tag{8}$$

Combining (7) and (8) yields the investment rule

Proposition 3: Consider a project with positive net-present-value V, discount rate ρ and cashflow volatility σ. If this project has payback period less than both

$$\frac{1}{\rho(V+1)} \text{ and } \frac{2}{\sigma^2}\left(\frac{V}{V+1}\right)^2$$

then it should be launched immediately. If this project has payback period greater than both

$$\frac{1}{\rho(V+1)} \text{ and } \frac{2}{\sigma^2}\left(\frac{V}{V+1}\right)^2$$

then it should be delayed.

Although the rule specified in Proposition 3 cannot evaluate all projects, it does provide a significant simplification of the optimal investment policy for a subset of projects. The first critical value $\frac{1}{\rho(V+1)}$ is the payback period for a project with net-present-value V and zero cashflow growth ($\mu = 0$). For such a project, there are no timing benefits of delay, so if cashflow volatility is low, then it should be launched immediately. However, if cashflow volatility is high, then delay can be optimal even for a zero-growth project and a more stringent payback test is required. Specifically, immediate launching requires that payback also be less than the second critical value $\frac{2}{\sigma^2}\left(\frac{V}{V+1}\right)^2$ which is decreasing in cashflow volatility. Proposition 3 therefore states that a project with net-present-value V should definitely be launched if it has faster payback than the corresponding zero-growth project *and* it has low cashflow volatility. Similarly, a project with net-present-value V should definitely be delayed if it has slower payback than the corresponding zero-growth project *and* it has high cashflow volatility. In the former case, timing considerations outweigh uncertainty considerations while the reverse is true in the latter case.[7]

5. Concluding remarks

The findings of Summers (1987) and others that firms require project net-present-values to be significantly greater than zero (or hurdle rates well in excess of the cost of capital) are frequently cited as evidence of managerial awareness of the value of the option to delay. In this paper, we have shown that the equally-puzzling use of payback is also consistent with managerial attempts to incorporate irreversibility and delay in their investment decisions. In general, a project with a relatively short payback period generates a relatively high proportion of its future cashflows early in its economic life. For such a project, the benefits of delay are relatively low while the costs are relatively high. Consequently, the value of a project's option to delay is an increasing function of the project payback period. From this central observation, it follows that any positive net-present-value project should be launched immediately if and only if payback does not exceed a critical value P^*. In other words, a two-stage investment policy using net-present-value and payback sequentially is equivalent to the optimal investment policy. Hence, the widespread use of payback has a rational basis.

Our formal analysis has concentrated on the case where the only source of uncertainty is project cashflows. However, as Ross (1995) has pointed out, interest rate uncertainty represents an even more ubiquitous source of project option value. Moreover, firms operating in an international environment are subject to uncertainty about foreign laws, regulations, and currency values. The intuition for our analysis would suggest that payback also has a role to play in these situations.

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[7] Even if the manager lacks information on the volatility parameter σ, payback can still be used to identify projects that should be delayed. To see this, simply note that P^* is bounded above by $1/\rho$, so any project with positive net-present-value V and discount rate ρ should be delayed if it has payback period greater than $1/\rho$.

Appendix

Proof of (2)

As shown by Dixit and Pindyck (1994, pp 128-130), the optimal investment rule has the general form: Install the project when the current cashflow x is greater than or equal to some critical value x*; otherwise delay. The solution for x* can be obtained as follows. Let R(x; x*) denote the value of the option to invest in the project when the current cashflow is x, i.e., R(x; x*) is the expected present value of the net payoff from installing the project given that installation takes place the first time x is greater than or equal to x*. Note first that if x = 0, then, by (1), all future cashflows are also zero and the option to invest is therefore worthless. Hence:

$$R(0; x^*) = 0 \tag{A1}$$

Second, if x = x*, then the project is launched and the present value of the net payoff is:

$$E\left[\int_0^\infty x_t e^{-\rho t} dt - 1\right] = \int_0^\infty x^* e^{(\mu-\rho)t} dt - 1 = \frac{x^*}{\rho - \mu} - 1 \tag{A2}$$

where $\rho > \mu$ is the discount rate ascribed to the project. Therefore, to preclude arbitrage:

$$R(x^*; x^*) = \frac{x^*}{\rho - \mu} - 1 \tag{A3}$$

For $x \in (0, x^*)$, Bellman's Principle of Optimality implies:

$$E[dR] = \rho R dt \tag{A4}$$

i.e., the total expected return on the investment option E[dR] is exactly equal to the total required return ρR over some time interval dt. Since R is a function of x, Ito's Lemma implies:

$$dR = R_x dx + \frac{1}{2} R_{xx} (dx)^2 \tag{A5}$$

where subscripts indicate partial differentiation with respect to the indicated variable. Substitution of (1) into (A5) yields:

$$dR = (\mu x R_x + \frac{1}{2}\sigma^2 x^2 R_{xx})\, dt + (\sigma x R_x)\, dz_t$$

As E[dz] = 0, (A4) can therefore be rewritten as:

$$\mu x R_x + \frac{1}{2}\sigma^2 x^2 R_{xx} = \rho R \qquad \text{(A6)}$$

which is a second-order homogeneous differential equation. Given the boundary conditions (A1) and (A3), equation (A6) has the unique solution:

$$R(x; x^*) = \left(\frac{x^*}{\rho - \mu} - 1\right)\left(\frac{x}{x^*}\right)^{\delta} \qquad \text{(A7)}$$

where:

$$\delta = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) + \sqrt{\frac{2\rho}{\sigma^2} + \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2} \qquad \text{(A8)}$$

The holder of the opportunity to invest in the project wishes to maximize R(x; x*). Hence x* satisfies the first-order condition:

$$\left(\frac{1-\delta}{\rho-\mu}\right)\left(x^*\right)^{-\delta} + \delta\left(x^*\right)^{-(1+\delta)} = 0$$

Solving this equation for x* yields (2).

Proof of (7) and (8)

Suppose $\theta \geq 0$. Then, by definition:

$$\rho \geq \frac{\sigma^2(V+1)}{2V^2} \qquad \text{(A9)}$$

and, by the properties of the log function

$$\frac{1}{1+\theta} \leq \frac{\log(1+\theta)}{\theta} \leq 1. \qquad \text{(A10)}$$

Hence:

$$\frac{1}{\rho(V+1)} = \frac{V+1}{\rho(V+1)^2}$$

$$= \frac{1}{\rho(\frac{2V+1}{V+1}) + \frac{\rho V^2}{V+1}}$$

$$\leq \frac{1}{\rho(\frac{2V+1}{V+1}) + \frac{\sigma^2}{2}} \qquad \text{by (A9)}$$

$$= \frac{2V(V+1)}{2\rho V(2V+1) + \sigma^2 V(V+1)}$$

$$= \left(\frac{2V}{2\rho V + \sigma^2(V+1)}\right) \frac{1}{1+\theta}$$

$$\leq \left(\frac{2V}{2\rho V + \sigma^2(V+1)}\right) \frac{\log(1+\theta)}{\theta} \qquad \text{by (A10)}$$

$$= P^*$$

$$\leq \frac{2V}{2\rho V + \sigma^2(V+1)} \qquad \text{by (A10)}$$

$$\leq \quad \frac{2}{\sigma^2}\left(\frac{V}{V+1}\right)^2 \qquad \text{by (A9)}$$

This proves (7); the proof of (8) is similar.

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