Stabilising Properties of Discretionary Monetary Policies in a Small Open Economy: Domestic vs CPI Inflation Targets

By

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Abstract:

This paper sets out a simple New Keynesian open-economy model and shows that the conduct of discretionary monetary policy in an open economy differs substantially from the closed-economy framework. The paper shows analytically that the existence of the direct exchange rate channel in the open economy Phillips Curve impairs the perfect stabilizing property of monetary policy in the face of demand-side disturbances under domestic inflation targeting. If CPI inflation is instead the target, then the perfect stabilizing property of monetary policy breaks down even in the absence of the direct exchange rate channel in the Phillips Curve.

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1. Introduction

In small open economies, the pricing decisions of domestic firms are affected by movements in the exchange rate and other developments abroad such as changes in the pricing behaviour of foreign firms. This is well-understood. The challenge is to show how domestic producers react to these changes. This paper introduces an open economy Phillips Curve that is derived on the assumption that domestic firms adjust their optimal product price in line with changes in the domestic currency price of the foreign consumption good. The characterising feature of this open economy Phillips Curve is the presence of the real exchange rate.

The existence of a direct real exchange rate channel in the Phillips Curve has an important consequence for the conduct of discretionary monetary policy in the open economy: the perfect stabilising property of discretionary monetary policy in the wake of demand-side disturbances and exchange rate disturbances under flexible domestic inflation targeting disappears.[1] Both domestic inflation and real output deviate from their respective target in the face of such disturbances because the change in the policy instrument causes a change in the real exchange rate that in turn has a direct effect on domestic inflation. This result contrasts sharply with the stabilising features of optimal policy in more conventional open and closed economy models where the policymaker need merely adjust the setting of the policy instrument to control aggregate demand. These standard models feature a conventional Phillips curve where domestic inflation responds to expected future inflation, the output gap, a cost-push shock but not to the real exchange rate. As a result, the policymaker is unable to perfectly stabilise the economy only in the face of cost-push shocks.[2]

The insight that both demand-side and supply-side parameters determine policy under flexible domestic inflation targeting is developed analytically in a framework consisting of optimising households and firms. We also consider the stabilising property of flexible CPI inflation targeting. Under this strategy, the perfect stabilising property fails to hold in the face of all stochastic disturbances, even if the direct exchange rate channel in the Phillips Curve is not operative.

A comparison of the merits of domestic inflation as opposed to CPI inflation targeting constitutes the final part of the paper. We examine how sensitive the variances of the variables of the model, in particular real output and the two rates of inflation, are to the degree of openness of the economy. Our results indicate that flexible domestic inflation targeting dominates flexible CPI inflation targeting from the standpoint of real output stabilisation. Moreover, greater openness does not materially affect the ability of flexible domestic inflation targeting to stabilise real output. However, fluctuations in the CPI rise sharply under flexible domestic inflation targeting as the economy becomes more open. In contrast, flexible CPI inflation targeting is more attractive than flexible domestic inflation targeting from the standpoint of inflation stabilisation and exchange rate stabilisation. Indeed, flexible CPI inflation targeting affords the policymaker increasingly better control over both rates of inflation and the real exchange rate as the degree of openness increases.

The organisation of the paper is as follows. In Section *2* we derive the forward-looking IS relation and the forward-looking Phillips Curve from an explicit optimisation framework. Section *3* discusses the implications of targeting domestic inflation versus CPI inflation under flexible inflation targeting. In Section *4* we use numerical methods to compare and contrast the advantages and disadvantages of targeting either rate of inflation. Concluding comments appear in Section *5*.

2. The Building Blocks of a Small Open Economy Model

The model proper consists of three equations: an IS relation that explains the behaviour of aggregate demand in the open economy, a Phillips Curve that illustrates the price setting behaviour of monopolistically competitive firms, and a standard uncovered interest rate parity condition. This section provides a step-by-step derivation of the open economy IS relation and the open economy Phillips Curve.[3]

The IS Relation

Consumers maximise a lifetime utility function that depends on the consumption level of the domestically produced final good and an imported final good.

The period utility function takes the following form:

$$U(C_t^h, C_t^f) = \frac{C(C_t^h, C_t^f)^{1-\frac{1}{\sigma}} - 1}{1-\sigma} \tag{1}$$

where $\sigma > 0$ is the intertemporal elasticity of substitution and C measures aggregate consumption while C_t^h and C_t^f measure the quantity of the domestic and foreign consumption good, respectively.

From the standard *intertemporal* utility maximisation problem, the following first-order condition obtains (lower case letter denotes deviation from steady state value):

$$c_t = E_t c_{t+1} - \sigma(R_t - E_t \pi_{t+1}^{CPI}) \tag{2}$$

where c_t denotes aggregate consumption and $R_t - E_t \pi_{t+1}^{CPI}$ denotes the real rate of interest, defined as the difference between the nominal rate of interest and the expected rate of CPI inflation.

The *intratemporal* first-order condition yields the following relationship: the demand for the domestic consumption good is proportional to aggregate consumption and depends inversely on its relative price:

$$c_t^h = -\eta(p_t^h - p_t^{CPI}) + c_t \tag{3}$$

η measures the elasticity of substitution between the domestic and the foreign consumption good. p_t^{CPI} and p_t^h are defined as the consumer price index and the price of the domestic consumption good, respectively.

With η taken to equal unity, the consumer price index can be written as a weighted average of the price of the domestic and the imported foreign consumption good, respectively:

$$p_t^{CPI} = (1-\gamma)p_t^h + \gamma(p_t^f + s_t). \quad (4)$$

p_t^f represents the price of the foreign consumption good, s_t is the spot exchange rate at time t, defined as the units of domestic currency required to buy one unit of foreign currency, and γ denotes the weight of the price of the foreign good in the CPI.

Substituting (4) into (3) yields the following expression:

$$c_t^h = \eta\gamma q_t + c_t \quad (5)$$

where q_t represents the real exchange rate and is defined as $q_t = p_t^f + s_t - p_t^h$.

The next step consists of substituting (5) into (2):

$$c_t^h - \eta\gamma q_t = E_t c_{t+1}^h - \eta\gamma E_t q_{t+1} - \sigma(R_t - E_t \pi_{t+1}^{CPI}) \quad (6)$$

Expressing the resource constraint as a log-linearised equation around the steady state levels yields:

$$y_t = (1-\gamma)c_t^h + \gamma c_t^{hf} \quad (7)$$

where y_t is the real output gap and c_t^{hf} is foreign consumption of domestic goods, i.e. domestic exports.

Foreign demand for the domestic consumption good evolves in accordance with equation (8):

$$c_t^{hf} = c_t^f + \eta^f \gamma^f q_t \quad (8)$$

Foreign consumption is proportional to foreign real output, i.e. $c_t^f = \beta^f y_t^f$. Hence (8) can be written as:

$$c_t^{hf} = \beta^f y_t^f + \eta^f \gamma^f q_t \quad (9)$$

Updating and taking expectations of the resource constraint (Equation (7)) yields:

$$E_t y_{t+1} = (1-\gamma)E_t c_{t+1}^h + \gamma E_t c_{t+1}^{hf} \quad (10)$$

After solving for $E_t c_{t+1}^h$, we can restate the above equation as follows:

$$\frac{E_t y_{t+1} - \gamma E_t c_{t+1}^{hf}}{1-\gamma} = E_t c_{t+1}^h \quad (10')$$

Next, substitute (10') into (6):

$$c_t^h = \frac{E_t\, y_{t+1} - \gamma E_t c_{t+1}^{hf}}{1-\gamma} + \gamma\eta(q_t - E_t q_{t+1}) - \sigma(R_t - E_t \pi_{t+1}^{CPI}) \tag{11}$$

Expression (11) can then be substituted back into expression (7):

$$y_t = E_t y_{t+1} + (1-\gamma)\left[\gamma\eta(q_t - E_t q_{t+1}) - \sigma(R_t - E_t \pi_{t+1}^{CPI})\right] + \gamma(c_t^{hf} - E_t c_{t+1}^{hf}) \tag{12}$$

Making use of equation (9), we can restate equation (12) as:

$$y_t = E_t y_{t+1} + \gamma\left[(1-\gamma)\eta + \eta^f \gamma^f\right](q_t - E_t q_{t+1}) - (1-\gamma)\sigma(R_t - E_t \pi_{t+1}^{CPI}) + \gamma\beta^f (y_t^f - E_t y_{t+1}^f) \tag{13}$$

or

$$y_t = E_t y_{t+1} - a_1(R_t - E_t \pi_{t+1}^{CPI}) + a_2(q_t - E_t q_{t+1}) + a_3(y_t^f - E_t y_{t+1}^f) \tag{14}$$

where

$a_1 = (1-\gamma)\sigma > 0$

$a_2 = \gamma[(1-\gamma)\eta + \eta^f \gamma^f] > 0$

$a_3 = \gamma\beta^f > 0$

Equation (14) represents the open economy IS relation. The forward-looking characteristic of aggregate demand is self-evident: current real output depends not only on the current real exchange rate and current foreign real output but also on their expected values next period. More specifically, the difference between real output in the current period and expected real output in the next period depends on the difference between the real exchange rate in the current period and the expected real exchange rate in the next period. Exactly the same pattern governs the response of real output to variations in foreign real output. The standard real interest rate channel is defined in terms of expected CPI inflation.

It is common to interpret γ as reflecting the degree of openness of the economy. All structural coefficients of the IS equation are thus very sensitive to the degree of openness of the economy.

Phillips Curve

Monopolistically competitive firms aim to minimise menu costs weighed against the cost of being away from the optimal price they would charge in the absence of those menu costs. This optimal price is denoted p^{OPT}. The objective function faced by the typical firm is:

$$\min_P \Omega_t = E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t}\left[\left(p_\tau - p_\tau^{OPT}\right)^2 + c\left(p_\tau - p_{\tau-1}\right)^2\right] \tag{15}$$

where:[4]

Ω_t = the total cost at time t

p_t = the price of the domestic good at time t

p^{OPT} = the optimal price a firm charges.

δ = the constant discount factor

c = the parameter that measures the ratio of the costs of changing prices to the costs of deviating from the optimal price

E_t = the expectations operator conditional on information available at time t.

After taking and rearranging the first-order condition for the above cost-minimisation problem (where we have assumed δ to equal one for simplicity), we can characterize the relationship between past, current, and future price levels as:

$$p_t - p_{t-1} = E_t(p_{t+1} - p_t) - \frac{1}{c}(p_t - p_t^{OPT}) \tag{16}$$

The optimal price p^{OPT} is:

$$p_t^{OPT} = \hat{p}_t + \kappa y_t + \varsigma_t \qquad \kappa > 0 \tag{17}$$

where all variables are as previously defined. In addition:

$\hat{p}_t$ = the price charged by foreign firms at time *t*

ς_t = a stochastic disturbance.

The optimal price responds to changes in marginal cost. But marginal cost and real output are positively related.[5] Hence it is innocuous to replace marginal cost with the output gap in (17).

So far our analysis of price-setting behaviour has been very much in the spirit of the closed economy "New Keynesian Framework". In a small open economy, however, the price-setting behaviour of domestic firms also takes into consideration developments abroad. Being a small player in world markets, the typical firm is guided in its pricing decision by the prevailing conditions in world markets.[6] More specifically, there exists a benchmark price $\hat{p}_t$ that the firm faces in world markets. This benchmark price affects the optimal price charged by the firm. Indeed, the firm adjusts its optimal price in line with the domestic currency price of the final goods charged by its foreign competitors. Thus $\hat{p}_t$ becomes:

$$\hat{p}_t = p_t^f + s_t \tag{18}$$

where

p_t^f = the price of the foreign good in foreign currency at time *t*

Using this specification for p_t^{OPT}, we can rewrite equation (16) as:

$$p_t - p_{t-1} - E_t(p_{t+1} - p_t) = -\frac{1}{c}(p_t - p_t^f - s_t - \varsigma_t) + \frac{\kappa}{c} y_t \tag{19a}$$

If aggregated over all firms, equation (19a) represents a Phillips Curve relation for an open economy. The same equation can also be expressed as:

$$\pi_t = E_t\pi_{t+1} + ay_t + bq_t + u_t \tag{19b}$$

where

$$\pi_t = p_t - p_{t-1}$$

$$E_t\pi_{t+1} = E_t p_{t+1} - p_t$$

$$q_t = p_t^f + s_t - p_t$$

$$a = \frac{\kappa}{c}$$

$$b = \frac{1}{c}$$

$$u_t = \frac{1}{c}\varsigma_t .$$

Equation (19b) differs from the standard forward-looking Phillips Curve by allowing the real exchange rate to affect domestic inflation directly.[7] In the wake of a depreciation of the domestic currency, domestically produced goods become cheaper. Hence domestic production is stepped up. In addition, the domestic currency price of the imported foreign consumption good rises. Both the rise in domestic production and in the price of the import-competing good cause the optimal price to increase. Facing an increase in the optimal price, firms raise the price of their output so as to minimise the deviation between the actual price charged and the optimal price. At the aggregate level, the increase in the domestic price level causes the rate of domestic inflation to rise. Thus we observe the positive link between the real exchange rate and the rate of domestic inflation.

The Complete Model

The model that will serve as the foundation for the analysis of the monetary policy issues in Sections *3* and *4* comprises the following three equations:

$$y_t = E_t y_{t+1} - a_1(R_t - E_t\pi_{t+1}^{CPI}) + a_2(q_t - E_t q_{t+1}) + a_3(y_t^f - E_t y_{t+1}^f) + v_t \tag{14}$$

$$\pi_t = E_t\pi_{t+1} + ay_t + bq_t + u_t \tag{19b}$$

$$R_t - E_t\pi_{t+1} = R_t^f - E_t\pi_{t+1}^f + E_t q_{t+1} - q_t + \varepsilon_t \tag{20}$$

Equation (20) represents the uncovered interest rate parity condition. Stochastic disturbances have been added to the three relations to reflect the existence of uncertainty in the economy.[8] The degree of persistence is the same for all disturbances and fixed at ρ. More formally,

$$
\begin{aligned}
u_t &= \rho u_{t-1} + \hat{u}_t & \hat{u}_t &\sim N(0,\sigma^2_{\hat{u}}) \\
v_t &= \rho v_{t-1} + \hat{v}_t & \hat{v}_t &\sim N(0,\sigma^2_{\hat{v}}) \\
\varepsilon_t &= \rho \varepsilon_{t-1} + \hat{\varepsilon}_t & \hat{\varepsilon}_t &\sim N(0,\sigma^2_{\hat{\varepsilon}}) \\
R^f_t &= \rho R^f_{t-1} + \hat{R}^f_t & \hat{R}^f_t &\sim N(0,\sigma^2_{\hat{R}^f}) \\
\pi^f_t &= \rho \pi^f_{t-1} + \hat{\pi}^f_t & \hat{\pi}^f_t &\sim N(0,\sigma^2_{\hat{\pi}^f}) \\
y^f_t &= \rho y^f_{t-1} + \hat{y}^f_t & \hat{y}^f_t &\sim N(0,\sigma^2_{\hat{y}^f})
\end{aligned}
\tag{21}
$$

3. Discretionary Monetary Policy: The Case of Flexible Inflation Targeting

Central to any discussion of the properties of different monetary policy strategies is the specification of the objective function of the central bank. Following Svensson (2000), we specify an objective function for the policymaker that varies in accordance with the chosen strategy for monetary policy and the target variable for inflation.

3.1 *Domestic Inflation Targeting*

The policymaker has a standard objective function consisting of squared deviations of the real output gap and the rate of inflation, respectively. The rate of inflation is defined in terms of changes in the level of domestic prices. The explicit objective function that he attempts to minimise is given by

$$\frac{1}{2}E_t[\sum_{i=0}^{\infty}\beta^i[y^2_{t+i} + \mu\pi^2_{t+i}]]. \tag{22}$$

All variables are as previously defined. β is the discount rate and μ represents the relative weight the policymaker attaches to the squared deviations of the rate of domestic inflation. Equation (22) implies that the policymaker's sole concern rests with real output and domestic inflation. Fluctuations in the real exchange rate do not enter explicitly the loss function.[9,10]

To set the stage for illustrating how discretionary policymaking in the open economy is carried out, it is helpful at the outset to reduce the dimension of the optimisation problem to one involving only one constraint. A few simple steps need to be taken. First, we solve the UIP condition for the real exchange rate and substitute it into both the IS equation and the Phillips curve relation. Next, after substituting for the rate of CPI inflation in Equation (14), we solve the IS relation for the expected real rate of interest ($R_t - E_t\pi_{t+1}$). Following this, we insert the expression for the expected real rate of interest into the Phillips curve relation. The following expression results:

$$\pi_t = (a + \frac{b}{a_1 + a_2^*})y_t + E_t\pi_{t+1} + \frac{b}{a_1 + a_2^*}\left[a_1(R_t^f - E_t\pi_{t+1}^f + \varepsilon_t) - (E_t y_{t+1} + v_t) - a_3(y_t^f - E_t y_{t+1}^f)\right] + bE_t q_{t+1} + u_t \tag{23}$$

where $a_2^* = a_2 - \gamma a_1$.

When setting policy with discretion, the policymaker takes the expectations of the endogenous variables y_t, π_t, q_t and the remaining terms as given.[11] Hence we can rewrite the above as

$$\pi_t = (a + \frac{b}{a_1 + a_2^*})y_t + f_t \tag{23'}$$

where

$$f_t = E_t\pi_{t+1} + \frac{b}{a_1 + a_2^*}\left[a_1(R_t^f - E_t\pi_{t+1}^f + \varepsilon_t) - (E_t y_{t+1} + v_t) - a_3(y_t^f - E_t y_{t+1}^f)\right] + bE_t q_{t+1} + u_t$$

Notice further that the objective function can be neatly broken up into two separate components as future values of the endogenous variables are independent of today's policy action:[12]

$$\frac{1}{2}[y_t^2 + \mu\pi_t^2] + F_t \tag{22'}$$

where $F_t = \frac{1}{2}E_t[\sum_{i=1}^{\infty}\beta^i(y_{t+i}^2 + \mu\pi_{t+i}^2)$

The problem of setting policy under discretion thus reduces to the following simple one-period optimisation problem:

$$\underset{y_t, \pi_t}{Min}\ \frac{1}{2}[y_t^2 + \mu\pi_t^2] + F_t \tag{24}$$

subject to

$$\pi_t = (a + \frac{b}{a_1 + a_2^*})y_t + f_t$$

Replacing a_2^* with $a_2 - \gamma a_1$ and combining the first-order conditions produces a systematic negative relationship between real output and the rate of inflation:

$$y_t = -\mu(a + \frac{b}{a_1(1-\gamma)+a_2})\pi_t \qquad (25)$$

The coefficient on the rate of inflation indicates the loss of output that the policymaker is prepared to sustain if the rate of inflation exceeds its zero target level.

What is striking about the above optimising condition is the relationship between the degree of openness (γ) and the sensitivity of domestic inflation to the real exchange rate in the Phillips Curve (b). The degree of openness matters only to the extent that the direct exchange rate channel is operative in the Phillips Curve. In case this channel is absent from the Phillips Curve, i.e. if $b=0$, then the optimal relationship between real output and the rate of inflation is independent of γ and the same for both the open and the closed economy framework: $-\mu a$.[13] There is a straightforward explanation for this result. Shutting off the direct exchange rate channel in the Phillips Curve enables the policymaker to offset any disturbances arising on the demand side of the economy by simply adjusting the setting of the policy instrument. Thus, demand-side factors should not have any role to play in the determination of the optimal relationship between real output and the rate of inflation. The degree of openness is, however, a characteristic of the demand-side of the economy as it denotes the share of the imported foreign consumption good in total consumption. Thus, if the policymaker is in a position to offset any demand-side disturbance, then γ should not matter in the determination of the optimising condition.

In the more likely case of $b > 0$, the optimality condition depends on *all* parameters – except a_3 - of the model.[14]

Combining the above optimising condition with the Phillips Curve, the IS, and the UIP relation allows us to solve for the reduced form equations and the variances of the endogenous variables that appear in the policymaker's loss function. The expected loss function under a domestic inflation objective appears in Table 1. We observe readily that the perfect stabilising property of optimal discretionary policymaking in the face of demand-side disturbances that is typically found in models of both open and closed economies does not carry over to the present framework. In the face of demand-side disturbances (v_t, y_t^f), a UIP disturbance (ε_t), and a foreign interest rate disturbance (R_t^f), the policymaker is unable to keep the real output gap and the rate of inflation at their respective target. As all disturbances are autocorrelated, a foreign inflation shock will also cause the targets for output and inflation to be missed. In his attempt to offset the impact of any one of the aforementioned disturbances on real output by varying the nominal interest rate, the policymaker causes the real exchange rate to change. The change in the real exchange rate in turn directly affects the domestic rate of inflation in the Phillips Curve. Thus the policymaker sees himself confronted with changes in both the rate of inflation and real output. Inspection of the numerators of the coefficients of the variances of IS, UIP, and foreign interest rate disturbances and foreign inflation shocks reveals that the magnitude of b, which captures the potency of the direct exchange rate channel in the Phillips Curve, is instrumental in transmitting the effects of these disturbances on both real output and the rate of inflation. In short, all disturbances that impinge upon the economy – not just cost-push disturbances – cause the rate of inflation and real output to deviate from their respective target and hence cause the variances of both variables to increase if a direct exchange rate channel is operative in the Phillips Curve.

Examining the loss function further, we find that it is positively related to the size of the persistence parameter ρ. Again there is a straightforward explanation for this positive relationship. The greater the degree of persistence in the disturbances, the more closely the current expectations of future values of both endogenous (and exogenous variables) follow their current values. This property in turn implies that both current real output and the rate of domestic inflation react more sensitively to the disturbances that impinge upon the economy. Hence both variables have a tendency to deviate more from their target values, causing their respective variance to increase. What is striking, however, is that the origin of a particular disturbance is crucial in determining the extent to which the loss function increases as the degree of persistence in the shocks increases. Closer inspection reveals that the loss function increases dramatically as persistence increases because the increase in persistence augments the effects of the variances of IS, foreign interest rate disturbances, UIP disturbances, and shocks to foreign inflation. The variances of all these shocks - but not the variances of cost-push shocks and foreign output shocks - are multiplied by the factor $\left(\frac{1}{1-\rho}\right)^2$. For comparatively high values of ρ this factor tends to become rather large.

3.2. *CPI Inflation Targeting*

Most central banks seek to ensure stability in the rate of CPI inflation. Indeed, the performance of central banks is judged in general on the basis of their ability to keep the rate of CPI inflation within specific bounds. Thus, a case can be made for including CPI inflation as a target variable in the objective function.

The rate of inflation is now defined in terms of changes in the level of the consumer price index:

$$\pi_t^{CPI} = (1-\gamma)\pi_t + \gamma(s_t - s_{t-1} + \pi_t^f) \tag{26}$$

Given the definition of the real exchange rate (q_t) and the assumption of complete exchange rate pass-through, the above can be restated as:

$$\pi_t^{CPI} = \pi_t + \gamma \Delta q_t \tag{27}$$

In view of the fact that the policymaker is concerned about stabilising the rate of CPI inflation, it is necessary to recast the model of the economy in terms of the rate of CPI inflation. This is accomplished by solving the above definition of the rate of CPI inflation for π_t and substituting the right-hand side of this expression for π_t in both the Phillips Curve and the uncovered interest rate parity condition. The amended equations take the following form:

$$\pi_t^{CPI} = E_t \pi_{t+1}^{CPI} + (2\gamma + b) q_t - \gamma E_t q_{t+1} - \gamma q_{t-1} + a y_t + u_t \tag{28}$$

$$R_t - E_t \pi_{t+1}^{CPI} = R_t^f - E_t \pi_{t+1}^f + (1-\gamma)(E_t q_{t+1} - q_t) + \varepsilon_t \tag{29}$$

The IS relation remains unchanged:

$$y_t = E_t y_{t+1} - a_1 (R_t - E_t \pi_{t+1}^{CPI}) + a_2 (q_t - E_t q_{t+1}) + a_3 (y_t^f - E_t y_{t+1}^f) + v_t \tag{14}$$

Again, the first step of the optimisation routine is to reduce the dimension of the problem. This is accomplished by following the same procedure as outlined in Section *3.1*. With the rate of CPI inflation rate entering the objective function, the minimisation exercise that the policymaker undertakes can be stated formally as:

$$\underset{y,\pi^{CPI}}{Min} \ \frac{1}{2} E_t [\sum_{i=0}^{\infty} \beta^i [y_{t+i}^2 + \mu \pi^{CPI}{}_{t+i}^{2}]] \tag{30}$$

subject to

$$\pi_t^{CPI} = \left[a + \frac{2\gamma + b}{a_1(1-\gamma) + a_2} \right] y_t + f_t$$

where

$$f_t = E_t \pi_{t+1}^{CPI} + u_t - \gamma q_{t-1} + (\gamma + b) E_t q_{t+1} + \left[\frac{2\gamma + b}{a_1(1-\gamma) + a_2} \right] \left[a_1 \mathrm{Z}_t - (E_t y_{t+1} + a_3 (y_t^f - E_t y_{t+1}^f) + v_t) \right]$$

and

$\mathrm{Z}_t = (R_t^f - E_t \pi_{t+1}^f + \varepsilon_t)$.

The minimisation exercise gives rise to two first-order conditions. Combining these first-order conditions establishes the following optimal systematic relationship between the rate of CPI inflation and real output:

$$y_t = -\mu (a + \frac{b + 2\gamma}{a_1(1-\gamma) + a_2}) \pi_t^{CPI} \tag{31}$$

Comparing the coefficient on CPI inflation in Equation (31) to the coefficient on domestic inflation in Equation (25), we observe that, ceteris paribus, the size of the former exceeds that of the latter. A one percentage point increase in CPI inflation evokes a greater negative response in real output than a one percentage point increase in domestic inflation. This is an important result as it foreshadows that focusing on CPI inflation as opposed to domestic inflation is associated with greater fluctuations in real output.

Owing to the appearance of a lagged endogenous variable in the model (q_{t-1}), analytical solutions for the endogenous variables are difficult to establish in the current context.[15] Nevertheless the presence of IS and Phillips Curve parameters in the optimising condition makes it evident that the perfect stabilising properties of discretionary policy fail to hold in the present case just as they fail when the policymaker targets domestic inflation. Indeed, inspection of Equation (31) reveals that the focus on CPI inflation deprives the policymaker of his ability to insulate the economy against demand-side disturbances even if the direct exchange rate channel is shut off. Even if $b = 0$, demand-side parameters are involved in determining the optimising condition adhered to by the policymaker.

To get a more concrete idea of the implications of selecting domestic rather than CPI inflation as a target variable, we employ a numerical solution technique. With the help of this tool we evaluate the behaviour of the endogenous variables and the policy instrument for the two policy problems. The results of this exercise are discussed in the next section.

4. Applying the Numerical Solution Technique to the Policy Problem under Flexible Inflation Targeting: A Domestic Inflation versus a CPI Inflation Target.[16]

Throughout the analysis the following values of the parameters and the variances of the stochastic disturbances are employed:

$$\sigma = 0.5 \qquad \eta = 1 \quad \eta^f = 2 \qquad \gamma^f = 0.15 \qquad \beta^f = 0.9 \qquad \rho = 0.8$$

$$a = 0.25 \qquad b = 0.25 \qquad \mu = 1 \quad \sigma_i^2 = 1 \text{ for } \quad i = \hat{u}, \hat{v}, \hat{\varepsilon}, \hat{R}^f, \hat{\pi}^f, \hat{y}^f$$

The unitary weight on inflation represents the operational definition of flexible inflation targeting employed in this paper. The degree of persistence is fixed at $\rho = 0.8$.

The first step of the solution procedure is to rewrite the model in the following form:

$$\mathbf{A_0 Y_t} = \mathbf{A}_1 \mathbf{Y_{t-1}} + \mathbf{A}_2 \mathbf{E_t Y_{t+1}} + \mathbf{A}_3 \mathbf{R_t} + \mathbf{A}_4 \mathbf{E_t R_{t+1}} + \mathbf{A}_5 \mathbf{V_t} \tag{33}$$

$\mathbf{Y_t}$ = vector of endogenous variables in period *t*.

$\mathbf{Y_{t-1}}$ = vector of lagged endogenous variables.

$\mathbf{E_t Y_{t+1}}$ = vector of expectations of endogenous variables in period *t+1*.

$\mathbf{R_t}$ = policy instrument in period *t (scalar)*.

$\mathbf{E_t R_{t+1}}$ = expected setting of policy instrument in period *t+1 (scalar)*[17].

$\mathbf{V_t}$ = vector of stochastic disturbances of the model.

$\mathbf{A_j}$ = coefficient matrices, $j = 1, \ldots 5$.

Solving the model for the endogenous variables produces solutions that depend on the lagged endogenous variables and the exogenous disturbances:

$$\mathbf{Y_t} = \mathbf{H}_1 \mathbf{Y_{t-1}} + \mathbf{H}_2 \mathbf{V_t} \tag{34}$$

$\mathbf{H_j}$ = coefficient matrices, $j = 1, 2$.

The solution for the policy instrument also depends on the vector of lagged endogenous variables and the exogenous disturbances:

$$\mathbf{R_t} = \mathbf{F}_1 \mathbf{Y_{t-1}} + \mathbf{F}_2 \mathbf{V_t} \tag{35}$$

$\mathbf{F_j}$ = coefficient matrices, $j = 1, 2$.

The variances of the endogenous variables and the policy instrument are the diagonal elements of the respective variance and covariance matrix:

$$\mathbf{\Phi Y} = \mathbf{E}\left[\mathbf{Y_t Y_t'}\right] \tag{36}$$

$$\mathbf{\Phi R} = \mathbf{E}\left[\mathbf{R_t R_t'}\right] \tag{37}$$

The algorithm described in the preceding paragraphs is applied first to the model of Section 3.1. where the policymaker targets domestic inflation and then to the model of Section 3.2. where he targets CPI inflation. In each case, we examine the sensitivity of the variances of the variables of the model to the degree of openness.

We now turn to a visual inspection of the variability of the endogenous variables of the model and the policy instrument.[18] Figure 1 illustrates the case of flexible domestic inflation targeting. This targeting strategy leads to relative stability in real output: the variance of real output is always less than the variances of domestic and CPI inflation. Notice that the variance of CPI inflation is always greater than the variance of domestic inflation; indeed, the gap between the two variances widens as openness (γ) increases. Both the variance of domestic inflation and the variance of real output actually decline initially as γ increases. At higher values of γ the variance of domestic inflation continues to decrease while the variance of real output begins to increase.

When the policymaker targets CPI inflation, he succeeds in increasingly mitigating fluctuations in both CPI inflation and domestic inflation as the degree of openness rises. But there is a substantial cost. For $\gamma \geq 0.2$ the variance of real output lies above the variance of CPI inflation and keeps on rising as the economy becomes more open. This case is illustrated in Figure 2. Notice also that under CPI targeting the variance of domestic inflation is always lower than the variance of CPI inflation.

Figures 3 and 4 capture the behaviour of the real exchange rate and the nominal rate of interest under the varying policy objectives. Domestic inflation targeting causes a dramatic difference between the variances of the real exchange rate and the policy instrument. Figure 3 illustrates that the variance of the real exchange rate is more than three times the size of the variance of the policy instrument. Attention to minimising fluctuations in the domestic rate of inflation results in dramatic swings in the real exchange rate. This problem can be eased somewhat by the policymaker if he focuses his attention on CPI inflation. Figure 4 illustrates this case. The variability of the nominal rate of interest and, in particular, the variability of the real exchange rate are lower under CPI inflation targeting than under domestic inflation targeting. Finally, we find the variance of the nominal rate of interest declining throughout as the degree of openness increases irrespective of which rate of inflation is chosen as a target variable. In contrast, the variance of the real exchange decreases monotonically as γ increases only if the policymaker targets CPI inflation.

5. Conclusion

This paper presents a simple optimising open-economy model to explain why the conduct of discretionary monetary policy in an open economy differs substantially from the closed-economy. The differences stem from the exchange rate channel in the Phillips Curve and the choice of the inflation target. The paper shows analytically that the existence of a direct real exchange rate channel in the open economy Phillips Curve impairs the perfect stabilising property of monetary policy in the face of demand-side disturbances under a domestic inflation target. If CPI inflation is targeted, then the perfect stabilising property of discretionary policy breaks down even in the absence of the direct real exchange rate channel in the Phillips Curve.

The paper compares the attractiveness of flexible CPI versus flexible domestic inflation targeting under varying degrees of openness. Given the central bank's objectives, our findings suggest that in an open economy framework a flexible inflation targeting strategy centred on domestic inflation can stabilise real output, but at the expense of substantial fluctuations in the exchange rate, CPI inflation, and the policy instrument. In contrast, targeting CPI inflation assures better control over both measures of inflation and the real exchange rate, albeit at a cost of pronounced swings in real output.

Appendix

An open-economy Phillips Curve can also be derived from the Calvo (1983) model that emphasizes stochastic price adjustment. The single most important parameter of this framework is η as it governs the extent of price stickiness. More specifically, η represents the probability that a firm cannot adjust the price of its product in a given period.[19] As there are a large number of firms in the economy, the fraction of firms changing prices corresponds to the probability of price adjustment: $1-\eta$. The fraction of firms that in the current period charge the same price set j periods ago is given by $(1-\eta)\eta^{j}$.

The Calvo framework reduces to three equations. All variables are in logs.

$$p_t = \eta p_{t-1} + (1-\eta)p_t^* \qquad 0 < \eta < 1 \qquad \text{(A1)}$$

$$p_t^* = (1-\beta\eta)[\omega(\psi_t + p_t) + (1-\omega)(p_t^f + s_t)] + \beta\eta E_t p_{t+1}^* \qquad 0 \le \omega \le 1 \qquad \text{(A2)}$$

$$\psi_t = hy_t \qquad h>0 \qquad \text{(A3)}$$

where

p_t = current aggregate price

p_t^* = current price chosen by firms adjusting prices

ψ_t = real marginal cost

$p_t^f + s_t$ = price of foreign consumption good in domestic currency

y_t = real output gap

β = discount factor ($0 < \beta \le 1$)

E_t = expectations operator conditional on information available at time t.

Equation (A1) shows that the aggregate price level is a weighted average of the price level of the previous period and the current price chosen by those firms that adjust their product price. Price stickiness is embodied by the presence of ηp_{t-1}.

Equation (A2) reflects a key characteristic of price-setting firms: forward-looking behaviour. These firms are aware of the possibility that they may not be able to adjust prices for a while. Hence they look at the future evolution of the factors that govern the determination of their product prices. These factors are nominal marginal cost and the domestic currency price of the foreign consumption good. The latter represents the price that prevails in world markets where domestic firms compete. The relative importance of nominal marginal cost and the domestic currency price of the foreign consumption good in the process of price adjustment is given by ω and $1-\omega$, respectively. Clearly $1-\omega$ indicates the extent to which the current "world price" influences the price setting behaviour of domestic firms.

Equation (A3) conveys the positive relationship between real marginal cost and the real output gap that is a standard feature of general equilibrium models.[20] To derive the open economy Phillips Curve, update Equation (A1) by one period. Taking conditional expectations of updated expression and multiplying by $\beta\eta$ yields:

$$\beta\eta E_t p_{t+1} = \beta\eta^2 p_t + \beta\eta(1-\eta)E_t p^*_{t+1} \tag{A1'}$$

Subtract the above equation from (A1) to obtain:

$$p^*_t - \beta\eta E_t p^*_{t+1} = \frac{1}{1-\eta}\left[p_t - \beta\eta E_t p_{t+1} + \eta(\beta\eta p_t - p_{t-1})\right] \tag{A1''}$$

Substituting (A1'') into Equation (A2) and rearranging the resulting expression yields:

$$\eta(p_t - p_{t-1}) = \beta\eta(E_t p_{t+1} - p_t) + (1-\beta\eta)(1-\eta)(\omega\psi_t + (1-\omega)q_t) \tag{A4}$$

q_t is the real exchange rate and defined as $q_t = p^f_t + s_t - p_t$.

Finally, insert Equation (A3) into Equation (A4). After rearranging the resulting expression and letting $\beta = 1$, we can specify the open economy Phillips Curve as follows:

$$\pi_t = E_t \pi_{t+1} + \phi\omega h y_t + \phi(1-\omega)q_t \tag{A5}$$

where $\phi = \dfrac{(1-\eta)^2}{\eta}$

$$\pi_t = p_t - p_{t-1}$$

$$E_t \pi_{t+1} = E_t p_{t+1} - p_t$$

This Phillips Curve is very similar to the one employed in the main part of the paper. Most importantly, the level of the real exchange rate directly influences domestic inflation. A stochastic disturbance could be easily introduced into the above equation by appending a random element to Equation (A2).

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Footnotes

[1] Flexible inflation targeting is defined in this paper as a strategy where the central bank is equally concerned about deviations of both the rate of inflation and the real output gap from their respective target level.

[2] See, for instance, the analytical findings reported by Clarida, Gali, and Gertler (1999, 2001).

[3] The contributions by McCallum and Nelson (1997), Gali and Monnacelli (1999), Svensson (2000), and Clarida, Gali, and Gertler (2001, 2002) represent earlier attempts to model optimising behavior by economic agents in the open economy in ways that are directly comparable to the microfoundations established in this section. The derivation of the IS curve in the current model is in parts similar to that of Svensson (2000).

[4] See Roberts (1995) for further details on the derivation of the closed-economy Phillips Curve. For convenience we drop the superscript ("h") on domestic prices.

[5] Within a general equilibrium framework, the co-movement between marginal cost and economic activity can be established by combining the labour supply and demand relations with the market clearing condition in the goods market. On this point see Clarida, Gali, and Gertler (2001, 2002) or Gali and Monacelli (1999) who derive a similar relation that stresses the positive relation between real marginal cost and domestic consumption. The positive link between output and marginal cost is also characteristic of earlier models of monopolistic competition such as Blanchard and Kiyotaki (1987). The link features also prominently in Mankiw and Reis (2002) who propose an alternative Phillips curve that is based on slow dissemination of information.

[6] In a general equilibrium setting, for the pricing decision of domestic firms to be sensitive to the prevailing price charged by foreign competitors, it is necessary to drop the assumption of constant elasticity of substitution in the utility function. Bergin and Feenstra (2000) and Taylor (2000) show how a *translog* specification for preferences or a *linear* demand relation yields an

optimal pricing rule that responds to competitors' prices *in addition* to marginal cost. Equation (17) embodies this idea.

7 In the appendix we show that the Calvo (1983) framework, where price adjustment occurs randomly, can be extended to the open economy too. The extension produces a Phillips curve for the open economy that also includes the real exchange rate. Walsh (1999) also presents an open economy Phillips Curve where the real exchange rate enters as a result of wage demands being dependent on the CPI.

8 $R_t^f, y_t^f, \text{ and } \pi_t^f$ are considered to be exogenous stochastic variables. The home country is too small to affect prices, interest rates, and real output abroad. For simplicity, we also assume that all foreign shocks are independent of each other.

9 Adopting equation (22) as the welfare criterion ignores the effects on welfare of changes in the real exchange rate in the open economy framework. Including only real output and the rate of inflation in the loss function is rather typical in the literature and thus facilitates comparing the results of this paper to earlier contributions (e.g. Clarida, Gali, and Gertler (1999, 2001) or Svensson (2000)).

10 The target level for real output is the potential level of output. The target for the rate of inflation is assumed to be zero.

11 Here we adopt the convention of describing the conduct of discretionary policy along the lines of Clarida, Gali, and Gertler (1999).

12 Future values of y_t and π_t are not affected by policy today as the effect of policy is contemporaneous and the absence of persistence in the endogenous variables.

13 This is rather unlikely though as $b = \frac{1}{c}$. Thus c would have to become infinitely large for b to approach zero.

14 The optimising condition can be written in terms of all deep parameters of the model:

$$y_t = -\frac{\mu}{c}\left[\kappa + \frac{1}{(1-\gamma)^2\sigma + \gamma((1-\gamma)\eta + \gamma^f\eta^f)}\right]\pi_t$$

where all parameters are as previously defined.

[15] Due to the presence of a lagged endogenous variable (q_{t-1}), the current expectations of all endogenous variables for period $t+1$ depend on q_t. This fact complicates the derivation of analytical solutions immensely.

[16] This section draws on Dennis (2001) who has written an algorithm that produces numerical solutions to the optimisation problem under discretion. The algorithm has been adapted to fit the models described in Sections 3.A. and 3.B.

[17] This scalar is irrelevant in the current context. Hence its coefficient matrix (**A$_4$**) consists of zeros only.

[18] The numerical scores are available upon request from the author.

[19] For a lucid exposition of the Calvo model, see King (2000).

[20] See, for instance, Equation (35) in Clarida, Gali, and Gertler (2002).

Table 1: Domestic Inflation Targeting:

Flexible Domestic Inflation Targeting: $\mu = 1;\ \pi^* = 0$	
AR(1) Disturbances $(\rho > 0)$	$E(L_t) = \left[\frac{(b+aA)^2 + A^2}{(1-\rho^2)}\right] \frac{b^2}{\left[(b+aA)^2 + A^2(1-\rho)\right]^2} \left[\left(\frac{1}{1-\rho}\right)^2 \left(\sigma_{\hat{v}}^2 + a_1^2\left(\sigma_{R\hat{f}}^2 + \sigma_{\hat{\varepsilon}}^2 + \rho^2\sigma_{\pi\hat{f}}^2\right)\right) + \left(\frac{A}{b}\right)^2 \sigma_{\hat{u}}^2 + a_3^2\sigma_{y\hat{f}}^2\right]$

$A = (1-\gamma)a_1 + a_2$

Fig. 1: The Variances of Inflation and Real Output under Domestic Inflation Targeting

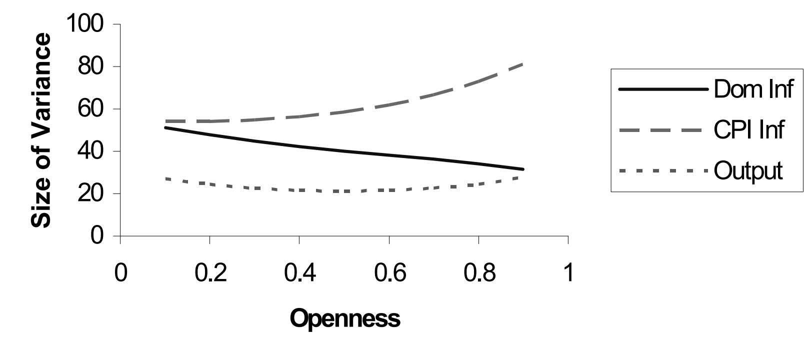

Fig. 2: The Variances of Inflation and Real Output under CPI Inflation Targeting

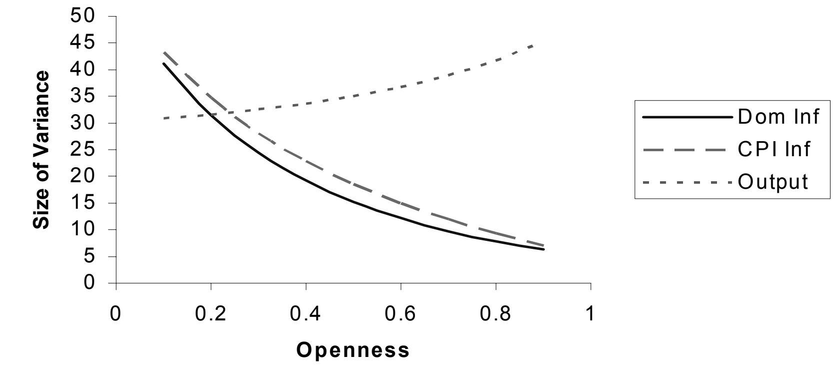

Fig. 3: The Variances of the Real Exchange Rate and the Nom. Rate of Interest under Dom. Inflation Targeting

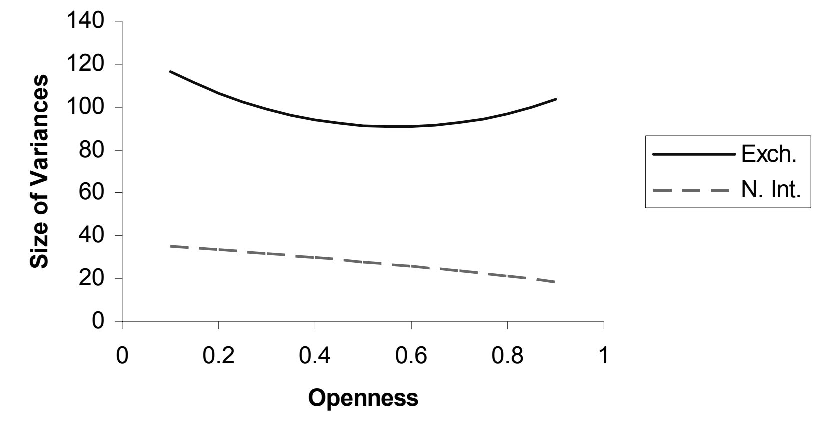

Fig. 4: The Variances of the Real Exchange Rate and the Nom. Rate of Interest under CPI Inflation Targeting