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## **Instrument Versus Target Rules As Specifications of Optimal Monetary Policy: What are the Issues, If Any?**

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### **Abstract:**

One issue in the literature on monetary policy in New Keynesian models has been the relative merits of instrument versus target rules. This paper focuses on *optimal* instrument and target rules within three workhorse models in the literature: *IS-LM*, *AS-AD* and the New Keynesian model. The focus on optimal rules enables us to exploit the equivalence among alternative expressions of optimal policies for a given information set. We find that in the *AD-AS* model, characterized by the presence of observable information variables and unobservable target variables, an optimal explicit instrument rule, a combination policy, and a target rule produce identical outcomes for the target variables. In the New Keynesian model, the optimal explicit instrument rule achieves the same stabilization results as the globally optimal target rule. However, the latter approach provides a more direct rationale for introducing inertia into policymaking. This seems to be the key advantage of the target rule approach. Along other dimensions such as robustness and transparency, target rules offer no clear advantages over optimal instrument rules.

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## 1. INTRODUCTION

For over a decade one issue in the literature on monetary policy in New Keynesian models has been the relative merits of instrument versus target rules. Svensson (1997a,b), (1999), (2002), (2003), (2005), Svensson and Woodford (2005), and Woodford (2003), have argued for the superiority of target rules. McCallum (1999a,b) and McCallum and Nelson (2004), (2005) have questioned the case for target rules arguing that instrument rules may be preferable. In this paper we focus on *optimal* instrument and target rules within a series of widely-used models for monetary policy analysis. This focus enables us to exploit the equivalence between expressions for optimal policies as settings for an instrument or as a linear combination of currently observable variables. Instrument rules are obviously of the former class. Target rules are a limiting case of the latter class, for the case where the targets are observable.<sup>1</sup> Optimal target and instrument rules will for the models we consider be equivalent. By this we mean that a central bank will take the same actions whichever rule they choose. Thus, from a welfare point of view, target rules are not inherently superior to optimal instrument rules.

Questions about the usefulness or practicality of optimal instrument rules have arisen in the literature. McCallum and Nelson (2005) agree with Svensson (2003, p. 439), that “commitment to an optimal instrument rule has no advocates.....” Despite this criticism we believe that comparisons of suboptimal instrument rules with the optimal one serve a useful purpose as they clarify why the former are suboptimal - - for example, “What is Wrong with Taylor Rules?”

Moreover, the focus on *optimal* instrument versus target rules and the fact of their equivalence enables us to clarify issues in the debate over their relative advantages. Their equivalence eliminates some issues. Still, there might be advantages to one or another for a number of reasons. Target rules are viewed by their proponents as dominating optimal instrument rules along three dimensions. Target rules offer greater robustness to model changes, afford greater transparency in the conduct of monetary policy, and are of “higher order”. An important message of this paper is that the alleged clear superiority of target rules to optimal instrument rules along these dimensions is, on balance, overstated.

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<sup>1</sup> More generally, in models where certainty equivalence holds optimal policies will remain unchanged when expected values replace actual values in target rules, as in Svensson and Woodford (2003).

A focal point in our analysis of alternative specifications of optimal monetary policy in current and past workhorse macroeconomic models is the information set. The information available to the policymaker conditions the policy setting. The equivalence of the optimal instrument setting and a policy setting as a linear combination of observables results from their being equivalent ways to respond to the same information set - - the same optimal conditional expectation of the target variables. Suboptimal policies will result from failure to fully exploit available information. Policy rules will change when information sets change. This implies that if optimal target rules and instrument rules are equivalent they will change in exactly the same circumstances.

Our examination of these issues proceeds as follows. Section 2 sets out the terminology adopted for the analysis. Section 3 considers target and instrument rules in two standard Keynesian models: the *IS-LM* model and the *AS-AD* model. A comparison of target versus instrument rules in these simple models provides useful background for the discussion of more modern models. Section 4 focuses on the canonical forward-looking New Keynesian model. Optimal instrument and target rules are derived and compared under different assumptions about policy conduct: discretion, simple commitment and global commitment (the “timeless perspective”). Section 5 examines issues including robustness and transparency which may lead to a preference of one type of rule over the other within variations of the New Keynesian model. Section 6 contains concluding comments.

## **2. TERMINOLOGY**

We begin by defining terms. By a rule for monetary policy we mean, as do Svensson and other participants in this debate, a prescribed guide for the conduct of monetary policy. For each type of rule we consider there is a loss function that specifies the central bank’s objectives - - the target variables. This specification of the objectives of monetary policy can be called a “targeting regime” [Svensson (2005, p. 622); Walsh (2003)], for example, inflation targeting, or just an objective function. We use the latter term. The term *target rule* is reserved for a condition to be fulfilled by the target variables (or forecasts thereof). Svensson calls this a *specific* target rule.

An instrument is defined as a variable administered by the central bank or as a financial market variable the central bank controls so closely that control error can be ignored.

The U.S. federal funds rate is an example of the latter type of instrument. We consider explicit and implicit instrument rules. An explicit instrument rule will be defined as one that sets the instrument directly as a function of elements of a given information set. An implicit instrument rule will be one where the instrument setting is derived from an associated target rule. An optimal instrument rule will be one that minimizes the central bank's loss (objective) function. If it is an optimal explicit instrument rule then it must employ all available information. An optimal implicit instrument rule is simply the setting that implements the optimal target rule.<sup>2</sup>

A final characterization of policy we will consider is as a linear combination of currently observable variables. A prominent early example of this type of policy setting is Poole's (1970) combination policy in the *IS-LM* model. LeRoy (1975) and LeRoy and Waud (1977) clarify the relationship of Poole's combination policy and instrument rules that exploit currently observable financial market information. Target rules are examples of this type of policy setting for the case where the current values of the target variables are observable.<sup>3</sup>

### 3. OPTIMAL POLICY SPECIFICATIONS IN KEYNESIAN MODELS

As background to the later discussion of optimal policy rules in the New Keynesian model we consider two versions of the earlier Keynesian model.

These earlier models, *IS-LM* and the aggregate supply – aggregate demand model have been superseded by the New Keynesian framework. A consideration of instrument and target rules within these frameworks is still useful for an understanding of the role that informational assumptions play in the target versus instrument rule debate. Many papers working within the New Keynesian target rule framework have made use of the *separation principle* that applies under certainty equivalence, [Svensson and Woodford (2003; p. 692)] “according to which the selection of the optimal policy (the optimization problem) and the estimation of the current state of the economy (the estimation or signal extraction problems) can be treated as separate problems.” This is a long-standing practice in the literature on monetary policy under

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<sup>2</sup> It should be noted that these definitions differ from those of Svensson (1999).

<sup>3</sup> Another characterization of optimal policy is where an instrument rule is written to include a target rule as an extreme (or limiting case). Such a policy specification is proposed by McCallum (1999b) and examined in McCallum and Nelson (2004)(2005). Svensson (2005) is critical of this specification. While we do not examine this type of specification, it is an alternative route to demonstrating the equivalence of optimal target and instrument rules for the same information sets.

uncertainty and hence defensible. What is less defensible, if not objectionable, is that the current literature tends to concentrate only on the optimization problem under full-current information whereby the current values of the variables in the target rule, the inflation rate and output gap, are observed. The signal extraction problem is ignored. Once limited information becomes a defining feature of the policy environment, the policymaker must confront the problem of forming estimates of the relevant target variables. Without the solution to the signal extraction problem, the target rule as well as the accompanying implicit instrument rule can not be implemented.

Apart from the full-information target rule, a second widely used specification in New Keynesian model is a Taylor rule which is an instrument rule but for which no general claims of optimality are made.

Optimal explicit instrument rules were developed within the earlier Keynesian framework with which we begin our analysis. The structure of these models was simpler and the optimization and signal extraction problems were solved together. Financial market prices and quantities were treated as observables; policy goals such as output and inflation were not.

It is when *optimal* target and instrument rules are considered for the same information set that we see their equivalence. Some advantages of target rules, for example along the dimension of robustness, are seen simply to result from different assumptions about information sets.

### A. The IS-LM Model

A stochastic version of the *IS-LM* model, as in Poole (1970), is given by

1.  $y = a_0 - a_1 i + v$
2.  $m = b_0 + b_1 y - b_2 i + \eta$

with  $E(v) = E(\eta) = 0$

$$E(v^2) = \sigma_v^2 \quad E(\eta^2) = \sigma_\eta^2 \quad E(v\eta) = 0$$

Where we ignore time subscripts and where:  $y$  = output;  $i$  = the interest rate;  $m$  = money supply.

The policymaker's objective function is

$$3. \quad \text{Min } E[L_t] = E[(y_t - E[y_t])^2] = \sigma_y^2$$

### The Optimal Instrument Rule

Substitution of (1) into (2) yields

$$2.' \quad m = b_0 + a_0 b_1 - (b_2 + a_1 b_1) i + (\eta + b_1 v)$$

The assumption in Poole (1970) was that the financial market variables ( $m$  and  $i$ ) were currently observable but the target variable ( $y$ ) was not.<sup>4</sup> With this assumption the central bank can observe  $(\eta + b_1 v)$ . Using this information, the policymaker can form the optimal conditional expectation of the target variable ( $y$ ) and construct the optimal instrument rule.

If we assume that the interest rate is chosen as the instrument, the optimal rule will be

$$4. \quad i^* = -a_1^{-1} [y^* - a_0 - E_t v]$$

where the subscript  $t$  is used on the expectations operator to indicate the expectation conditional on available current information. Using the standard formula for conditional expectations this term is

$$5. \quad E_t (v | \eta + b_1 v) = \frac{b_1 \sigma_v^2 (\eta + b_1 v)}{b_1^2 \sigma_v^2 + \sigma_\eta^2}$$

### Optimal Policy as a Linear Combination of Observable Variables

An alternative expression for optimal policy is as a linear combination of observable variables. If we continue to assume that the target variable ( $y$ ) is unobservable, while  $m$  and  $i$  are observed, this results in Poole's (1970) combination policy which is of the form

$$6. \quad m = \lambda_0 + \lambda i$$

The optimal combination policy can be derived by substituting equation (6) into equation (2') for  $m$  and proceeding to calculate the values of  $\lambda_0$  and  $\lambda$  that minimize the loss function (3). Alternatively, the optimal combination policy can be expressed directly from the optimal instrument rule (4). Using (5) in (4) we solve for  $(\eta + b_1 v)$  and substitute the result into (2'). With some rearrangement this yields

$$7. \quad m = b_0 + y^* b_1 + \frac{\sigma_\eta^2}{b_1 \sigma_v^2} (y^* - a_0) + \frac{a_1 \sigma_\eta^2 - b_2 b_1 \sigma_v^2}{b_1 \sigma_v^2} i$$

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<sup>4</sup> Papers such as Friedman (1975) make the more realistic assumption that a reserve aggregate is currently observable but the money supply is not.

Equation (7) and the optimal combination policy derived directly from equation (6) are identical. This is the result in LeRoy (1975) and LeRoy and Waud (1977). Both specifications exploit the information content of observable variables to learn about the realizations of the error terms affecting the target variable.

### A Target Rule

In the fixed price *IS-LM* model a target rule would simply be

$$8. \quad y = y^*$$

To be operational, contrary to the assumption in the Poole framework, output must be observed. There is no substantive policy problem.

### **B. Aggregate Supply – Aggregate Demand Framework**

For two decades a workhorse model for optimal monetary policy analysis was the aggregate demand and supply framework. A version of the model consists of the following three equations:

$$9. \quad y_t = c_0 + c_1(p_t - p_{t,t-1}^e) + u_t$$

$$10. \quad y_t = a_0 - a_1(i_t - (p_{t+1,t-1}^e - p_{t,t-1}^e)) + v_t$$

$$11. \quad m_t = p_t + b_0 + b_1 y_t - b_2 i_t + \eta_t$$

where new variables and error terms are:

$p_t$  = aggregate price level

$p_{t+j,t-1}^e$  = rational expectation of  $p$  for  $t+j$ ;  $j = 0, 1$  taken at  $t-1$ .

$u_t$  = supply shock with mean zero variance  $\sigma_u^2$  and zero covariances

with other disturbances in the model.

Equations (10) and (11) are modified *IS* and *LM* equations from the previous sub section. The real interest rate replaces the nominal rate in the *IS* equation (10). The price level now appears in the *LM* equation. The new equation (9) is a Lucas-type supply specification.

The loss function to be minimized is

$$12. \quad E[L_t] = V(y_t) + \mu V(p_t)$$

We consider the same three types of optimal policy specifications that we did for the simple *IS-LM* model. We do this in a simplified set-up of the model:

$$13. \quad p_t = ky_t + u_t$$

$$14. \quad y_t = -a_1 i_t + v_t$$

$$15. \quad m_t = p_t + b_1 y_t - b_2 i_t + \eta_t$$

In (13-15) each variable is replaced by the deviation from its unconditional expected value.<sup>5</sup>

The supply specification (13) is rearranged with the price level (in deviation form) on the left-hand side. This rearrangement facilitates comparisons with the New Keynesian model in Section 4.

### Policy as an Optimal Instrument Rule

As with the *IS-LM* model we assume that the financial market variables  $m$  and  $i$  are observed. Output ( $y$ ) and the price level ( $p$ ) are assumed to be unobserved. Substitution of (13) and (14) into (15) yields

$$16. \quad m_t = (b_2 - a_1(k + b_1))i_t + (k + b_1)v_t + u_t + \eta_t$$

and thus an observation on a linear combination of the model's three structural disturbances.

With the interest rate as the chosen instrument, the optimal instrument rule will then be of the form

$$17. \quad i_t = \gamma_1 E_t v_t + \gamma_2 E_t u_t$$

where the expectation of each of the disturbances relevant to the loss function (12) are conditional on the linear combination of the error terms in (16).

Combining the instrument rule (17) with the *IS* equation (14) and substituting for the conditional expectations yields the reduced form equation for output.

$$18. \quad y_t = \frac{1}{D}((W + Z)v_t - a_1 X(\eta_t + u_t))$$

From the (13) reduced form for the price level is:

$$19. \quad p_t = \frac{k}{D}((W + Z)v_t - a_1 X\eta_t) + \frac{1}{D}(D - ka_1 X)u_t$$

where  $W = (k + b_1)^2 \sigma_v^2 (1 - a_1 \gamma_1)$   $Z = (1 - a_1 \gamma_2 (k + b_1)) \sigma_u^2 + \sigma_\eta^2$

$$X = \gamma_1 (k + b_1) \sigma_v^2 + \gamma_2 \sigma_u^2 \quad D = (k + b_1)^2 \sigma_v^2 + \sigma_u^2 + \sigma_\eta^2$$

<sup>5</sup> We use the same symbols for variables in deviation form.

From (18) and (19) we calculate the variances: of  $y$  and  $p$  under an instrument rule.

$$20. \quad V(y_t) = \frac{1}{D^2} \left[ (W + Z)^2 \sigma_v^2 + (a_1 X)^2 (\sigma_\eta^2 + \sigma_u^2) \right]$$

$$21. \quad V(p_t) = \frac{1}{D^2} \left[ k^2 (W + Z)^2 \sigma_v^2 + (ka_1 X)^2 \sigma_\eta^2 + (D - ka_1 X)^2 \sigma_u^2 \right]$$

Then, minimizing the loss function (12) with respect to the parameters of the rule yields:

$$\gamma_1^* = \frac{1}{a_1} \quad \gamma_2^* = \frac{\mu k}{a_1(1 + \mu k^2)}$$

### Policy as a Linear Combination of Observable Variables

As in the previous sub section, we assume that the financial market variables  $m$  and  $i$  are observable while output and the price level are not. In this case the optimal combination policy is still of the form given by (6), now with time subscripts.

$$(6') \quad m_t = \lambda_0 + \lambda i_t$$

The model is then composed of equations (13) – (15) and (6').

We solve for  $y_t$  and  $p_t$ , take their variances and substitute the result into the loss function (12). Minimization with respect to  $\lambda$  yields the optimal value of the combination parameter:

$$22. \quad \lambda^* = \frac{a_1(1 + \mu k^2)\sigma_\eta^2 + (a_1 - (a_1 b_1 + b_2)k\mu)\sigma_u^2 - b_2(b_1 + k)(1 + \mu k^2)\sigma_v^2}{\mu k \sigma_u^2 + (b_1 + k)(1 + \mu k^2)\sigma_v^2}$$

The optimal combination parameter is a function of all the parameters of the model and the variances of the model's structural disturbances. It is further the case that at this optimal value of  $\lambda$ , the variances of  $p_t$  and  $y_t$  are equal to those at the optimal values of  $\gamma_1$  and  $\gamma_2$  for the instrument rule. The two policies are equivalent.

### Policy as a Target Rule

Policy as an optimal target rule is of more interest in the *AS-AD* framework than in the *IS-LM* model. In the *AS-AD* model there is a substantive policy choice even if price and output are observed; there is a price-output tradeoff.

In line with the literature on New Keynesian models, a target rule would be of the form:

$$23. \quad \theta y_t + p_t = 0$$

The Optimal Target Rule if  $p_t$  and  $y_t$  are Observed:

An optimal target rule can be viewed as an optimal policy conditioned on a linear combination of observable variables where the target variables themselves are observable. The observability of  $p_t$  and  $y_t$  is required to enable the policymaker to achieve a non random linear combination of two endogenous variables subject to stochastic shocks.

If  $p_t$  and  $y_t$  are observable and the interest rate is the policy instrument, the model comprises (13), (14), (23) and the loss function (12). Substituting (13) and (14) into the target rule and solving for the interest rate yields the instrument setting

$$24. \quad i_t = \frac{1}{a_1} v_t + \frac{1}{(\theta + k)a_1} u_t$$

Substituting (24) back into the *IS* equation (14) and solving for  $y_t$  and  $p_t$  yields

$$25. \quad y_t = -\frac{u_t}{\theta + k}$$

$$26. \quad p_t = \frac{\theta u_t}{\theta + k}$$

If we then compute the variance of the two target variables, substitute the expressions into the loss function (12) and minimize with respect to  $\theta$ , we find the optimal parameter of the target rule

$$27. \quad \theta^* = \frac{1}{\mu k}$$

This result is analogous to the optimal policy setting in New Keynesian models. Notice that the only parameter that matters (apart from the preference parameter  $\mu$ ) is  $k$ . Neither *IS* nor *LM* parameters affect the optimal policy setting. The policymaker moves the output gap, via movements in the policy instrument, to a degree that depends on his ability to affect the price level and the relative weight he attaches to the variance of the price level in the expected loss function. The relative size of the variance of the stochastic disturbances also play no role in the optimal target rule.

The Optimal Target Rule if  $p_t$  and  $y_t$  are Not Observed:

If  $p_t$  and  $y_t$  are not observed then the target rule (23) cannot be implemented.<sup>6</sup> Such a target rule is not a monetary policy rule in accord with our definition in Section 2 (or Svensson's) as a prescribed guide for the conduct of monetary policy.

An implementable target rule is

$$28. \quad \theta E_t y_t + E_t p_t = 0$$

We assume, as in previous sections, that while  $p_t$  and  $y_t$  are not observable, the financial market variables  $i_t$  and  $m_t$  are observed.

Combining the *IS* equation (14) with the *LM* and aggregate supply equations [(13) and (15)] yields a relationship between the observable variables  $m_t$  and  $i_t$  and a linear combination of the error terms.

$$29. \quad m_t = -((k + b_1) a_1 + b_2) i_t + (k + b_1) v_t + \eta_t + u_t$$

From (29) we observe  $((k + b_1) v_t + \eta_t + u_t)$ .

Forming conditional expectations of the disturbances yields:

$$30. \quad E_t v_t = \frac{(k + b_1) \sigma_v^2}{(k + b_1)^2 \sigma_v^2 + \sigma_\eta^2 + \sigma_u^2} ((k + b_1) v_t + \eta_t + u_t)$$

$$31. \quad E_t u_t = \frac{\sigma_u^2}{(k + b_1)^2 \sigma_v^2 + \sigma_\eta^2 + \sigma_u^2} ((k + b_1) v_t + \eta_t + u_t)$$

We now have the reaction function

$$32. \quad i = \frac{1}{a_1} E_t v_t + \frac{1}{(\theta + k) a_1} E_t u_t$$

There are several points to note with reference to the policy set-up given by the target rule (28) and equations (30) – (32) which complete the description of monetary policy. First, if we proceed to solve the model for  $y_t$  and  $p_t$ , compute their variances and substitute into the loss function (12), we can optimize to find the optimal  $\theta$ . This value will be the same value as in (25) where  $p_t$  and  $y_t$  are observed. This follows from the certainty equivalence property of the model due to the quadratic loss function and additive disturbances. Second, while the

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<sup>6</sup> That current aggregate output and prices were not observable to the policymaker was a common assumption in the literature on these earlier Keynesian models [e.g., Benavie and Froyen (1983), Turnovsky (1980)]. Some papers did assume aggregate price was observed [e.g., Aizenman and Frankel (1985)(1986)]. The separation found in the literature on the New Keynesian models between the information problem and the optimization problem was not a feature of this literature. In fact the interrelation of the two problems was a key feature [e.g., Turnovsky (1987)].

optimal value of theta is unchanged and depends only on  $\mu$  and  $k$ , all the parameters of the model and variances of the shocks enter the policy rule via the formation of expectations. These expectations are a necessary component of the policy rule – a *target rule* (28). Finally, substitution of the optimal value of  $\theta$  into the reaction function (32) for the target rule yields the implicit instrument rule corresponding to the target rule (28). This results in

$$33. \quad i_t = \frac{1}{a_1} E_t v_t + \frac{\mu k}{a_1(1 + \mu k^2)} E_t u_t$$

which is optimal explicit instrument rule derived earlier. With the same information assumptions, the three policies - - an explicit instrument rule, a combination policy, and a target rule - - are identical.

#### 4. THE NEW KEYNESIAN MODEL

The debate about instrument versus target rules has taken place mostly within the framework of the New Keynesian model. Here we use a simple version of that model that has been termed the *canonical form* to examine alternative specifications of the optimal policy rule.

##### A. The Case where the Inflation Rate and Output Gap are Currently Observed

The canonical version of the New Keynesian model is given by the following structural equations

$$34. \quad y_t = -a_1 (i_t - E_t \pi_{t+1}) + E_t y_{t+1} + v_t$$

$$35. \quad \pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t$$

where  $y_t$  is now the output gap and  $\pi_t$  is the rate of inflation. The other symbols are as defined in previous models. The policymaker's loss function is given by

$$36. \quad E[L_t] = V(y_t) + \mu V(\pi_t)$$

Equation (34) is the standard forward-looking *IS* equation and equation (35) is a forward-looking Phillips curve. The loss function can be justified by a pragmatic argument that inflation and the output gap are recognized policy objectives. Alternatively, Rotemberg and Woodford (1997) provide a formal justification of (36) as an approximation of a utility-based loss function.

### The Optimal Target Rule

First consider the case where the disturbances are white noise processes and the discount factor ( $\beta$ ) is set to one. Additionally, we restrict our attention to a target rule of the form:<sup>7</sup>

$$37. \quad \theta y_t + \pi_t = 0$$

This target rule represents *simple* commitment, where as Clarida, Gali and Gertler (1999, p. 1678) explain this restricts “the form of the policy rule to the general form that arises in equilibrium under discretion.” We consider a broader class of globally optimal rules in the next section.

Substitution of (34) and (35) into the target rule (37) and solution for the implicit instrument rule yields

$$38. \quad r_t = i_t - E_t \pi_{t+1} = \frac{1}{(\theta + \kappa)a_1} E_t \pi_{t+1} + \frac{1}{a_1} (E_t y_{t+1} + v_t) + \frac{1}{(\theta + \kappa)a_1} u_t$$

where  $r_t$ , the real interest rate, will be assumed to be the policy instrument.

Substituting (38) into the *IS* relationship (34), using the result in the Phillips curve (35) and the fact that with white noise processes

$$E_t(\pi_{t+1}) = E_t(y_{t+1}) = 0$$

yields

$$39. \quad \pi_t = \frac{\theta}{\theta + \kappa} u_t$$

$$40. \quad y_t = -\frac{1}{\theta + \kappa} u_t$$

Putting these solutions into the loss function (36) and minimizing over  $\theta$  yields

$$41. \quad \theta^* = \frac{1}{\mu\kappa}$$

The optimal target rule depends only upon the relative weight given to inflation in the policymakers loss function ( $\mu$ ) and the parameter giving the terms of the output gap – inflation tradeoff ( $\kappa$ )

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<sup>7</sup> We will continue to use  $\theta$  as the parameter in the target rule.

### The Optimal Explicit Instrument Rule

Consistent with the assumption under the target rule, namely that the current state of the economy is observed, the optimal instrument rule is of the form

$$42. \quad r_t = \gamma_1 v_t + \gamma_2 u_t$$

where constant terms in the model are ignored and the error terms are assumed to be white noise processes.<sup>8,9</sup>

Combining (42) with the *IS* equation (34) and the Phillips curve (35), and noting that with white noise disturbances  $E_t \pi_{t+1} = E_t y_{t+1} = 0$ , yields the solutions for the output gap and inflation rate

$$43. \quad y_t = (1 - a_1 \gamma_1) v_t - a_1 \gamma_2 u_t$$

$$44. \quad \pi_t = \kappa(1 - a_1 \gamma_1) v_t + (1 - \kappa a_1 \gamma_2) u_t$$

Computing the variances of the output gap and rate of inflation, substituting these values into the loss function (36), and minimizing yields the optimal  $\gamma_s$ :

$$45. \quad \gamma_1^* = \frac{1}{a_1} \quad \gamma_2^* = \frac{\mu \kappa}{a_1(1 + \mu \kappa^2)}$$

The optimal explicit instrument rule [(42) with the optimal  $\gamma_s$  inserted] is identical to the implicit reaction function corresponding to the optimal target rule (38) in the previous

subsection, once we insert  $\theta^* = \frac{1}{\mu \kappa}$  and  $E_t y_{t+1} = E_t \pi_{t+1} = 0$ .

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<sup>8</sup> As with the target rule, in this section we restrict the analysis to the class of rules typically considered in analysis of policy under discretion. Globally optimal rules under commitment are considered in the next section. With white noise disturbances, equation (42) is the optimal form of the instrument rule under discretion or simple commitment. As stated in the text, the real rate of interest is the policy instrument. This simplification has no bearing on the results reported.

<sup>9</sup> Whether explicit instrument rules of the type (42) represent sensible policy prescriptions is a matter of controversy. The results presented in this paper are based on the method of undetermined coefficients. Application of this method produces well-defined solutions for the target variables if the explicit instrument rule responds only to the exogenous shocks of the model. For a different view, see Svensson and Woodford (2003). They argue that a non-explosive rational expectations equilibrium exists only if implicit or explicit instrument rules contain one or more lagged endogenous variables.

## **B. The Case of Less than Complete Information about the Output Gap and Inflation**

It is unrealistic to assume that the current state of the economy is perfectly observable. Feasible versions of optimal target and instrument rules will employ optimal conditional expectations of the target variables or realizations of the error terms. The feasible target rule would be

$$46. \quad \theta^* E_t y_t + E_t \pi_t = 0$$

The optimal explicit instrument rule is

$$47. \quad r_t = \gamma_1^* E_t v_t + \gamma_2^* E_t u_t$$

The  $\theta^*$  and  $\gamma_s^*$  will be the same values as in (41) and (45), respectively. This follows from the certainty equivalence property of the model.

One possible informational assumption is that financial market variables are observed and can be used to form the conditional expectations in (46) and (47). A variant of this assumption would be that the model contained a portfolio balance schedule, e.g., an *LM* equation. But the presence of observable financial variables in addition to money and of other information (or indicator) variables leads to the general approach set out in Svensson and Woodford (2003), (2004). The solution of this problem leads to conditional expectations of the output gap and inflation rate that depend on the parameters of the model, variances of structural shocks: and of the error terms attached to information variables as estimates of the state variables.

## **C. Globally Optimal Rules: The Timeless Perspective**

Previously in this section attention was confined to the class of rules that arise in considering policy under discretion. Specifically, these rules do not allow for policies to manage expectations in forward-looking models. For policy under commitment this case has been termed simple commitment. In this section we consider target and instrument rules that are globally optimal in the sense that they allow for managing expectations for future periods. For target rules, Woodford has termed this the *timeless perspective*.

### The Optimal Target Rule

The discussion of optimal policy from the timeless perspective necessitates casting the policy problem into an intertemporal framework. The policymaker minimizes an intertemporal loss function that includes the target variables: the output gap and the rate of inflation

$$48. \quad \underset{\pi, y}{Min} \quad E_t \sum_{j=0}^{\infty} \beta^j (y_{t+j}^2 + \mu \pi_{t+j}^2) \quad 0 \leq \beta \leq 1$$

The parameter  $\beta$  is the discount factor. As in the previous section, the respective target value for the output gap and the rate of inflation is zero. The policymaker minimizes the loss function (48) with respect to the target variables subject to the constraint imposed by the Phillips curve (35), rewritten here

$$49. \quad \pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t$$

Hence the Lagrangean for the policy problem becomes:

$$50. \quad \Gamma_t = E_t \left\{ \sum_{j=0}^{\infty} \beta^j [(y_{t+j}^2 + \mu \pi_{t+j}^2) + \lambda_{t+j} (\beta \pi_{t+j+1} + \kappa y_{t+j} + u_{t+j} - \pi_{t+j})] \right\}$$

The global or timeless perspective indicates that the policymaker ignores the optimizing condition for the rate of inflation that prevails in period  $I$ . Accordingly, combining the first-order condition for inflation ( $t=2,3,\dots$ ) with that for the output gap ( $t=1,2,3,\dots$ ) yields the target rule that guides optimal policy from a timeless perspective:<sup>10</sup>

$$51. \quad \frac{1}{\mu \kappa} (y_t - y_{t-1}) + \pi_t = 0$$

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<sup>10</sup> For a step-by-step derivation of the globally optimal target rule see Froyen and Guender (2007), McCallum and Nelson (2004) or Woodford (1999).

In this target rule the “past” - represented by the lagged output gap – matters for setting optimal policy in the present. The relative weight on the *change* in the output gap in (51) is the same as the relative weight on the output gap proper that guides policy under simple commitment (equation 37).

Next, substitute the Phillips curve (setting  $\beta = 1$ ) and the *IS* relation into the optimal policy rule and solve for the interest rate:

$$52. \quad r_t = \frac{1}{a_1}(E_t y_{t+1} + v_t) + \frac{1}{a_1(\theta + \kappa)}(E_t \pi_{t+1} + u_t - \theta y_{t-1}) \quad \theta^* = \frac{1}{\mu\kappa}$$

Equation (52) is the policymaker’s reaction function (or implicit instrument rule) under the timeless perspective. The above equation is in turn substituted into the *IS* relation to obtain the reduced form equation for the output gap:

$$53. \quad y_t = \frac{\theta}{\theta + \kappa} y_{t-1} - \frac{1}{\theta + \kappa} (E_t \pi_{t+1} + u_t)$$

Combining equation (53) with the policy rule (equation 51) yields the reduced form equation for the rate of inflation:

$$54. \quad \pi_t = \frac{\kappa\theta}{\theta + \kappa} y_{t-1} + \frac{\theta}{\theta + \kappa} (E_t \pi_{t+1} + u_t)$$

Application of the method of undetermined coefficients results in the final form equation for the output gap and the rate of inflation. For the case of white-noise disturbances, the output gap and the rate of inflation under policy from the timeless perspective evolve as follows:

$$55. \quad y_t = \frac{1}{\tau} y_{t-1} - \frac{\mu\kappa}{\tau} u_t$$

$$56. \quad \pi_t = \frac{(\tau - 1)}{\tau\mu\kappa} y_{t-1} + \frac{1}{\tau} u_t$$

where

$$\tau = 1 + \frac{\kappa^2 \mu}{2} + \frac{\kappa \mu \sqrt{\kappa^2 + \frac{4}{\mu}}}{2} > 1$$

After using (55) and (56) to eliminate the conditional expectations of the output gap and the rate of inflation in (52), we can restate the reaction function (implicit instrument rule) as:

$$57. \quad r_t = \frac{1}{a_1} (v_t + \Lambda (\mu \kappa u_t - y_{t-1}))$$

where

$$\Lambda = \frac{1}{\tau^2 (1 + \mu \kappa^2)} (\tau (\tau - 1) - \mu \kappa^2)$$

The implicit instrument rule has the interest rate respond mechanically to an *IS* disturbance. The policymaker's preference parameter conditions the interest rate response to the cost-push shock and to the lagged output gap.

### The Optimal (Explicit) Instrument Rule

Inspection of (57) reveals that the implicit instrument rule responds to the shocks of the model economy and the lagged output gap. The lagged output gap appears in the implicit instrument rule because it forms part of the optimal target rule. The question arises of why (or how) the policymaker choosing an explicit instrument rule would arrive at this specification. Why would the rule under simple commitment (42, with 45 inserted) not still be followed?

The target rule is what Woodford (2003) terms a "higher level" description of policy. It comes directly from the first-order conditions derived in the previous subsection. The choice of the information set for the instrument rule is less clear cut.

In particular, the New Keynesian model we are considering contains no lagged endogenous variables and the shocks are white noise processes. So there is no obvious reason for an inertial component in the explicit instrument rule. Still, a number of papers [e.g. Turnovsky (1980), Weiss (1980), Canzoneri, Henderson and Rogoff (1983)] consider instrument rules with inertial elements. Moreover, in these papers the role of the inertial

elements is, as in the case of the target rule under the timeless perspective, to influence the formation of expectations. We return to the issue of advantages of a “higher level” policy formulation in Section 5. Here we simply assume that the explicit instrument rule responds to the same information set as the globally optimal target rule.

This point of departure yields the following specification of the explicit instrument rule:

$$58. \quad r_t = \gamma_1 v_t + \gamma_2 u_t + \gamma_3 y_{t-1}$$

The above explicit instrument rule is substituted into the *IS* relation. The resulting equation is then combined with the Phillips curve. Applying the method of undetermined coefficients allows us to solve the model for the variances of the endogenous variables:

$$59. \quad V(y_t) = \frac{\phi_{12}^2 \sigma_u^2}{1 - \phi_{13}^2}$$

$$60. \quad V(\pi_t) = \left( \phi_{22}^2 + \frac{\phi_{23}^2 \phi_{12}^2}{1 - \phi_{13}^2} \right) \sigma_u^2$$

where

$$\phi_{12} = -\frac{a_1 \gamma_2}{1 - \phi_{13}} \quad \phi_{13} = \frac{1 + \sqrt{1 + a_1 \gamma_3}}{2} \quad \phi_{11} = \phi_{21} = 0$$

$$\phi_{22} = -\kappa a_1 \gamma_2 \left( \frac{\phi_{13} - a_1 \gamma_3}{(1 - \phi_{13})^2} + 1 \right) + 1 \quad \phi_{23} = \kappa \left( \frac{\phi_{13}^2 - a_1 \gamma_3}{1 - \phi_{13}} \right)$$

and

$$61. \quad y_t = \phi_{11} v_t + \phi_{12} u_t + \phi_{13} y_{t-1}$$

$$62. \quad \pi_t = \phi_{21}v_t + \phi_{22}u_t + \phi_{23}y_{t-1}$$

The policymaker's objective can then be stated as follows:

$$63. \quad \text{Min}_{\gamma_1, \gamma_2, \gamma_3} E[L_t] = V(y_t) + \mu V(\pi_t)$$

The optimal settings of the policy parameters that minimize the above expected loss function are:

$$\gamma_1^* = \frac{1}{a_1} \quad \gamma_2^* = \frac{(1-Z)^3(3+Z)\kappa\mu}{8a_1(1+2a_1\gamma_3 - Z + 2\kappa^2\mu)} \quad \gamma_3^* = \frac{2\mu\kappa^2 - (X - \mu\kappa^2)(1 + \mu\kappa^2)}{2a_1}$$

$$Z = \sqrt{1 + 4a_1\gamma_3} \quad X = \kappa\sqrt{\mu(4 + \mu\kappa^2)}$$

It remains to be seen whether the  $\gamma_i^*$   $i = 1, 2, 3$  produce the optimal response to the *IS* shock, the cost-push, and the lagged output gap that emerges under the optimal target rule. Given the complexities of the coefficients in the two rules, the performance check is best done by numerical analysis. The results of this exercise appear in the top two panels of Table I.<sup>11</sup> Two important results stand out. First, the explicit instrument rule (labeled Case I) elicits the same response to the shocks and lagged output gap as the implicit instrument rule under the target rule (the top panel). Second, as a direct consequence, the variances of the endogenous variables and of the policy instrument are the same under both types of rules. In sum, the optimal explicit instrument rule replicates the behavior of the target variables and the policy instrument under the target rule. There is no difference between the two approaches as long as the specification of the optimal explicit instrument rule relies on the same information set as the target rule approach.

## 5. KEY ISSUES

Previous sections illustrate the equivalence of optimal target and instrument rules in a number of models. Taking a monetary policy rule as a prescribed guide for the conduct of

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<sup>11</sup> The remaining three panels on the table will be discussed in Section 5.

monetary policy, we mean by “equivalence” that within the models considered the two types of policy rules result in the same policy actions by a central bank. Given this equivalence, on what grounds could a preference for one type of rule versus the other be based? Along what other dimensions may there be relative advantages to one type of rule? The other dimensions we consider are robustness, transparency, and the “higher order” of one type of rule. We begin with a different type of comparison: that of optimal target rules with simple (or suboptimal) instrument rules. This issue has received attention most often with regard to so-called Taylor rules.<sup>12</sup>

### **A. What is Wrong with Taylor Rules?**

This sub section heading is taken from Svensson’s (2003) paper. Our analysis leads to a simple answer which does not seem contradictory to Svensson’s. Taylor’s rule and other simple rules will in many models not be optimal instrument rules because they do not efficiently exploit all available information. Svensson (2000) provides an example of this in the context of an open-economy model. A Taylor rule responds to only a small subset of the model’s 11 state variables. The optimal reaction function responds to all of them.<sup>13</sup>

Moreover, even in models where inflation and the output gap are the only state variables, it is as Svensson (1999, p. 608) points out “usually inefficient to let the instrument respond to target variables, compared to letting the instrument respond to the determinants of the target variables.” In the forward-looking New Keynesian model in Section 4.A, for example, the implicit instrument rule derived from the optimal target rule (equation 38) responds to the current values of the *IS* and Phillips curve disturbances. The *IS* disturbance is simply offset and the response to the Phillips curve disturbance results in the optimal trade-off in the response of the output gap and inflation rate. A Taylor type rule for the model is

$$(38') \quad r_t = \bar{r} + \lambda_1(\pi_t - \pi^*) + \lambda_2 y_t$$

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<sup>12</sup> A key reason for bringing in a comparison to Taylor rules is that in many places when Svensson turns to a comparison of the merits of *optimal* target rules versus instrument rules, he considers Taylor rules as an example of the latter category. See for examples: Svensson (1997, p. 111), (2002, p. 774).

<sup>13</sup> As an illustration of this point, in Ball (1999b), which is a simple backward-looking New Keynesian model, the optimal reaction function is a Taylor-type rule. In Ball (1999a), an open-economy version of the same model, the exchange rate is an additional information variable and the optimal policy becomes an MCI (monetary conditions index) which contains the real exchange rate.

With this rule, to completely offset the effect of the *IS* disturbance requires  $\lambda_2 \rightarrow \infty$ . This rules out the optimal response to the Phillips curve disturbance. There is less information content in the realized value of the target variables than in the realized values of the individual shocks.

A third inefficiency in the Taylor rule arises when we relax the assumption that the target variables are observed and assume that the instrument can respond only to the expectations of the disturbances or target variables. The Taylor rule allows only a response to a past observation of the targets. The optimal (implicit) instrument rule, in this case equation (46), allows a response to all available information.<sup>14</sup>

There are, however, two points to keep in mind. First, the *optimal* explicit instrument rule for either information assumption (equations 42 and 47) is, as previously shown, identical to the implicit instrument rule for the optimal target rule. The comparison is with a simple instrument such as a Taylor rule. Second, Taylor (e.g., Taylor, 1993, 1999, 2009) asks whether a simple rule might not have resulted in better economic performance than resulted from actual policy. Taylor's proposal is in the spirit of Milton Friedman's (1968, p. 16) argument that in monetary policy formation, "the best is likely to be the enemy of the good." In contrast Svensson (2003) worries that the good might be enemy of the best.

## **B. Robustness**

A key advantage cited by advocates of target rules is greater robustness. Svensson (2002, p. 778) states that "The specific targeting rule is relatively robust, in that it only depends on the marginal tradeoffs between the target variables, that is, the derivatives of the loss function and aggregate-supply relation with respect to the target variables." Svensson (1998, p. 22) argued that "As concerns the difference between targeting rules and instrument rules, the optimal instrument rules generally depend on all model parameters, whereas even rather specific targeting rules are arguably more model-robust than instrument rules in that they may depend on only parts of the model."<sup>15</sup> More generally Svensson (2005, p. 622) praises a target rule as "a compact, robust, structural and therefore practical representation of goal-directed monetary policy."

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<sup>14</sup> Here, by a Taylor rule we mean the type of simple instrument rule used by Taylor (1993), (1999) and analyzed by Svensson (2003). The term is used loosely in the literature with adjectives such as "forward looking." These modified Taylor-types rules may respond to additional information.

<sup>15</sup> See also Woodford (2003, p. 614).

To appraise the robustness of target rules it is useful to consider separately the case where the target variables are observed and the case where they are not observed.

### Inflation and Output Gap are Observed

If the target variables are observed, the optimal target rule is then given by

$$37. \quad \theta^* y_t + \pi_t = 0$$

where  $\theta^* = 1/\mu\kappa$ .

The target rule depends only on the weight on inflation in loss function relative to the output gap and on the slope of the Phillips curve.

As in the case of earlier models such as Poole's (1970) combination policy (equation 6), the target rule (37) can be represented as the response of one observable to the other. In the case of the new Keynesian model:

$$y_t = -(\mu\kappa)\pi_t$$

The output gap responds to the inflation rate. This assumes both target variables are observable. The optimal policy problem is to see that output and inflation respond in optimal proportions to a Phillips curve shock.

A comparison to the aggregate demand-aggregate supply framework in section 3.B is helpful. Within that framework the optimal target rule (equation 23) specifies a policy response of output to price,

$$(23') \quad y_t = -\frac{1}{\theta^*} p_t.$$

Within the aggregate demand-aggregate supply framework, this can be seen as the policymaker moving the aggregate demand curve in response to a supply shock to achieve the optimal  $p$ - $y$  combination. All demand-side shocks are simply offset. In this case and in the New Keynesian model it is therefore not surprising that policy might be robust to changes in the demand-side of the model. Within the target rule framework a change on the demand side - an additional *IS* shock for example - would affect the implicit instrument rule that implements the target rule (equation 38 in the New Keynesian model). This additional *IS* shock would also change the optimal explicit instrument rule (equation 42); the two rules are identical.

Still the robustness of even the target rule itself is limited. The marginal tradeoffs between the target variables, on which the parameter of the target rule depends, will involve parameters of the demand side of the model in several not unrealistic cases. In the open economy the presence of a direct exchange rate channel in the Phillips curve, will result in the optimal  $\theta$  depending on all the parameters in the model (see Froyen and Guender (2007, pp. 245-253)).<sup>16</sup> The presence of a cost channel for monetary policy (see, for example, Ravenna and Walsh, 2006) will also result in the target rule parameter ( $\theta^*$ ) depending on the demand-side of the model.

### Inflation and the Output Gap not Observed

The target variables are in reality not currently observable. That case is of interest only because in the class of models we consider certainty equivalence holds. When the current state of the economy is not perfectly observable, the optimal target rule is

$$46. \quad \theta^* E_t y_t + E_t \pi_t = 0$$

Here any change in the model disturbances or parameters, even if it did not affect  $\theta^*$ , would affect the target rule via an effect on the optimal conditional expectations of the target variables. The optimal explicit instrument rule

$$47. \quad r_t = \gamma_1 E_t v_t + \gamma_2 E_t u_t$$

would be altered in an analogous way. It is hard to see how the two frameworks differ in robustness. A new shock such as a credit market shock would directly affect the explicit instrument rule (47) while changing the target rule (46) via the optimal expectations of the targets.

As noted in Section 3, Svensson and Woodford (2003) highlight the separation principle “according to which the selection of the optimal policy (the optimization problem) and the estimation of the current state of the economy (the estimation or signal-extraction problem) can be treated as separate problems. But policy formulation consists of *both* problems. Robustness of a policy rule must encompass both aspects of the policy rule (46 or 47).

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<sup>16</sup> A direct exchange rate channel in the Phillips curve is a property of some open-economy models and not others. This channel is present in Ball (1999a), Svensson (2000), and Guender (2006) but not in Clarida, Gali and Gertler (2001,2002) or Gali and Monacelli (2005).

### **C. Transparency**

Isolating a part of a rule, the choice of  $\theta$ , still may be useful for the next issue we consider – transparency. There seems no doubt that the desire for effective communication with the public – commonly referred to as transparency – has been a goal of central banks in moving to inflation targeting. A target rule such as (equation 37) conveys an important part of the monetary policy rule to the public in a compact form. It tells the public the policymaker’s marginal rate of substitution between the policy goals. How does this compare with the transparency of instrument rules?

If the comparison is with a simple instrument rule such as a Taylor rule, the target rule is likely to be more transparent. A Taylor rule for example specifies the relative weight for the policy objectives but it seems unlikely that a central bank would commit to fixed weights in such a simple rule. The policy responses are likely to be state dependent.

A rule specifying the tradeoffs between target variables is especially transparent if one target, the inflation rate, has the dominant role in the loss function. As  $\mu \rightarrow \infty$ , the policymaker simply announces the target inflation rate and policy is transparent. This fact, is probably sufficient to explain the movement to inflation targeting by many central banks that wanted to be transparent and to “get the mean right” for inflation. A targeting rule such as equation (37) can still be transparent if there are multiple goals for the central bank as long as the central bank is willing to specify the relative weight put on the competing goals ( $\theta^*$ ).

If the comparison over the dimension of transparency is between a central bank following a target rule and one following an optimal explicit instrument rule, the situation is different. The optimal instrument rule is derived from the policymaker’s loss function which can be communicated to the public. The only additional information conveyed by the target rule is the tradeoff that policymakers believe they face ( $\kappa$ ) in pursuing the two targets.

In practice it does not seem that inflation targeting central banks communicate the perceived tradeoff between multiple goals. The gain in transparency is rather the commitment to an announced target range for one important goal of policy. Mervyn King (2005), governor of the Bank of England, argues that under inflation targeting “it is easier to ... to influence inflationary expectations.” Ben Bernanke (2004), proposed an “incremental step towards inflation targeting,” namely, the announcement of a long-run inflationary objective, on the same grounds. Critics of inflation targeting such as Benjamin Friedman (2004, 2008) argue

that for a central bank interested in stabilization of real variables as well as price stability, inflation targeting leads to “obfuscation and opacity” and possibly a “distorted set of policy objectives” if the central bank believes its own rhetoric.<sup>17</sup>

#### **D. A Higher-Order Policy Rule**

Discussing a target rule, Woodford (2003, p. 533) states that “A rule of this kind represents a ‘higher-level’ description of policy than an explicit specification of the instrument setting in each possible state of the world ...” The targeting rule as Svensson (1999, p. 617) explains “can be expressed as a system of equations representing a first-order condition for a minimum of the loss function.” The targeting rule is structural and specifies an equation that “the targets must fulfill.” Higher level then refers to closeness to the policymaker’s loss function; a rule closely based on optimality given model structure. In part, this question of higher-order rules is related to robustness. Because it is only a step removed from the loss function, the target rule will be robust to some changes at more removed levels that would have an impact on an instrument rule. These issues have been addressed in Section 5.B.

Where this issue of higher-level policy rules has additional implications is with consideration of globally optimal policy rules in Section 4.C in particular concerning the “timeless perspective” and role of policy inertia in optimal policy. As explained there, the target rule approach by focusing on the first-order conditions leads directly to the role of an inertial component in monetary policy. Traditional applications of the instrument variable approach do not so clearly define a role for a reaction to past deviations of the output gap from its optimal level. Papers in the literature that consider optimal instrument variables have taken into account the role that policy inertia might have in stabilization (e.g. Turnovsky (1980), Weiss (1980), Canzoneri, Henderson and Rogoff (1983). The choice of an inertial rule in this framework can be considered more ad hoc - - not coming directly from the optimization process.

A general solution procedure for rational expectations models going back to Muth (1961) begins by postulating a solution of the model containing an infinite distributed lag on

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<sup>17</sup> Faust and Henderson (2004) discuss the positive and negative aspects of inflation targeting as a communications strategy for monetary policy when a central bank has multiple objectives. Walsh (2009, p.22) argues, that inflation targeting central banks in practice provide a great deal of information about their projections for real output.

the disturbances in the model along with any trends or constant terms. Following this procedure to choose the optimal instrument rule would in some models also lead to the inclusion of lagged responses to model errors. This is the procedure followed by Turnovsky (1980) which led him to a rule that allows for *prospective* monetary policy and thus a response of the instrument, in his model the money supply, to lagged disturbances.

This leads us to examine the degree to which inclusion of lagged disturbances in the instrument rule leads it to mimic the optimal target rule under the timeless perspective. The results of this exercise are shown in the lower three panels of Table 1, labeled Case 2-4. Recall that Case 1 simply shows that an instrument rule including the current shocks and lagged output gap can duplicate the performance of the optimal target rule.

In these panels, Case 2 is an explicit instrument rule that responds to the current and the lagged cost-push shock in addition to the current-period demand shock. The stabilization performance of this instrument rule is clearly worse than the optimal instrument rule. The welfare loss amounts to 4.6 percent.<sup>18</sup> The primary reason for the sub-par performance is the weak response of the instrument to the current cost-push shock. This suboptimal adjustment of the policy instrument brings about a stabilization bias. Compared to the optimal instrument rule, application of the explicit instrument rule reduces the variance of the output gap – from 0.02139 to 0.0044 - but only at the expense of increasing the variance of the rate of inflation from 0.83471 to 0.89108. The variance of the policy instrument is approximately the same under both instrument rules. Cases 3 and 4 consider explicit instrument rules that respond to shocks further in the past. Several findings are noteworthy. As the number of lagged shocks in the explicit instrument rule increases

- the policymaker reacts more emphatically to the current-period cost push shock;
- the stabilization bias decreases, i.e. the variance of the output gap increases but the variance of inflation decreases;
- the welfare loss decreases: 3.6 percent for 5 lags and 2.7 percent for 9 lags.

Implementing monetary policy via an explicit instrument rule that draws on past realizations of the cost-push shock activates the expectations channel of monetary policy in

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<sup>18</sup> The welfare loss is calculated as  $\frac{E[L_t^{Case i}] - E[L_t^{OPT}]}{E[L_t^{OPT}]} \times 100 \quad i=2,3,4.$

much the same manner as optimal policy under a target rule. The extent to which an explicit instrument rule can replicate the outcomes associated with the target rule improves with the number of lagged cost-push shocks included.

In sum, previous studies of optimal instrument rules have taken into account an expectations channel that has features in common with the channel under the timeless perspective. The criteria for choosing such an inertial instrument rule are more of a trial and error process than with the target rule approach.

## 6. CONCLUSION

We believe that a comparison of optimal instrument rules with optimal target rules clarifies the issues in the instrument – target rule debate. This comparison exploits the equivalence of the two types of optimal rules to limit the issues that stem from the inherent properties of the rules. In the models we consider the target rule approach leads to an implicit instrument rule that is equivalent to the optimal explicit instrument rule. By this we mean that a central bank following either of these rules would take the same actions.

This fact narrows the range of advantages one approach can have over the other. Relative advantages still might exist. We discuss such advantages along the dimensions of robustness, transparency and the “higher order “or more “structural” nature of the target rule procedure.

Many issues in the literature arise from a comparison of target rules with simple not optimal explicit instrument rules. Our approach leads to the conclusion that many advantages claimed for the target rule approach are because these simple instrument rules are suboptimal instrument rules - - not because they are instrument rules per se. A Taylor rule will, for example, in general fail to exploit all available relevant information because of its simplicity not because it is an instrument rule. A Taylor rule might in practice be a “good” rule but that requires a historical or other empirical type of examination. A simple target rule such as “the inflation forecast should equal 2%” might be a good rule but in general would not be optimal.

But is the examination of optimal instrument rules of interest in itself? McCallum and Nelson (2004, p. 600) agree with Svensson’s statement that commitment to optimal instrument rule is “completely impractical” and “has no advocates.” The optimal instrument rule will in

general be complex and will change with any changes in the parameters of the model and shocks that affect the model equations. We have seen in Sections 3 and 4 that the same is true of the optimal target rule once the unobservable nature of the target variables is recognized. Imperfect information about target variables complicates the construction of target rules and hence detracts from their appeal.

From a theoretical perspective, the application of optimal instrument or target rules leads to equivalent results. Yet, in practice, relative advantages might still exist. We discuss such advantages along the dimensions of robustness, transparency, and the “higher order” or more “structural” nature of the target rule in Section 5. The target rule has an advantage in communicating the tradeoff between the policy goals to the public in a compact way. Including this piece of information in the optimal policy framework also helps to define the form of the optimal instrument rule. Balanced against this is the fact that central banks with multiple goals may be unwilling to specify this tradeoff. Instead they may specify only an inflation target, therefore losing the transparency advantage to the approach. Indeed, a clear focus on the inflation target seems to be the norm in practice. The target rule approach may also place too much emphasis on the tradeoff between the target variables in the presence of Phillips curve shocks and too little on the stabilization of aggregate demand in the face of *IS* and financial market shocks. The one clear advantage of the target rule is its ability to *motivate* inertia in policy-making from a timeless perspective. Whether this sole distinct advantage warrants the conclusion that target rules are unambiguously superior to optimal instrument rules is debatable.

<b>Table I: Target vs Instrument Rules from the Timeless Perspective: <math>a_1 = .5, \kappa = .05, \sigma_u^2 = .9, \sigma_v^2 = .9, \mu = 1</math></b>				
<b>Target Rule</b>				
$r_t = c_v v_t + c_u u_t + c_y y_{t-1}$ $c_v = \frac{1}{a_1}, c_u = \frac{(\tau(\tau-1) - \mu\kappa^2)\mu\kappa}{a_1\tau^2(1 + \mu\kappa^2)}, c_y = -\frac{(\tau(\tau-1) - \mu\kappa^2)}{a_1\tau^2(1 + \mu\kappa^2)}$ $c_v = 2, c_u = .004639, c_y = -.092775$	$V(y_t)$	$V(\pi_t)$	$E(L_t^{OPT})$	$V(r_t)$
	.021396	.834715	.856111	3.60020
<b>Instrument Rules</b>				
<b>Case 1: <math>r_t = \gamma_1 v_t + \gamma_2 u_t + \gamma_3 y_{t-1}</math></b>				
$\gamma_1^* = \frac{1}{a_1} = 2 \quad \gamma_2^* = \frac{(1-Z)^3(3+Z)\kappa\mu}{8a_1(1+2a_1\gamma_3 - Z + 2\kappa^2\mu)} = .004639 \quad \gamma_3^* = \frac{2\mu\kappa^2 - (X - \mu\kappa^2)(1 + \mu\kappa^2)}{2a_1} = -.092779$ $Z = \sqrt{1+4a_1\gamma_3} \quad X = \kappa\sqrt{\mu(4 + \mu\kappa^2)}$	$V(y_t)$	$V(\pi_t)$	$E(L_t)$	$V(r_t)$
	.021397	.834716	.856114	3.60013
<b>Case 2: <math>r_t = \gamma_1 v_t + \gamma_2 u_t + \gamma_4 u_{t-1}</math></b>				
$\gamma_1^* = \frac{1}{a_1} = 2 \quad \gamma_2^* = \frac{\kappa^3\mu^2}{a_1(1+3\kappa^2\mu + \kappa^4\mu^2)} = .00024 \quad \gamma_4^* = \frac{\kappa\mu}{a_1(1+3\kappa^2\mu + \kappa^4\mu^2)} = .09925$	$V(y_t)$	$V(\pi_t)$	$E(L_t)$	$V(r_t)$
	.004444	.891084	.895528	3.60886
<b>Case 3: <math>r_t = \gamma_1 v_t + \gamma_2 u_t + \gamma_4 u_{t-1} + \gamma_5 u_{t-2} + \gamma_6 u_{t-3} + \gamma_7 u_{t-4} + \gamma_8 u_{t-5}</math></b>				
$\gamma_1^* = \frac{1}{a_1} = 2 \quad \gamma_2^* = .001199 \quad \gamma_4^* = .000955 \quad \gamma_5^* = .000714$ $\gamma_6^* = .000475 \quad \gamma_7^* = .000237 \quad \gamma_8^* = .094972$	$V(y_t)$	$V(\pi_t)$	$E(L_t)$	$V(r_t)$
	.012538	.874453	.886991	3.60812
<b>Case 4: <math>r_t = \gamma_1 v_t + \gamma_2 u_t + \gamma_4 u_{t-1} + \gamma_5 u_{t-2} + \gamma_6 u_{t-3} + \gamma_7 u_{t-4} + \gamma_8 u_{t-5} + \gamma_9 u_{t-6} + \gamma_{10} u_{t-7} + \gamma_{11} u_{t-8} + \gamma_{12} u_{t-9}</math></b>				
$\gamma_1^* = \frac{1}{a_1} = 2 \quad \gamma_2^* = .002039 \quad \gamma_4^* = .001799 \quad \gamma_5^* = .001565 \quad \gamma_6^* = .001334 \quad \gamma_7^* = .001106 \quad \gamma_8^* = .000882$ $\gamma_9^* = .000659 \quad \gamma_{10}^* = .000438 \quad \gamma_{11}^* = .000219 \quad \gamma_{12}^* = .087671$	$V(y_t)$	$V(\pi_t)$	$E(L_t)$	$V(r_t)$
	.018792	.860658	.87945	3.60693

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