The Timeless Perspective vs. Discretion: Theory and Monetary Policy Implications for an Open Economy

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Abstract:

Compared to the standard Phillips curve, an open-economy version that features a real exchange rate channel leads to a markedly different target rule in a New Keynesian optimizing framework. Under optimal policy from a timeless perspective (TP) the target rule involves additional history dependence in the form of lagged inflation. The target rule also depends on more parameters, notably the discount factor as well as two IS and two Phillips curve parameters. Stabilization policy in this open economy model is no longer isomorphic to policy in a closed economy. Because of the additional history dependence in an open economy target rule price level targeting is no longer consistent with optimal policy. The gains from commitment are smaller in economies where the real exchange rate channel exerts a direct effect on inflation in the Phillips curve.

Key Words: Timeless Perspective, Discretion, Price Level Targeting, Exchange Rate Channel.

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Introduction

For some time now central banks and academics have been preoccupied with the way monetary policy ought to be conducted in an era of relative price stability. Woodford (1999a) proposes that the course of monetary policy in a forward-looking New Keynesian framework be set from a timeless perspective (commitment). This form of policy has a number of desirable features. To begin with, policy from a timeless perspective introduces history-dependence into the conduct of monetary policy because it is based on an optimal policy rule that depends on the *change* in the output gap. The policy instrument responds to a cost-push shock in the current and subsequent periods until the target variables return to their original targets. The gradual adjustment process gives rise to persistence in the behavior of the output gap and the rate of inflation. Because the conduct of policy is history-dependent under policy from a timeless perspective, this strategy dominates pure discretion under which the response of the target variables to the cost-push shock is confined to the current period. Moreover, since policy from a timeless perspective is a time-consistent form of optimal policy under commitment it serves as a standard of comparison for forms of discretionary policy that also inject an element of history-dependence into policymaking such as price-level targeting, a speed limit policy, nominal income growth targeting, average inflation targeting or money growth targeting. Jensen (2002), Néssen and Vestin (2005), Soederstroem (2005), Vestin (2006), and Walsh (2003) evaluate the aforementioned discretionary strategies in a closed economy setting and verify to what extent these policies achieve the optimal stabilization results under policy from a timeless perspective.

This paper focuses on two key differences between policy in an open and a closed economy. First, it analyzes optimal monetary policy from a timeless perspective in a simple forward-looking framework to show that the open-economy target rule is far more complex than its closed-economy counterpart. Optimal stabilization policy in this open economy model is no longer isomorphic to policy in a closed economy. Second, the paper examines the connection between optimal policy and price level targeting and finds that the latter is not synonymous with the former in the proposed open economy model. Central to our discussion of policy in an open economy is a Phillips curve that features a real exchange rate channel. This exchange rate channel appears in the aggregate Phillips curve because domestic firms are concerned about their competitiveness at home and in world markets where their products compete with those produced by foreign firms. Briefly, an important objective of the typical cost-minimizing domestic firm is to avoid fluctuations in its firm-specific terms of trade. Hence an incipient rise in the foreign price of the competing foreign good or a rise in the nominal exchange rate leads the typical domestic firm to raise the price of its output. Thus external factors induce a firm to alter the price of output, which it sets in domestic currency.

Earlier contributions that examine the implications of the existence of an exchange rate channel for the conduct of monetary policy are Ball (1999), Walsh (1999), Svensson (2000), and Guender (2006). Ball motivates the real exchange rate channel in the Phillips curve in a backward-looking framework by assuming that foreign producers are concerned only about receipts in their home currency. Any change in the nominal exchange rate is offset by adjusting the nominal price of the good in the foreign country. Positing a linear target rule, Ball finds that optimal policy requires a central bank to follow a monetary conditions index rather than a Taylor-type rule. Walsh (1999) derives an open-economy Phillips curve in a forward-looking model where the nominal wage demands are tied to the CPI. In a model that mixes elements of backward- and forward-looking behavior, Svensson (2000) introduces a Phillips curve where the expectation, formed in the past, of the change in the real exchange rate affects the current rate of inflation. He discusses a number of different policy strategies under discretion but does not derive the underlying endogenous target rules. Such an explicit target rule is derived in a forward-looking open economy model by Guender (2006) where firms are guided in their domestic pricing decisions by a benchmark price that is set in the world market. Such pricing behavior at the firm-level gives rise to an aggregate Phillips curve that depends on the real exchange rate and a target rule guiding optimal monetary policy that includes demand-side parameters. Guender considers only optimal policy under discretion. [1]

Other contributions downplay or dismiss the importance of a real exchange rate channel in the Phillips curve. Drawing on empirical evidence that shows only weak correlations between changes in the nominal exchange rates and inflation rates for a number of countries, McCallum and Nelson (2000, p. 89) are sceptical about the existence of a direct exchange rate channel and its relevance in policymaking. Moreover, in their theoretical set-up of an open economy the effect of the real exchange rate on the output gap is neutralized as it affects both the level of actual output and the level of potential output in the same way. As a consequence, their Phillips curve is the same as in a closed economy. Clarida, Gali, and Gertler (2001, 2002) and Gali and Monacelli (2005) derive essentially a closed-economy Phillips curve too except that the coefficient on marginal cost is sensitive to the degree of openness of the economy. These papers assume that firms that operate in small open economies set domestic prices without reference to world market prices or consideration of the effects of terms of trade changes on their competitiveness.

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1 The design of optimal monetary policy is sensitive to the degree of exchange rate pass-through. Monacelli (2005) argues that incomplete exchange rate pass-through drives a wedge between policymaking in open versus closed economies. He derives an open economy Phillips curve where the deviation of the world price from the domestic currency price of imports affects domestic and CPI inflation.

This paper takes a contrary view. It emphasizes that domestic firms value stability of their terms of trade in addition to stable prices. This concern forces cost-minimizing domestic firms to take account of expected changes in their firm-specific terms of trade when altering the domestic price of output. Changes in the nominal exchange rate and changes in the price charged by competing foreign firms are beyond the control of the typical domestic firm. Yet such changes affect its competitiveness. The only way that a domestic firm can counteract such pressure is to adjust its domestic price in such a way so that the overall cost to the firm is minimized. If such pricing behavior applies to the typical firm, aggregation over all firms leads to a Phillips curve where the expected change in the real exchange rate impacts on domestic inflation.[2]

The implications for optimal monetary policy are stark. Once the existence of a real exchange rate channel in the Phillips curve is acknowledged, the optimal target rule under policy from a timeless perspective becomes vastly different from the standard target rule.[3] The new target rule depends on multiple parameters that appear in both the IS relation and the Phillips curve as well as on the discount factor. The proposed target rule is history-dependent too but differs from the standard target rule in a critical way: under policy from a timeless perspective the proposed target rule also features the lagged rate of inflation in addition to the lagged output gap. However, unlike the output gap, the rate of inflation does not enter the target rule in first-difference form.

Optimal stabilization policy changes dramatically. In the absence of a real exchange rate channel in the Phillips curve it is optimal for the policymaker to fix the output gap as doing so ensures an optimal response to demand-side disturbances and leaves domestic inflation unaffected. If this channel does exist, however, the policymaker can no longer perfectly offset demand-side disturbances. Specifically, the policymaker's relative aversion to inflation variability shapes his response to demand-side disturbances. Both inflation and the output gap deviate from target.

Further analysis shows that a real exchange rate channel in the Phillips curve matters greatly in other policy-related contexts. Comparing policy from a timeless perspective to discretion, we find that the gains from commitment are smaller in an open economy where a real exchange rate channel is operative in the Phillips curve. Under policy from a timeless perspective, an adverse cost-push shock prompts the policymaker to "lean with the wind", i.e. lower the nominal interest rate if there is no exchange rate channel in the Phillips curve. If this channel exists, then the policymaker raises the interest rate. Such an ambiguous response cannot occur under discretion irrespective of whether a real exchange channel exists or not.

2 As in Ball (1999) and Svensson (2000), the derivation of the open-economy Phillips curve is laid out in the appendix.
3 By standard target rule we mean the target rule that is associated with a standard Phillips curve where only the output gap exerts pressure on domestic inflation (abstracting from cost-push shocks and the expectation of future inflation).

The final noteworthy finding concerns price level targeting in an open economy framework. Woodford (1999a) and Vestin (2006) argue that in a simple closed economy forward-looking model price level targeting is consistent with optimal policy from a timeless perspective. This result does not carry over to the open economy framework proposed in this paper. There is a simple explanation for this result. Because of a real exchange rate channel in the Phillips curve the target rule under policy from a timeless perspective depends on the lagged rate of inflation. This has the effect of augmenting the history-dependence of optimal policy and rules out expressing the target rule governing price level targeting in such a way so as to be consistent with the target rule underpinning optimal policy from a timeless perspective. With the targeting rules being incongruous, the delegation of a price level target to a central banker with the requisite aversion to price level variability does not conform to optimal policy from a timeless perspective in an open economy even if the shocks follow a white noise process.

The organization of the remaining parts of the paper is as follows. Section 2 introduces the model. Section 3 analyzes the conduct of optimal monetary policy from a timeless perspective. Section 4 examines the case of pure discretion. Section 5 compares and contrasts the two forms of optimal policy. Section 6 takes up the discussion of the compatibility of price level targeting with optimal policy from a timeless perspective in an open economy framework. Section 7 offers a brief conclusion.

2. The Model

The model that will serve as the foundation for the analysis of the monetary policy issues consists of three equations:

$$y_t = E_t y_{t+1} - a_1(R_t - E_t\pi^{CPI}_{t+1}) + a_2(q_t - E_t q_{t+1}) + a_3(y^f_t - E_t y^f_{t+1}) + v_t \qquad (1)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + b(q_t - E_t q_{t+1}) + u_t \qquad (2)$$

$$R_t - E_t\pi_{t+1} = R^f_t - E_t\pi^f_{t+1} + E_t q_{t+1} - q_t + \varepsilon_t \qquad (3)$$

$\pi_t =$ the rate of domestic inflation

$E_t\pi^{CPI}_{t+1} =$ the expected rate of CPI inflation

$q_t =$ the real exchange rate[4]

$y_t =$ the output gap

4 The real exchange rate is defined as the difference between the domestic currency price of the foreign good and the price of the domestic good: $q_t = s_t + p^f_t - p_t$. s_t = the nominal exchange rate, expressed in terms of domestic currency per unit of foreign currency. Thus the real exchange rate and the terms of trade are identical.

$R_t =$ the nominal rate of interest (policy instrument)

$R_t^f =$ the foreign nominal rate of interest

$E_t \pi_{t+1}^f =$ the expected foreign rate of inflation

y_t^f = the foreign output gap

Lower case variables represent logarithms. All parameters are positive. The discount factor β is less than or equal to one.

Equation (1) is the forward-looking open economy IS relation that features a real interest rate and real exchange rate channel. A foreign output shock and an idiosyncratic shock also affect the demand for domestic output.[5] Equation (2) represents the open-economy Phillips curve. The current rate of inflation moves not only in response to positive or negative realizations of the output gap but also in response to deviations of the current real exchange rate from its expectation next period. Equation (3) represents the uncovered interest rate parity (UIP) condition. Stochastic disturbances have been added to the three relations to reflect the existence of uncertainty in the economy.[6] More formally,

$$u_t \sim N(0, \sigma_u^2) \qquad v_t \sim N(0, \sigma_v^2) \qquad \varepsilon_t \sim N(0, \sigma_\epsilon^2)$$
$$R_t^f \sim N(0, \sigma_{R^f}^2) \qquad \pi_t^f \sim N(0, \sigma_{\pi^f}^2) \qquad y_t^f \sim N(0, \sigma_{y^f}^2) \qquad (4)$$

To simplify the analysis, we treat all foreign variables as exogenous random variables that are independent of each other.

3. Optimal Monetary Policy from a Timeless Perspective

The policymaker has a standard objective function consisting of squared deviations of the real output gap and the rate of inflation, respectively. The rate of inflation is defined in terms of changes in the level of domestic prices. The explicit objective function that he attempts to minimize is given by

$$E_t [\sum_{i=0}^{\infty} \beta^i [y_{t+i}^2 + \mu \pi_{t+i}^2]] . \qquad (5)$$

All variables are as previously defined. β is the discount rate and μ represents the relative weight the policymaker attaches to the squared deviations of the rate of domestic inflation from target.

5 The derivation of the forward-looking IS relation is explained in Guender (2006). A separate appendix, available from the author, shows how the shocks that appear in the IS relation can be motivated.

6 The shock in the UIP condition can be thought of as a risk-premium. The property that all shocks are white noise follows Woodford (1999). Its purpose is to show that gradual adjustment of the output gap, the rate of inflation, etc. and the policy instrument is not exclusively tied to the presence of autocorrelated disturbances in the model.

Equation (5) implies that the policymaker's sole concern rests with the output gap and domestic inflation. Fluctuations in the real exchange rate do not enter explicitly the loss function.[7,8]

To set the stage for illustrating how optimal policy in the open economy is carried out, it is helpful at the outset to reduce the dimension of the optimization problem to one involving only one constraint. A few simple steps need to be taken. First, substitute for the rate of CPI inflation in Equation (1).[9] Next, solve the UIP condition for the difference between the current and the expected real exchange rate and substitute this expression into both the IS equation and the Phillips curve relation. Then solve the IS relation for the expected real rate of interest ($R_t - E_t\pi_{t+1}$). Following this, insert the expression for the expected real rate of interest into the Phillips curve relation. The following expression results:

$$\begin{aligned}\pi_t &= (\kappa + \frac{b}{a_1(1-\gamma)+a_2})y_t + \beta E_t\pi_{t+1} \\ &+ \frac{b}{a_1(1-\gamma)+a_2}\left[a_1(R_t^f - E_t\pi_{t+1}^f + \varepsilon_t) - (E_t y_{t+1} + v_t) - a_3(y_t^f - E_t y_{t+1}^f)\right] + u_t\end{aligned} \quad (6)$$

Equation (6) serves as the constraint that the policymaker faces in determining optimal policy. It is apparent that this constraint is different from the standard closed-economy Phillips curve.

The optimization exercise thus reduces to the following:

$$\underset{y_t,\pi_t}{Min} \quad E_t[\sum_{i=0}^{\infty}\beta^i[y_{t+i}^2 + \mu\pi_{t+i}^2]] \text{ subject to} \quad (7)$$

$$\begin{aligned}\pi_t &= (\kappa + \frac{b}{a_1(1-\gamma)+a_2})y_t + \beta E_t\pi_{t+1} \\ &+ \frac{b}{a_1(1-\gamma)+a_2}\left[a_1(R_t^f - E_t\pi_{t+1}^f + \varepsilon_t) - (E_t y_{t+1} + v_t) - a_3(y_t^f - E_t y_{t+1}^f)\right] + u_t\end{aligned}$$

Let the time period in which the policy problem is formulated be denoted by $t=0$. Then the Lagrangean can be written in the following form:

7 Adopting (5) as the welfare criterion ignores the effects on welfare of shifts in the real exchange rate on the level of potential output. Including only the output gap and the rate of domestic inflation in the loss function is rather typical in the literature and thus facilitates comparing the results of this article to earlier contributions (e.g. Aoki (2001), Clarida, Gali, and Gertler (1999, 2001, 2002) or Svensson (2000)). Kirsanova, Leith, and Wren-Lewis (2006) also include domestic inflation in the objective function, arguing that the production of output requires only domestic labor. For a contrasting view the reader is referred to Allsopp, Kara, and Nelson (2006) who argue that CPI inflation is the relevant inflation target variable if production is based on a foreign intermediate input.

8 The target for the output gap and the rate of inflation is zero, respectively.

9 Under perfect exchange rate pass-through $E_t\pi_{t+1}^{CPI} = E_t\pi_{t+1} + \gamma E_t\Delta q_{t+1}$ where γ denotes the weight on the price of the imported good in the CPI.

$$
\begin{aligned}
\mathfrak{M}_{o} = E_0 [\, & y_0^2 + \mu\pi_0^2 + \beta(y_1^2 + \mu\pi_1^2) + \beta^2 (y_2^2 + \mu\pi_2^2) + ... \\
& + \lambda_0 ((\kappa + c) y_0 + \beta\pi_1 + c(-y_1 - a_3 (y_0^f - y_1^f) - v_0 + a_1 \Omega_0) + u_0 - \pi_0) \\
& + \beta\lambda_1 ((\kappa + c) y_1 + \beta\pi_2 + c(-y_2 - a_3 (y_1^f - y_2^f) - v_1 + a_1 \Omega_1) + u_1 - \pi_1) \\
& + \beta^2 \lambda_2 ((\kappa + c) y_2 + \beta\pi_3 + c(-y_3 - a_3 (y_2^f - y_3^f) - v_2 + a_1 \Omega_2) + u_2 - \pi_2) + ...]
\end{aligned} \tag{8}
$$

where $\Omega_t = R_t^f - E_t \pi_{t+1}^f + \varepsilon_t$ $\qquad c = \dfrac{b}{a_1 (1 - \gamma) + a_2}$

$t = 0,1,2,...$

Taking the first-order conditions with respect to the two target variables in each time period yields the following set of equations:

$$\frac{\partial \mathfrak{M}_{o}}{\partial y_0} = 2 y_0 + \lambda_0 (\kappa + c) = 0 \tag{9}$$

$$\frac{\partial \mathfrak{M}_{o}}{\partial y_1} = 2 \beta y_1 - \lambda_0 c + \lambda_1 \beta (\kappa + c) = 0 \tag{10}$$

$$\frac{\partial \mathfrak{M}_{o}}{\partial y_2} = 2 \beta^2 y_2 - \lambda_1 \beta c + \lambda_2 \beta^2 (\kappa + c) = 0 \tag{11}$$

…

…

$$\frac{\partial \mathfrak{M}_{o}}{\partial \pi_0} = 2 \mu \pi_0 - \lambda_0 = 0 \tag{12}$$

$$\frac{\partial \mathfrak{M}_{o}}{\partial \pi_1} = 2 \mu \pi_1 + \lambda_0 - \lambda_1 = 0 \tag{13}$$

$$\frac{\partial \mathfrak{M}_{o}}{\partial \pi_2} = 2 \mu \pi_2 + \lambda_1 - \lambda_2 = 0 \tag{14}$$

…

…

The remainder of this section draws attention to the importance of a real exchange rate channel in the Phillips curve and discusses its implication for optimal policymaking in an open economy framework. Part A takes up the case where the real exchange rate channel is suppressed while part B considers the case where the real exchange rate channel is operative. Part C analyzes the behavior of the endogenous variables of the model and the policy instrument with the help of impulse response functions.

A. No Real Exchange Rate Channel in the Phillips Curve: *b=0*

Setting $b=0$ implies $c=0$. Closer inspection of equations (9)-(11) then reveals that the first-order condition for the output gap in each period establishes the same systematic relationship between the output gap and the Lagrange multiplier:

$$2y_t + \lambda_t \kappa = 0 \qquad t = 0, 1, 2, 3,\ldots \qquad (15)$$

The invariant optimizing condition for the output gap is used below to substitute for the Lagrange multipliers to derive the optimal policy setting from a timeless perspective. Under this policy the policymaker ignores the start-up condition for the rate of inflation. Only the systematic relationship between the Lagrange multipliers and the rate of inflation from time $t=1$ onward is relevant:[10]

$$2\mu\pi_t + \lambda_{t-1} - \lambda_t = 0 \qquad t = 1, 2, 3,\ldots \qquad (16)$$

After using equation (15) to substitute for the Lagrange multipliers, one can express the target rule under optimal policy from the timeless perspective as:

$$\pi_t + \frac{y_t - y_{t-1}}{\mu\kappa} = 0 \qquad (17)$$

Thus in an open economy optimal policy from a timeless perspective is identical to optimal policy in a closed economy provided that there is no real exchange rate channel in the Phillips curve. This result is consistent with the observation by Clarida, Gali, and Gertler (2001), according to whom the conduct of optimal policy in an open economy is isomorphic to policy in a closed economy.

B. A Real Exchange Rate Channel in the Phillips Curve: *b>0*

With the direct exchange rate channel being operative, the structure of the first-order condition that applies to the output gap is no longer the same for all periods. More specifically, the condition in the initial period is different from the one that obtains in succeeding periods. In this respect the pattern followed by the output gap in the current context mirrors the pattern set by the

10 Equation (16) can also include the initial period, i.e. hold for $t=0,1,2,3\ldots$. In this case, to ensure dynamic consistency, the rate of inflation should be set in accordance with this optimizing condition in every period. But a special situation arises in the initial period. In period *0* the lagged Lagrange multiplier (λ_{-1}) must equal zero. This effectively rules out the Phillips curve in the previous period ($t=-1$) serving as a constraint in the optimization exercise in period *0*. Woodford (1999a,b), McCallum and Nelson (2004), Froyen and Guender (2007, Ch. 9) and Gali (2008, Ch.5) provide a detailed analysis of optimal policy from a timeless perspective in a closed economy framework.

rate of inflation in the previous section. In the start-up period, the first-order condition that applies to the output gap is given by

$$2y_0 + \lambda_0(\kappa + c) = 0 \tag{18}$$

while for period *1*and subsequent periods the first-order condition is given by

$$2\beta^t y_t - \lambda_{t-1}\beta^{t-1}c + \lambda_t\beta^t(\kappa + c) = 0 \quad \text{for } t=1,2,3\ldots \tag{19}$$

Notice that the Lagrange multiplier of the initial period (λ_0) appears in both first-order conditions. It appears in the first-order condition for the output gap in period *1* because the output gap in period *1* appears as an element in the constraint that the policymaker faces in period *0*. Thus forward-looking behavior on the *demand side* of the economy is now instrumental in determining the optimizing condition that applies to the output gap.[11]

The first-order conditions for the rate of inflation in the initial period and period *1* are unaffected by the existence of a real exchange rate channel in the Phillips curve:

$$2\mu\pi_0 - \lambda_0 = 0 \tag{20}$$

$$2\mu\pi_1 + \lambda_0 - \lambda_1 = 0 \tag{21}$$

Due to the existence of an exchange rate channel in the Phillips curve, optimal monetary from a timeless perspective cannot be conducted along the lines described in the previous section. The optimal behavior of the output gap in period *1* differs from that in the initial period. This being the case, the Lagrange multipliers λ_0 and λ_1 in equation (21) can no longer be substituted by drawing on the invariant initial optimizing condition for output for the initial period and the subsequent period.

To design a time-consistent policy rule in an open-economy framework, the policymaker must disregard the optimizing condition for *both* target variables in the initial period, i.e. period *0*. Thus, the policymaker must also ignore the initial optimizing condition for the output gap. The derivation of the target rule then requires the optimizing conditions from period *1* onward and proceeds as follows. Solve equation (14) for λ_2 , substitute the resulting expression into equation

11 Alternatively, the optimizing condition for the output gap can be expressed for all periods as $2\beta^t y_t - \lambda_{t-1}\beta^{t-1}c + \lambda_t\beta^t(\kappa + c) = 0$ for *t=0,1,2,3….* Again, to ensure dynamic consistency, the lagged Lagrange multiplier (λ_{-1}) equals zero in the optimizing condition for the initial period. The constraint in period -1 has no bearing on the optimization problem in period *0*.

(11), and solve for λ_1. Then substitute for λ_1 in equation (13) and solve for λ_0. Next replace both Lagrange multipliers in equation (10) with the expressions for λ_0 and λ_1 to obtain the optimal target rule:

$$\frac{y_2 - y_1}{\mu(\kappa + c)} + (\pi_2 - \frac{c}{\beta(\kappa + c)}\pi_1) = 0 . \qquad (22)$$

More generally,

$$\frac{y_t - y_{t-1}}{\mu(\kappa + c)} + (\pi_t - \frac{c}{\beta(\kappa + c)}\pi_{t-1}) = 0 \qquad c = \frac{b}{a_1(1-\gamma) + a_2} \qquad (23)$$

for all periods except the initial period.

Just like its counterpart in section 3, the target rule embodied by equation (23) is history-dependent as it is based on the change in the output gap. Notice, however, that the weight on the change in the output gap is smaller in case the Phillips curve features a real exchange rate channel as $c > 0$. Moreover, there is an additional source of history dependence as the target rule for an open economy is based not only on the lagged output gap but also on lagged inflation. Interestingly, it is not the *change* in inflation that matters in the open-economy target rule. The lagged rate of inflation bears a coefficient different from unity.

The presence of demand-side parameters in the target rule underscores the fact that the structure of the demand side of the economy plays a much more important role in the design of optimal monetary policy if a real exchange rate channel exists in the Phillips curve. Unlike the standard target rule (equation (17)), which depends only on κ apart from the policymaker's preference parameter, the optimal policy rule for the open economy depends also on parameters that appear in the Phillips curve and in the IS relation. The interest and exchange rate (semi) elasticities a_1 and a_2 as well as b determine the coefficients on the change in the output gap and on the lagged rate of inflation. The discount factor also plays an explicit role in the determination of the target rule.

It is instructive to examine how the size of the parameters of the model affects the relative weight on the change in the output gap and on the lagged inflation rate, respectively, in the target rule. If both κ and b are large relative to a_1 and a_2 then the rate of inflation responds sensitively to the output gap and the expected change in the real exchange rate while the output gap's response through the expected real interest rate and exchange rate channel, respectively is rather muted. As a consequence, the policymaker attaches a low relative weight to the change in the output gap when setting policy. Conversely, if a_1 and a_2 are large relative to κ and b then the relative weight on the output gap in the target rule increases as the output gap reacts sensitively to the expected real interest rate and an expected change in the real exchange rate. In this event, only a small increase in the policy instrument is required to engineer a desired reduction in inflation through a contraction of real output. The relative weight on lagged inflation decreases as well given the increased emphasis on the output gap in setting policy.

The coefficient $\frac{c}{\beta(\kappa+c)}$ measures the importance of lagged inflation in the target rule. Lagged inflation enters the target rule because of the existence of an exchange rate channel in the Phillips curve. The greater the sensitivity of the rate of inflation to expected changes in the real exchange rate in the Phillips curve, i.e. the greater the size of b for given values of a_1, a_2 and κ, the more important the lagged rate of inflation becomes in setting policy today. Conversely, the greater the sensitivity of the rate of inflation to the output gap in the Phillips curve, i.e. the greater the size of κ for given values of a_1, a_2, and b, the less significant the role of past inflation in the target rule.

C. Why a Real Exchange Rate Channel in the Phillips Curve Matters

The importance of the real exchange rate channel for the behavior of the endogenous variables of the model is brought out in Figures *1 – 4*.[12] These figures show how the output gap, domestic inflation, the real exchange rate, and the nominal interest rate respond to a cost-push shock, an IS shock, and a UIP shock.[13] The quantitative impact effect of these shocks on the four endogenous variables is also reported in Table *2*.

Figure *1* depicts the response of the four variables to a positive cost-push shock when the real exchange rate channel in the Phillips curve exists (*b=0.05)* while Figure *2* repeats the exercise for the case when the real exchange rate does not exist (*b=0).*[14] A comparison of the impulse response functions of the two figures reveals that

12 All four figures are based on $\mu = 10$. Table *1* lists the values of the parameters upon which the impulse response analysis in this section is based. The table also lists the size of the variances of the shocks that figure in the comparisons of policy under discretion and from a timeless perspective carried out in the next section.
13 The remaining shocks are only of minor interest as their effects are the same as or similar to a UIP or IS shock. For instance, a foreign interest rate shock causes the same effect on the variables of interest as a UIP shock while a shock to the foreign output gap behaves essentially like an IS shock.
14 So as to not overstate the importance of the real exchange rate channel relative to the output gap in the Phillips curve b and κ are of the same size. In their study of six countries Guender and Yu (2007) find support for an exchange rate channel in the Phillips curve only for Korea. The estimated size of b ranges from *0.04* to *0.1*.

- a positive cost-push shock has a much greater impact effect on the output gap, the real exchange rate and the policy instrument if a real exchange rate channel exists in the Phillips curve *(b>0).*
- a positive cost-push shock has a somewhat greater impact effect on the rate of inflation if a real exchange rate channel does not exist in the Phillips curve (*b=0).*
- policy tightens in response to a positive cost-push shock if *b > 0* according to Figure *1*. Notice the stark difference in Figure *2*: choosing the optimal response to a cost-push shock, the policymaker lowers the nominal interest rate if no real exchange rate channel exists in the Phillips curve.

The intriguing behavior of the nominal interest rate can be traced to the UIP condition. The positive cost- push shock causes the real exchange rate to fall, i.e. the right-hand side of the UIP condition increases. At the same time, the positive cost-push shock lowers expected inflation next period because of the decrease in the current output gap, thus also increasing the left-hand side of the UIP condition.[15] However, the reduction in the current output gap is sufficiently large to require a fall in the nominal interest rate so that the UIP condition is restored in the aftermath of the cost-push shock.[16]

Figure *3* shows that a domestic demand-side disturbance causes the rate of inflation and the output gap to deviate from their respective target value. The top two panels illustrate how the rate of inflation falls and the output gap rises in response to a positive IS shock for the case *b=0.05*. According to the top panel of Table *2*, these impact effects on inflation and the output gap are quantitatively small, at -*0.0574* and *0.0728*, respectively. Nevertheless, they underscore the fact that the policymaker is unable to prevent a one-time demand-side disturbance from affecting both target variables temporarily. The responses of the policy instrument and the real exchange rate, which are shown in the bottom panels, help explain why this is the case. In his effort to provide the optimal stabilization response, the policymaker raises the nominal interest rate. The positive change in the setting of the policy instrument causes the real value of the domestic currency to appreciate which

15 As shocks are white noise, expected inflation next period is given by: $E_t\pi_{t+1} = c_{20}y_t$ with $c_{20} > 0$

16 Inspection of the reduced form equation for the output gap and the reaction function of the policymaker yields the same insight. For *b=0*, $\mu = 10$ and values for the other parameters in accordance with Table *1*, the two equations are:

$$y_t = 0.853893y_{t-1} - 0.426946u_t$$

$$R_t = 1.82519y_t - 1.50094y_{t-1} + 0.750496u_t$$

Substituting the former into the latter results in a negative coefficient on the cost-push shock. The nominal interest rate decreases in the wake of a positive cost-push shock.

lowers domestic inflation through the Phillips curve.[17] As inflation changes, the output gap must adjust to be consistent with the target rule.

A closer look at the target rule also explains why the behavior of the two target variables is clearly different if the real exchange rate channel in the Phillips curve does not exist. If $b = 0$, the target rule is the same as in the standard closed economy model where demand-side disturbances do not affect the target variables because the policymaker simply adjusts the interest rate to offset the effect of the demand-side disturbance on the output gap. This keeps the output gap fixed at its target level. Because the output gap does not change and because of the absence of a direct exchange rate channel there is no spillover effect onto the Phillips curve. Thus domestic inflation does not deviate from its target level. The isomorphic character of optimal policy holds.

Figure *4* traces the effects of UIP shocks on the four variables. The reaction of the output gap, inflation, and the real exchange rate to a positive UIP shock is very similar to that of a positive IS disturbance, albeit in the opposite direction. In general, the closer a_1 is to one, the more the shape of the impulse response functions for an IS disturbance resemble those of a UIP disturbance.

4. Optimal Policy under Pure Discretion

Under pure discretion, the policymaker sets policy anew in every period. In carrying out the minimization exercise at the beginning of a given period, the policymaker treats the expectations of the endogenous variables that appear in the constraint as constants. Because the policymaker re-optimizes every period, the start-up optimizing conditions of the target variables form the basis for the optimal target rule under discretion. These optimizing conditions are given by equations (9) and (12):

$$2y_0 + \lambda_0(\kappa + c) = 0 \qquad (9) \qquad\qquad 2\mu\pi_0 - \lambda_0 = 0 \qquad (12)$$

Combining equation (9) with equation (12) yields the optimal target rule under discretion:

$$\frac{1}{\mu(\kappa + c)} y_0 + \pi_0 = 0 \qquad\qquad c = \frac{b}{a_1(1-\gamma) + a_2}.$$

More generally,

$$\frac{1}{\mu(\kappa + c)} y_t + \pi_t = 0 \qquad\qquad t = 0,1,2... \qquad (24)$$

Thus the optimal target rule under pure discretion does not introduce history dependence into the conduct monetary policy.[18] Compared to a closed economy framework, the weight on the output

17 Strictly speaking, the increase in the nominal rate of interest leads to, ceteris paribus, a decrease in the difference between the real exchange rate and the expected real exchange rate next period.

gap is smaller in an open economy provided that there is a real exchange rate channel in the Phillips curve, i.e. $c > 0$.

5. An Evaluation of the Two Policy Regimes

In this section we compare and contrast the performance of optimal policy from a timeless perspective relative to pure discretion in an open economy. To underscore the importance of a real exchange rate channel in the Phillips curve for policymaking, we again distinguish between two cases: $b=0.05$ and $b=0$. In addition, we employ two different values of the policymaker's relative aversion to inflation variability: $\mu = 1$ and $\mu = 10$.

Table *2* reports the coefficients on the three disturbances in the reduced form equations for the rate of inflation, the output gap, the real exchange rate, and the nominal interest rate. The coefficients in the top panel capture the contemporaneous response of the four variables to an IS shock, a cost-push shock, and a UIP shock for the case where a real exchange rate channel is operative in the Phillips curve: $b=0.05$. Inspection of these coefficients yields a few noteworthy insights.

- The impact effects of all three disturbances on the two target variables y_t and π_t are smaller under policy from a timeless perspective than under discretion. This is a direct consequence of the fact that commitment requires that the two target variables return gradually to their target levels in the wake of a shock while discretion requires adjustment of the variables to a shock within the period.
- The impact effect (in absolute terms) of a cost-push disturbance on all four variables is smaller under policy from a timeless perspective than discretion.
- The picture is less clear for IS and UIP shocks. Suffice it to say that both shocks evoke similar responses of the nominal rate of interest and the real exchange rate under both policy strategies.

The lower panel reports results for the case where a real exchange rate channel in the Phillips curve does not exist ($b=0$).

- Here policy type matters for the reaction of the policy instrument in the face of a cost-push shock as explained in the previous section.
- In the absence of a real exchange rate channel in the Phillips curve, IS and UIP shocks have no effect on the target variables under both forms of policy.

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18 Examining the impulse response functions under discretion is not as revealing as under policy from a timeless perspective. Because of the absence of history dependence under discretion, the effect of the shock is felt only in the current period.

- Both policy from a timeless perspective and discretion lead to the same contemporaneous response of the real exchange rate and the nominal rate of interest, respectively, to IS and UIP disturbances. This happens because the responses of both variables are independent of policy design and thus purely mechanical.[19]

Table *3* summarizes the overall performance of the two policy strategies and reports the variances of the three endogenous variables and the policy instrument for $\mu = 1$ and $\mu = 10$. The top panel considers the case where a real exchange rate channel is operative in the Phillips curve while the bottom panel considers the case where it is absent. The dominance of policy under a timeless perspective over discretion is evident. In all four cases expected losses under policy from a timeless perspective are lower than under discretion. The gain from commitment relative to discretion does vary, however, depending on whether a real exchange rate channel exists in the Phillips curve. Substantial gains are realized in case the Phillips curve is the standard closed-economy variant. Specifically, the percentage gain from commitment rises from *4.63* % to *12.47* % as the relative weight on the variance of inflation in the expected loss function rises from $\mu = 1$ to $\mu = 10$. The gain from commitment is more modest for an open-economy Phillips curve ($b>0$): as μ increases from *1* to *10* the percentage gain from being able to precommit rises from *4.25* % to *9.02* %.

Overall, the policymaker wields better control over the variability of the rate of inflation if a real exchange rate channel is operative in the Phillips curve. This comes at some cost though as the variability of the output gap increases if $b>0$.

6. Is Price Level Targeting Consistent with Optimal Monetary Policy from a Timeless Perspective in an Open Economy?

The presence of a real exchange rate channel in the Phillips curve has further implications for policymaking in an open economy. This section shows why the delegation mechanism fails in the open economy model proposed in this paper.

Woodford (1999a) argues that price level targeting is consistent with optimal policy from a timeless perspective in the forward-looking model of a closed economy. This is a direct consequence of the fact that price level targeting produces the same optimal response to

19 It is also worth pointing out that under discretion, the coefficient on the UIP shock in the reduced form equation for the real exchange equals one minus the coefficient on the UIP shock in the equation for the nominal interest rate. This result holds also under policy from a timeless perspective provided the real exchange rate channel in the Phillips curve is not operative. The real exchange rate and the nominal rate of interest respond alike to an IS shock albeit in opposite directions under discretion. This result also holds under commitment if $b=0$.

disturbances as policy from a timeless perspective. The literature on the delegation issue in the conduct of monetary policy has recognized the attractiveness of price level targeting in the forward-looking model.[20] That price level targeting under discretion is compatible with optimal monetary policy from a timeless perspective can be seen by comparing the target rules that underlie both strategies of monetary policy.

In the standard New Keynesian framework, the target rule under optimal policy from a timeless perspective relates the rate of inflation to the change in the output gap:

$$\frac{1}{\mu\kappa}(y_t - y_{t-1}) + \pi_t = 0 \tag{25}$$

This target rule can be rewritten in terms of the output gap and the price level:[21]

$$\frac{1}{\mu\kappa} y_t + p_t = 0 \tag{26}$$

The target rule under discretionary price-level targeting is given by:[22]

$$p_t + \frac{y_t(\beta(1-\phi_{22})+1)}{\kappa\hat{\mu}} - \frac{E_t y_{t+1}}{\kappa\hat{\mu}} = 0 \tag{27}$$

The parameter $\hat{\mu}$ denotes the weight the policymaker assigns to the squared deviations of the price level from its target value in the intertemporal loss function. After replacing the expectation of the output gap with $\phi_{12} p_t$ and some algebraic manipulation, we can restate equation (27) as:[23]

$$\frac{(\beta(1-\phi_{22})+1)y_t}{\kappa\hat{\mu} - \beta\phi_{12}} + p_t = 0 \tag{28}$$

Comparing (26) with (28), we find that the two target rules differ only to the extent of the relative weight on the output gap. This in turn implies that in a closed-economy framework a suitably chosen central banker who engages in discretionary price level targeting can replicate the behavior of the rate of inflation and the output gap that occurs under optimal policy from a timeless perspective.

20 See Vestin (2006) for an analysis of price level targeting and the delegation issue in the context of a closed economy.
21 In what follows we assume that the target for the price level (p^*) is zero.
22 The derivation of the target rule under price level targeting is laid out in Appendix C.
23 ϕ_{12} = the coefficient on the lagged price level in the putative solution for the current price level. ϕ_{22} = the coefficient on the lagged price level in the putative solution for the current output gap.

To examine whether society can achieve the same outcome in the open-economy framework proposed in this paper, we return to the target rule of section 2 that guides optimal policy from a timeless perspective:

$$\frac{y_t - y_{t-1}}{\mu(\kappa + c)} + (\pi_t - \frac{c}{\beta(\kappa + c)}\pi_{t-1}) = 0 \qquad c = \frac{b}{a_1(1-\gamma) + a_2} \qquad (23)$$

If we can show that the target rule under discretionary price level targeting is compatible with equation (23), then the delegation of monetary policy to a suitably chosen central banker also works to society's benefit in an open economy framework. The target rule that applies to discretionary price level targeting in an open economy is given by:

$$\frac{[\beta(1-\phi_{22}) + c\phi_{12} + 1]\, y_t}{(\kappa + c)\hat{\mu} - \beta\phi_{12}} + p_t = 0 \qquad (29)$$

Inspection of (23) and (29) reveals that the two target rules are incongruous. That is to say that the target rule under optimal policy from a timeless perspective cannot be manipulated so as to be expressed solely in terms of the output gap and the price level, the two variables that appear in the target rule under discretionary price level targeting. The direct implication of this result is that discretionary price level targeting in an open economy cannot replicate the behavior of the rate of inflation and the output gap that eventuates under optimal policy from a timeless perspective. The delegation of a price level target to a central banker, who acts with discretion, does not achieve the gains that accrue from commitment in an open economy.

The breakdown of the delegation process in an open economy is due to the lack of history dependence of policy under price level targeting. At the very least, policy would have to respond to the lag of the price level to make the target rule under discretionary price level targeting compatible with the target rule under optimal policy from a timeless perspective.[24] Stated differently, optimal policy from a timeless perspective makes the conduct of monetary policy depend on the lagged output gap and the lagged rate of inflation. Thus there are two sources that account for the history dependence and produce the optimal degree of inertia in the conduct of optimal policy under commitment.

Further inspection of the target rules under optimal policy from a timeless perspective and discretionary price level targeting shows that the breakdown of the delegation process in an open

24 This can best be seen by setting $\beta = 1$ and rewriting equation (23) in terms of the output gap and the price level:

$$\frac{1}{\mu(\kappa+c)} y_t + p_t - \frac{c}{\kappa+c} p_{t-1} = 0$$

economy framework is due to the existence of an exchange rate channel in the Phillips curve. Setting $b=0$ yields $c=0$. Most important, the lagged rate of inflation drops out of the target rule under optimal policy from a timeless perspective. This in turn restores the compatibility of discretionary price level targeting with optimal policy from a timeless perspective.

7. Conclusion

This paper has examined the design of optimal monetary policy in an open economy version of the forward-looking model. Its major finding is that the existence of a real exchange rate channel in the Phillips curve changes the design of optimal policy in no small measure. First and foremost, the target rule under policy from a timeless perspective becomes more complex. The lags of both target variables appear in it, thus making the conduct of policy more history-dependent. The discount factor as well as IS and Phillips curve parameters determine the weights attached to the change in the output gap and the lagged rate of inflation.

With the target rule being significantly different if a real exchange rate channel is operative in the Phillips curve, it is not surprising that the character of stabilization policy changes. The policymaker can no longer perfectly stabilize both the domestic rate of inflation and the output gap in the wake of IS or UIP disturbances by mechanically adjusting the policy instrument. Demand-side disturbances thus cause temporary effects on both variables. Optimal stabilization policy in such an open economy is no longer isomorphic to policy in a closed economy.

Whether or not a real exchange rate channel exists in the Phillips curve also matters for the delegation issue. Discretionary price level targeting is consistent with optimal policy from a timeless perspective in a closed economy because the target rules that govern both strategies are compatible in the sense that they can be expressed in terms of the same target variables. This result does not carry over to the open economy framework of this paper because the timeless perspective introduces additional history dependence through inflation into the conduct of monetary policy. This additional history dependence cannot be matched by price level targeting.

The paper also assesses the welfare gains under commitment relative to discretion. The results suggest that the welfare gains under policy from a timeless perspective are somewhat lower in an open economy framework where a real exchange rate channel exists in the Phillips curve.

Taken altogether, the findings reported in this paper warrant the conclusion that optimal policy in an open economy framework is substantially different from optimal policy in a closed economy. The mere existence of a real exchange rate channel in the Phillips curve suffices for the conduct of optimal monetary policy from a timeless perspective to be more complex and information-intensive. The realistic assumption that in a small open economy individual firms take account of fluctuations in the terms of trade when adjusting prices provides a conduit through which the real exchange rate enters the aggregate Phillips curve. Its presence there ensures a prominent role for the real exchange rate in the design of optimal monetary policy.

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Appendix A: Derivation of the Open Economy Phillips Curve.

The starting point is Rotemberg (1982). Monopolistically competitive firms aim to minimize menu costs weighed against the cost of being away from the optimal price they would charge in the absence of those menu costs. This optimal price is denoted p^{OPT}. In addition, being concerned about changes in their competitiveness vis-à-vis foreign firms, domestic firms wish to avoid changes in their terms of trade by engaging in terms-of-trade smoothing. Unstable terms of trade interfere with the domestic firms' competitiveness in the domestic market as well as world markets. Such fluctuations tend to unhinge the steady market share of firms. Specifically, at time *t* domestic firms attempt to minimize the squared difference between the current and next period's terms of trade.[25] The objective function faced by the typical firm *j* is:

$$\min_{p(j)_t} \Omega(j)_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\left(p(j)_{t+i} - p(j)_{t+i}^{OPT} \right)^2 + c\left(p(j)_{t+i} - p(j)_{t+i-1} \right)^2 + d(q(j)_{t+1+i} - q(j)_{t+i})^2 \right] \quad \text{(A1)}$$

where:[1]

$\Omega(j)_t$ = the total cost of firm *j* at time *t*

$p(j)_t$ = the price of the good produced by firm *j* at time *t*

$p(j)_t^{OPT}$ = the optimal price firm *j* charges

$q(j)_t = p_t^f + s_t - p(j)_t$ = firm-specific terms of trade

β = the constant discount factor

c = the parameter that measures the costs of changing prices relative to the costs of deviating from the optimal price

d = the parameter that measures the costs of changes in the firm's terms of trade relative to the costs of deviating from the optimal price

E_t = the expectations operator conditional on information available at time t.

After taking and rearranging the first-order condition for the above cost-minimization problem, we can characterize the relationship between past, current, and future price levels as well as the current and expected terms of trade as:

$$p_t(j) - p(j)_{t-1} = \beta E_t \left(p(j)_{t+1} - p(j)_t \right) - \frac{1}{c}\left(p(j)_t - p(j)_t^{OPT} \right) - \frac{d}{c}(E_t q(j)_{t+1} - q(j)_t) \qquad \text{(A2)}$$

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25 Alternatively, the firm could be concerned about minimizing the deviation between q_{t+i} and q_{t+i-1} in which case the first-order condition would contain the change in the current firm-specific terms of trade next to the change in the expected terms of trade. The resulting Phillips curve would then also feature the change in the real exchange rate (q_t – q_{t-1}). Including Δq_t, while defensible, complicates the analysis considerably as it rules out the derivation of an analytical target rule that lends itself to straightforward interpretation. For this reason, this case is not pursued here.

Here we see that the expected change in the firm-specific terms of trade matters in the pricing decision. The greater the relative weight of expected changes in the terms of trade in the total cost function compared to the relative weight on costly price changes, the more the expected change in the terms of trade factors in the decision to change the price of output in the current period. Changes in the current nominal exchange rate and changes in the price charged by competing foreign firms are beyond the control of the typical domestic firm – they are exogenous. Yet such changes affect its terms of trade, i.e. its competitiveness. The only way that a domestic firm can counteract such pressure is to adjust its domestic price in such a way so that overall costs are minimized.
Next, consider the formation of the firm's optimal price:

$$p(j)_t^{OPT} = \hat{p}_t + \vartheta y(j)_t + \varsigma(j)_t \qquad \vartheta > 0 \qquad \text{(A3)}$$

where all variables are as previously defined. In addition:
$\hat{p}_t$ = the price charged by competing firms at time *t*

$y(j)_t$ = output produced (relative to potential) by firm *j*

$\varsigma(j)_t$ = a stochastic disturbance.

Under imperfect competition, a firm sets its optimal price as a mark-up over marginal cost. But marginal cost and real output are positively related.[26] Hence it is innocuous to replace marginal cost with the output gap in (A3). To capture the idea of a time-varying mark-up factor, we treat it as a random element that enters into the process of setting the optimal price. Hence ς_t appears in equation (A3).[27]

The other important factor that influences the firm's optimal price is the benchmark price set by competing firms. This price, denoted by, $\hat{p}_t$ equals the aggregate domestic price level p_t .

Substituting equation (A3) into (A2) and aggregating over all firms yields equation (A4), an open-economy Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + b(q_t - E_t q_{t+1}) + u_t \qquad \text{(A4)}$$

where

26 Within a general equilibrium framework, the co-movement between marginal cost and economic activity can be established by combining the labor supply and demand relations with the market clearing condition in the goods market. On this point see Clarida, Gali, and Gertler (2001, 2002) or Gali and Monacelli (2005) who derive a similar relation that stresses the positive relation between real marginal cost and domestic consumption. The positive link between output and marginal cost is also characteristic of earlier models of monopolistic competition such as Blanchard and Kiyotaki (1987). The link features also prominently in Mankiw and Reis (2002) who propose an alternative Phillips curve that is based on slow dissemination of information.
27 For an investigation of the time-varying mark-up factor, see Ball and Romer (2003).

$$\pi_t = p_t - p_{t-1}$$

$$E_t \pi_{t+1} = E_t p_{t+1} - p_t$$

$$\kappa = \frac{\vartheta}{c}$$

$$b = \frac{d}{c}$$

$$u_t = \frac{1}{c} \varsigma_t .$$

Appendix B:

The solutions for the endogenous variables and the policy instrument of the model under policy from a *timeless perspective* are:

<u>Output Gap:</u>

$y_t = c_{10} y_{t-1} + c_{11} \pi_{t-1} + c_{12} u_t + c_{13} v_t + c_{14} \varepsilon_t$ where

$$c_{10} = \frac{1}{X + \theta Y} (\theta (Y + \Lambda)) \qquad c_{11} = \frac{c}{\beta} c_{10}$$

$$c_{12} = \frac{-\Lambda}{X + \theta Y} \qquad c_{13} = \frac{1}{X + \theta Y} (1 - \Lambda (\theta + \kappa))$$

$$c_{14} = \frac{1}{X + \theta Y} (a_2 - a_1 \gamma - \Lambda B)$$

<u>Inflation:</u>

$\pi_t = c_{20} y_{t-1} + c_{21} \pi_{t-1} + c_{22} u_t + c_{23} v_t + c_{24} \varepsilon_t$ where

$$c_{20} = \frac{\theta}{X + \theta Y} (X - \theta \Lambda) \qquad c_{21} = \frac{c}{\beta} c_{20}$$

$$c_{22} = \frac{\theta \Lambda}{X + \theta Y} \qquad c_{23} = -\frac{\theta}{X + \theta Y} (1 - \Lambda (\theta + \kappa))$$

$$c_{24} = -\frac{\theta}{X + \theta Y} (a_2 - a_1 \gamma - \Lambda B)$$

<u>Real Exchange Rate:</u>

$q_t = c_{30} y_{t-1} + c_{31} \pi_{t-1} + c_{32} u_t + c_{33} v_t + c_{34} \varepsilon_t$ where

$$c_{30} = \frac{(-c_{46} + c_{20}) c_{10} + V c_{20} - c_{40}}{1 - c_{10}} \qquad c_{31} = \frac{(-c_{47} + c_{21}) c_{21} + W c_{11} - c_{41}}{1 - c_{21}}$$

$$c_{32} = Wc_{12} + Vc_{22} - c_{42} \qquad c_{33} = Wc_{13} + Vc_{23} - c_{43}$$
$$c_{34} = Wc_{14} + Vc_{24} - (1 + c_{44})$$

Nominal Interest Rate:

$R_t = c_{40}y_{t-1} + c_{41}\pi_{t-1} + c_{42}u_t + c_{43}v_t + c_{44}\varepsilon_t$ where

$$c_{40} = c_{46}c_{10} + c_{47}c_{20} - \frac{\Lambda\theta}{A} \qquad c_{41} = c_{46}c_{11} + c_{47}c_{21} - \frac{c\Lambda\theta}{\beta A}$$
$$c_{42} = c_{46}c_{12} + c_{47}c_{22} + \frac{\Lambda}{A} \qquad c_{43} = c_{46}c_{13} + c_{47}c_{23} + \frac{\Lambda(\theta+\kappa)}{A}$$
$$c_{44} = c_{46}c_{14} + c_{47}c_{24} + \frac{\Lambda B}{A} \qquad c_{46} = \frac{\Lambda}{A}(\theta+\kappa)c_{10} + (\frac{\Lambda\beta}{A}+1)c_{20}$$
$$c_{47} = \frac{\Lambda}{A}(\theta+\kappa)c_{11} + (\frac{\Lambda\beta}{A}+1)c_{21}$$

$$A = (1-\gamma)a_1 + a_2 \qquad B = b + (\theta+\kappa)(a_2 - a_1\gamma)$$
$$\Lambda = \frac{A}{(\theta+\kappa)A+b} \qquad \theta = \frac{1}{\mu(\kappa+c)} \qquad c = \frac{b}{A}$$

$$Y = c_{11} - \Lambda((\theta+\kappa)c_{11} + \beta c_{21}) \qquad X = 1 - (c_{10} - \Lambda((\theta+\kappa)c_{10} + \beta c_{20}))$$

$$W = c_{30} - c_{46} + c_{20} \qquad V = c_{31} - c_{47} + c_{21}$$

The solutions for the endogenous variables and the policy instrument of the model under *Discretion* are:

Output Gap:

$$y_t = \frac{1}{D}(-Au_t + bv_t - a_1 b\varepsilon_t)$$

Inflation:

$$\pi_t = -\frac{\theta}{D}(-Au_t + bv_t - a_1 b\varepsilon_t)$$

Real Exchange Rate:

$$q_t = \frac{1}{A}((a_1 - \frac{a_1 b}{D})\varepsilon_t - \frac{A}{D}((\theta+\kappa)v_t + u_t))$$

Nominal Interest Rate:

$$R_t = \frac{1}{A}((\frac{a_1 b}{D} + a_2 - a_1\gamma)\varepsilon_t + \frac{A}{D}((\theta+\kappa)v_t + u_t))$$

where $D = b + A(\theta+\kappa)$

Appendix C:

Under discretionary price-level targeting, the policymaker minimizes the current and expected future deviations of the price level and the output gap from its respective target value. The minimization exercise is repeated every period and the process of expectations formation is taken as given. The constraint for the policy problem is obtained by combining the UIP condition and the IS relation with the Phillips curve.

$$\underset{y_t, p_t}{Min}\, E_t \sum_{j=0}^{\infty} \beta^j (y_{t+j}^2 + \hat{\mu}(p_{t+j} - p^*)^2)$$

$$s.t.\ p_t = \frac{1}{1+\beta}[\beta E_t p_{t+1} + (\kappa + c) y_t + u_t + p_{t-1} + c(a_1(R_t^f - E_t \pi_{t+1}^f + \varepsilon_t) - (E_t y_{t+1} + v_t) - a_3(y_t^f - E_t y_{t+1}^f)) + u_t] \quad (C1)$$

$\hat{\mu}$ = policymaker's aversion to price level variability.

Taking the first-order conditions with respect to the choice variables y_t and p_t yields:

$$2y_t + \frac{\lambda_t(\kappa + c)}{1+\beta} = 0 \quad (C2)$$

$$2\hat{\mu}(p_t - p^*) + \lambda_t [\frac{1}{1+\beta}(\beta\phi_{22} - c\phi_{12}) - 1] + \frac{\beta E_t \lambda_{t+1}}{1+\beta} = 0 \quad (C3)$$

Notice that the conditional expectations of the price level and the output gap in period *t+1* have been replaced with the conditional expectation of the respective putative solution. The undetermined coefficients ϕ_{12} and ϕ_{22} appear in the putative solution for the output gap and the price level, respectively:

$$y_t = \phi_{11} u_t + \phi_{12} p_{t-1} + \phi_{13} v_t + \phi_{14} \varepsilon_t + \phi_{15} R_t^f + \phi_{16} y_t^f + \phi_{17} \pi_t^f \quad (C4)$$

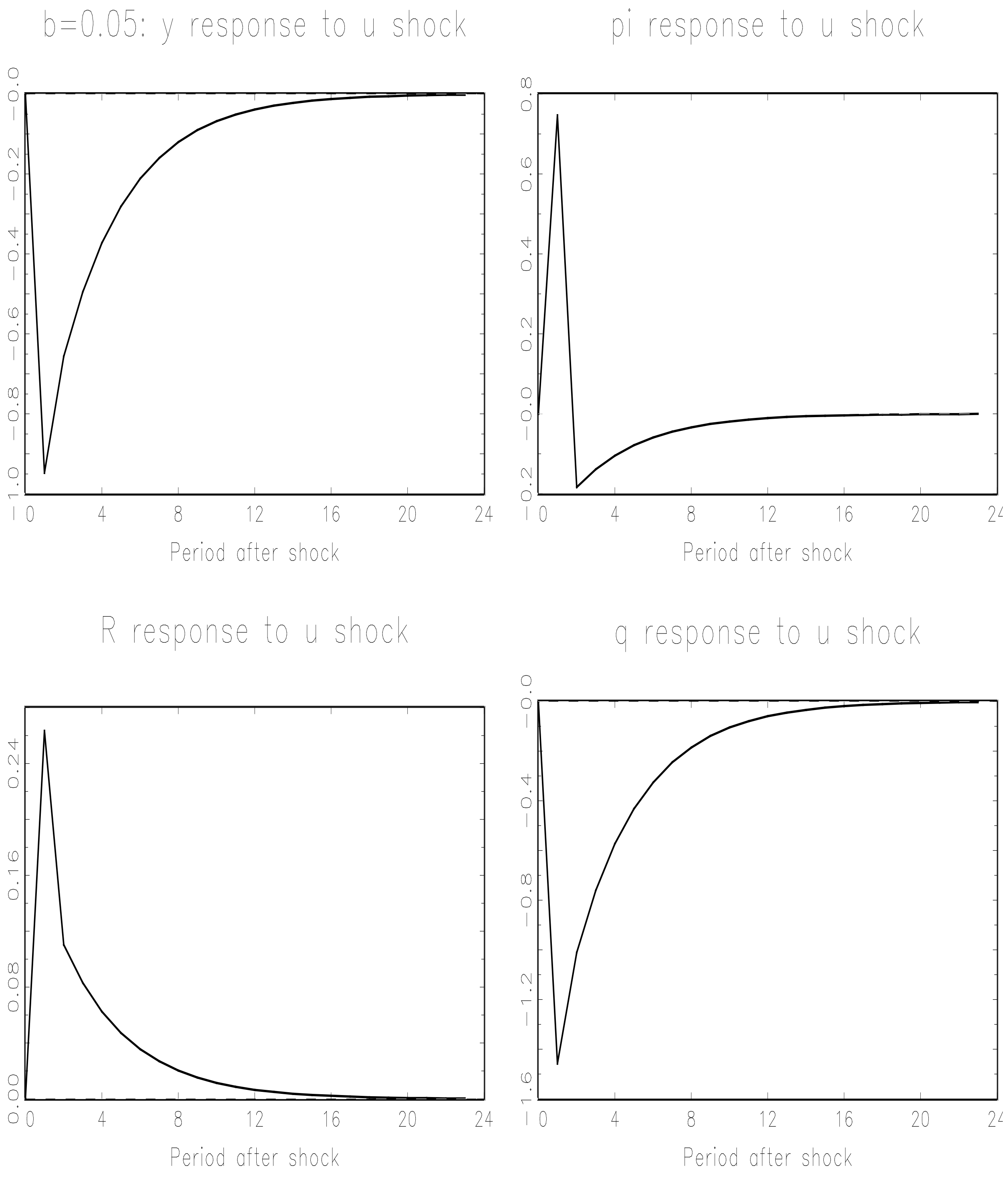
$$p_t = \phi_{21} u_t + \phi_{22} p_{t-1} + \phi_{23} v_t + \phi_{24} \varepsilon_t + \phi_{25} R_t^f + \phi_{26} y_t^f + \phi_{27} \pi_t^f \quad (C5)$$

Combining the first-order conditions (and setting $p^* = 0$) results in the target rule that systematically relates the price level to the current and the expected output gap in period *t+1*.

$$p_t + \frac{y_t(\beta(1 - \phi_{22}) + c\phi_{12} + 1)}{\hat{\mu}(\kappa + c)} - \frac{\beta E_t y_{t+1}}{\hat{\mu}(\kappa + c)} = 0 \quad (C6)$$

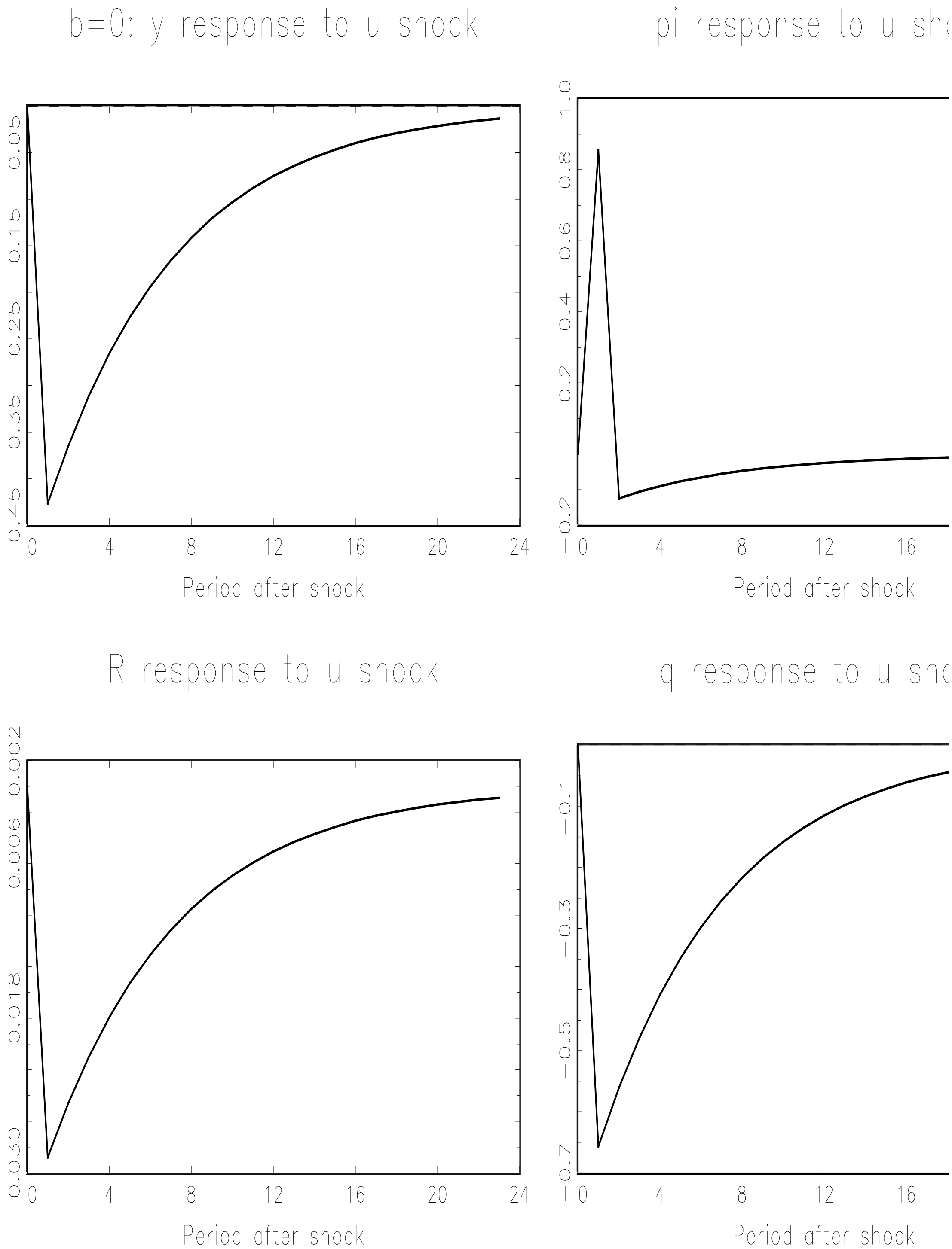
Letting $E_t y_{t+1} = \phi_{12} p_t$ and rearranging the above equation yields the target rule under price level targeting in the text (Equation (29)). Equation (27) obtains if $c = 0$, i.e. if the real exchange rate channel in the Phillips curve does not exist.

Figure 1:



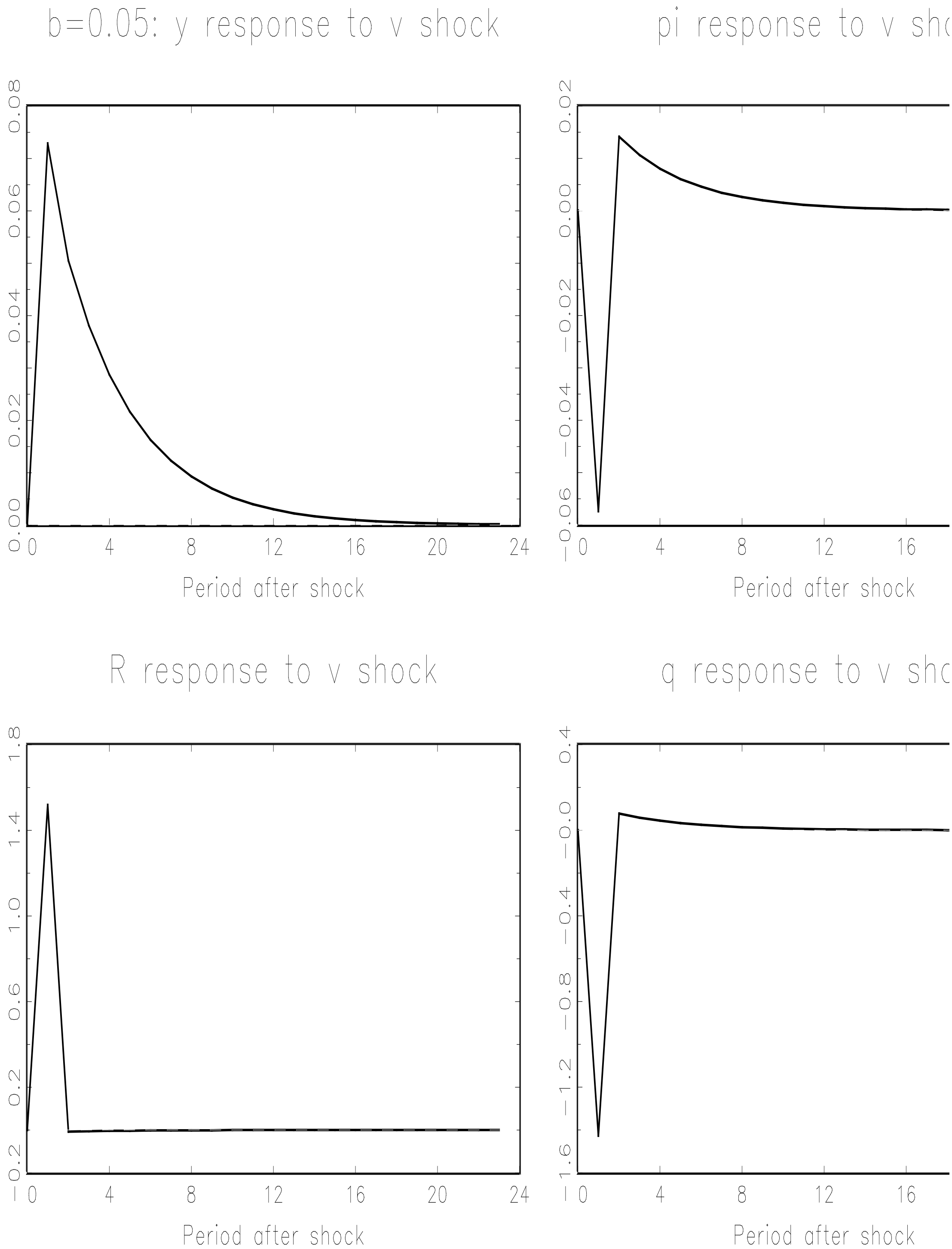
b=0.05: y response to u shock
-0.0
-0.2
-0.4
-0.6
-0.8
-1.0
0
4
8
12
16
20
24
Period after shock
pi response to u shock
0.8
0.6
0.4
0.2
-0.0
-0.2
0
4
8
12
16
20
24
Period after shock
R response to u shock
0.24
0.16
0.08
0.00
0
4
8
12
16
20
24
Period after shock
q response to u shock
-0.0
-0.4
-0.8
-1.2
-1.6
0
4
8
12
16
20
24
Period after shock

Figure 2:



b=0: y response to u shock
pi response to u sh
R response to u shock
q response to u sh
Period after shock
Period after shock
Period after shock
Period after shock

Figure 3:



b=0.05: y response to v shock
pi response to v sh
R response to v shock
q response to v sh
Period after shock
Period after shock
Period after shock
Period after shock

Figure 4:

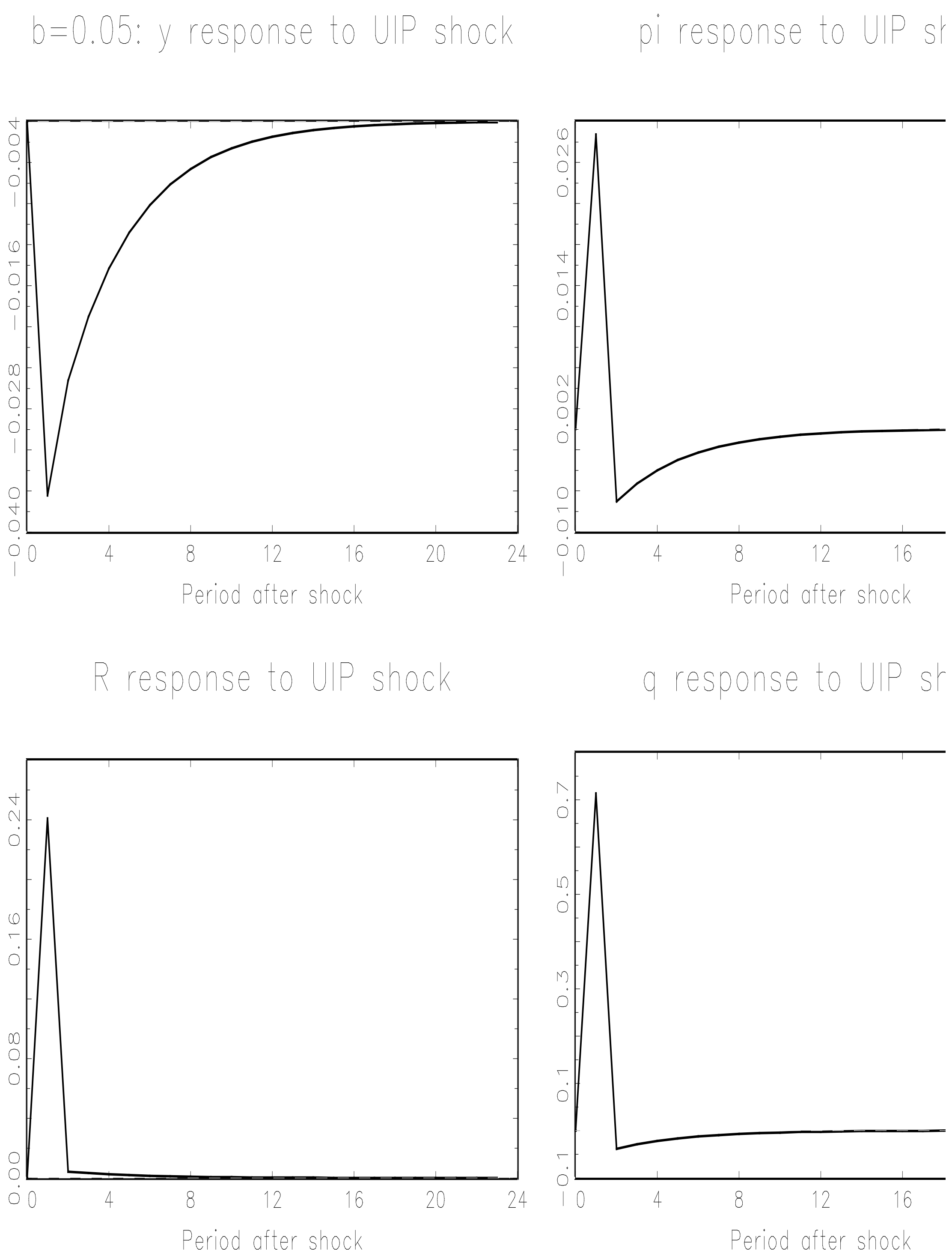

b=0.05: y response to UIP shock
pi response to UIP sh
R response to UIP shock
q response to UIP sh
Period after shock
Period after shock
Period after shock
Period after shock

Table 1: The Parameters of the Model

σ	η	η^f	γ^f	γ	β^f	a_1	a_2	a_3	κ	b	μ
5/7	*1*	*2*	*0.15*	*0.3*	*0.9*	*0.5*	*0.3*	*0.27*	*0.05*	$(0, 0.05)$	*(1,10)*

The variances of all random disturbances are fixed at *0.9*

Table 2: The Impact Effect of the Disturbances on the Variables of the Model

	Timeless Perspective				**Discretion**			
	$\mu = 10;\ b = 0.05$							
	π_t	y_t	q_t	R_t	π_t	y_t	q_t	R_t
u_t	0.7835	-0.9944	-1.5299	1.0317	0.8612	-1.0931	-1.6817	1.6817
v_t	-0.0602	0.0764	-1.4207	1.4591	-0.0662	0.0840	-1.4091	1.4091
ε_t	0.0301	-0.0382	0.7103	0.2704	0.0331	-0.0420	0.7045	0.2955
	$\mu = 1;\ b = 0.05$							
	π_t	y_t	q_t	R_t	π_t	y_t	q_t	R_t
u_t	0.9422	-0.1195	-0.1839	0.0747	0.9841	-0.1249	-0.1921	0.1921
v_t	-0.0724	0.0091	-1.5243	1.5327	-0.0757	0.0096	-1.5236	1.5236
ε_t	0.0362	-0.0045	0.7621	0.2336	0.0378	-0.0048	0.7618	0.2382

	Timeless Perspective				**Discretion**			
	$\mu = 10;\ b = 0$							
	π_t	y_t	q_t	R_t	π_t	y_t	q_t	R_t
u_t	0.8538	-0.4269	-0.6568	-0.0287	0.9756	-0.4878	-0.7504	0.7504
v_t	0	0	-1.5384	1.5384	0	0	-1.5384	1.5384
ε_t	0	0	0.7692	0.2308	0	0	0.7692	0.2308
	$\mu = 1;\ b = 0$							
	π_t	y_t	q_t	R_t	π_t	y_t	q_t	R_t
u_t	0.9512	-0.0475	-0.0731	-0.0428	0.9975	-0.0498	-0.0767	0.0767
v_t	0	0	-1.5384	1.5384	0	0	-1.5384	1.5384
ε_t	0	0	0.7692	0.2308	0	0	0.7692	0.2308

Table 3: Summary Measures of the Performance of Commitment and Discretion

b=0.05	Timeless Perspective			Discretion			
$\mu = 10$							
$V(\pi_t)$	$V(y_t)$	$V(q_t)$	$V(R_t)$	$V(\pi_t)$	$V(y_t)$	$V(q_t)$	$V(R_t)$
0.5985	1.1188	4.9035	2.9454	0.6725	1.0834	4.7791	4.4109
$E[L^{TP}] = 7.1040$				$E[L^{D}] = 7.8086$			
% Gain from Commitment: *9.02*							
$\mu = 1$							
$V(\pi_t)$	$V(y_t)$	$V(q_t)$	$V(R_t)$	$V(\pi_t)$	$V(y_t)$	$V(q_t)$	$V(R_t)$
0.8250	0.0293	2.6830	2.1840	0.8781	0.0141	2.6450	2.1737
$E[L^{TP}] = 0.8543$				$E[L^{D}] = 0.8922$			
% Gain from Commitment: *4.25*							
b=0	**Timeless Perspective**			**Discretion**			
$\mu = 10$							
$V(\pi_t)$	$V(y_t)$	$V(q_t)$	$V(R_t)$	$V(\pi_t)$	$V(y_t)$	$V(q_t)$	$V(R_t)$
0.7079	0.6056	4.0962	2.1808	0.8566	0.2141	3.1696	2.6849
$E[L^{TP}] = 7.6850$				$E[L^{D}] = 8.7804$			
% Gain from Commitment: *12.47*							
$\mu = 1$							
$V(\pi_t)$	$V(y_t)$	$V(q_t)$	$V(R_t)$	$V(\pi_t)$	$V(y_t)$	$V(q_t)$	$V(R_t)$
0.8347	0.02139	2.7133	2.1954	0.8955	0.0022	2.6680	2.1834
$E[L^{TP}] = 0.8561$				$E[L^{D}] = 0.8977$			
% Gain from Commitment: *4.63*							

Notes: Expected Loss = $E(L_t^S) = V(y_t^S) + \mu V(\pi_t^S)$ S=TP or D Gain from Commitment: $\frac{E(L_t^D) - E(L_t^{TP})}{E(L_t^D)} x100$

1. The above loss function obtains after premultiplying the intertemporal loss function by $(1-\beta)$ and letting $\beta \to 1$.
2. Only the variances of the cost-push shock, IS shock, and the UIP shock enter into the calculation of the variances of inflation, the output gap, the real exchange rate, and the policy instrument.