Practical Monetary Policies

Alfred V Guender*
Department of Economics
University of Canterbury
Christchurch, New Zealand

and

David R Gillmore
Reserve Bank of New Zealand
Wellington

Revised December 2009

JEL CLASSIFICATION: E5

KEY WORDS: Average Inflation Targeting, Speed Limit, Policy Lags, Backward-Looking Model, Forward-Looking Expectations

Abstract

This paper compares a monetary policy that targets average inflation with one that targets the change in the output gap. It shows that the stabilizing properties of monetary policy strategies are sensitive to both the existence of lags in the transmission mechanism and the design of target rules. A strategy focusing on the change in the output gap is likely to prove inferior to targeting the average rate of inflation in a model where monetary policy affects the real economy sooner than inflation. Even more favorable results for average inflation targeting emerge in a framework that also includes forward-looking expectations. These results stand in marked contrast to those in standard models where policy lags are absent. To ensure sound choice of policy, central banks are advised to examine the stabilizing properties of monetary policies in a variety of models.

The authors wish to thank Richard Froyen, Robin Harrison, John Tatom, seminar participants at La Trobe University, Universität Graz, Victoria University of Wellington, NZAE and WEA conference participants, and in particular two anonymous referees and the editor of this journal for helpful comments. The first author expresses his gratitude to the Swiss National Bank for its hospitality and excellent research support. We are obliged to Tommaso Mancini and Nicholas Cuche-Curti for introducing us to DYNARE. The authors take full responsibility for any errors. The views expressed in this paper are those of the authors alone and should not be interpreted as reflecting the official position of the Reserve Bank of New Zealand on policy matters.

* Corresponding Author. Mailing Address: Private Bag 4800, UofC, Christchurch, New Zealand.
E-mail: Alfred.Guender@Canterbury.ac.nz Phone: (64)-3-364-2519 Fax: (64)-3-364-2635

Over the past two decades, monetary policy in OECD countries has been spectacularly successful in reducing inflation from high and variable rates to relatively stable levels of around one to four per cent. The containment of inflation reflects the emergence of a general consensus that the ultimate goal of monetary policy is to achieve price stability. What price stability means in practice is open to interpretation, however. Some central banks like the Reserve Bank of New Zealand have a formal legislated mandate to keep inflation, defined as the percentage change in the CPI, within an announced target band. Other central banks such as the European Central Bank and the Bank of England aim at a specific target level for the percentage change in the price level, typically two percent or below. In the United States, the mission of the Federal Reserve Board is to achieve price stability without being bound by a formal inflation target.

There are also marked differences among central banks in the definition of the inflation target that monetary policy seeks to attain. The objective of a number of central banks, notably the Reserve Banks of Australia and New Zealand, is to maintain low *average* inflation over the cycle or medium term.[1] The Bank of Canada seeks to maintain low average inflation over longer horizons but aims to keep inflation at two percent annually. The term "average inflation" does not appear in the definition of the policy objectives of other central banks such as the European Central Bank, the Bank of England or the Federal Reserve Board.[2]

Doubts about a consensus on the target variables of monetary policy have also been expressed by Walsh (2003). He questions whether the Federal Reserve's target variable - other than the rate of inflation - is the output gap proper. According to his interpretation of recent Fed policy, the Fed has pursued a *speed* limit policy, i.e. focused on the growth rate of actual output relative to growth of potential output in the design of monetary policy. Analyzing such a speed limit policy in a forward-looking rational expectations model that also allows for output and inflation

---

[1] The Policy Target Agreement 2007 between the Minister of Finance and the Governor of the Reserve Bank of New Zealand stipulates that "[f]or the purpose of this agreement, the policy target shall be to keep future CPI inflation outcomes between 1 and 3 per cent on *average* over the medium term." In the description of its monetary policy framework, the Reserve Bank of Australia reports that "[i]n the Third Statement on the Conduct of Monetary Policy, issued in 2006, the Governor and the Treasurer agreed that the appropriate target for monetary policy is to achieve an inflation rate of 2-3 per cent on *average*, over the cycle, ….".

[2] The designers of the monetary policy strategy of the European Central Bank do emphasize, however, that the chief objective of policy is to maintain a rate of inflation close to (and preferably below) two percent over the *medium term* (Issing, Gaspar, Angeloni, and Tristani (2001)).

persistence, he finds that the focus on the change in the output gap in the design of monetary policy is warranted. A speed limit policy dominates flexible single-period inflation targeting unless agents are predominantly backward-looking. Nessén and Vestin (2005) assess the performance of average inflation targeting in a model similar to Walsh's and find that it is superior to flexible single-period inflation targeting. The reason that both types of policies work well in the forward-looking set-up is self-evident. Optimal policy dictates that monetary policy be history-dependent in the sense that past information is essential for setting current policy. Both a speed limit policy and average inflation targeting introduce a dynamic element into the policy-setting process which in turn establishes a conduit through which the monetary authorities can affect the forward-looking inflation expectations formed by agents. This expectations channel is not operative under flexible single-period inflation targeting. Söderström (2005) finds that a speed limit policy delivers a better stabilization performance than average inflation targeting in the standard forward-looking model upon which the current literature predominantly relies to analyze monetary policy strategies.

The workhorse model employed by Walsh (2003), Söderström (2005), and Nessén and Vestin (2005) is based on sound microeconomic underpinnings but has one potentially serious weakness. The model does not account for the lags in the transmission process of monetary policy.[3] The existence of these lags and their importance in the transmission process of monetary policy is widely acknowledged by practitioners, however. Blinder (1997) goes as far as saying that "failure to take account of lags is, I believe, one of the main sources of central bank error." Batini and Haldane (1999) formulate the basic challenge for policymakers as follows: "The monetary authorities need to be conscious of these lags when framing policy; they need to be able to calibrate them reasonably accurately; and they then need to embed them in the design of their policy rules."

What lag structure is observed in practice? According to conventional wisdom, a change in monetary policy affects output after 12 to 15 months and inflation after 18

[3] The absence of transmission lags in the forward-looking model enables the central bank to exercise perfect control over aggregate demand. A disturbance in the goods market that displaces output can be totally offset by the appropriate adjustment of the nominal rate of interest. Comprehensive treatments of the forward-looking model appear in Walsh (2003b), Woodford (2003), Froyen and Guender (2007), and Galí (2008).

to 24 months.[4] Thus in practice, the change in policy affects the real economy before it impacts on inflation.

Several recent contributions investigate the implications of lags in the effect of monetary policy.[5] Using a simple backward-looking model, Ball (1999) concludes that nominal income growth targeting, which is a special case of a speed limit policy, is a problematic monetary policy strategy as it causes the rate of inflation and the output gap to become excessively volatile. In sharp contrast, inflation-oriented policy strategies such as gradual and strict inflation targeting are efficient policies as they generate variances of the rate of inflation and the output gap on the policy frontier. Batini and Haldane (1999) emphasize that the existence of policy lags necessitates that central banks adopt a forward-looking, i.e. a pre-emptive approach in the design of monetary policy strategies. Central banks need to design simple forecast-based instrument rules that take proper account of the existing lag structure. Rudebusch and Svensson (1999) examine a number of instrument and targeting rules in a backward-looking model estimated with quarterly US data. They show that a simple forward-looking instrument rule that responds to expected inflation two years into the future does exceedingly well in stabilizing the economy. Flexible inflation targeting combined with interest rate smoothing achieves similarly good results relative to optimal policy. Goodhart (2001) provides a comprehensive account of the problems policymakers encounter in practice when choosing the optimal targeting horizon for monetary policy.

It is apparent then that competing views of the monetary policy transmission mechanism co-exist in the literature. No particular view is inherently superior or more

---

[4] Another view of the transmission mechanism dispenses with this distinctive *impact* lag pattern and argues instead that monetary policy shocks in the policymaker's reaction function have their *maximum* effect on the target variables at different points along the timeline. Such monetary policy shocks do not occur in our framework. The identification and measurement of monetary policy shocks in the policymaker's reaction function is investigated by Christiano, Eichenbaum, and Evans (1999). A later paper (2005) by the same authors featuring New Keynesian style wage and price setting finds an expansionary monetary policy shock having its *peak* effect on output after about 18 months and on inflation after 24 months. Woodford (2003, Chapter 3) arrives essentially at the same conclusion regarding the delayed effect of a monetary policy shock on real output.

[5] The literature on monetary policy of the 1950s and 1960 was acutely aware of the difficulties posed for policymakers by lags in the transmission process of monetary policy. See Culbertson (1960) and Friedman (1961). Using US and UK data, Batini and Nelson (2001) investigate the time lag in the effect of monetary policy. They confirm Friedman's earlier finding that the effect of policy on inflation peaks after one year. Other notable contributions are Svensson (1997) and Taylor (1994). Svensson derives the target rules that underpin flexible and strict inflation targeting strategies in a backward-looking model that is essentially the same as Ball's (1999). Taylor employs a simpler variant of the backward-looking model, one where a change in the real rate of interest affects the output gap in the same period.

plausible than another. The current paper does, however, take a firm stand on the modeling framework and the structure of the transmission mechanism of monetary policy. A purely backward-looking model where expectations are formed adaptively and the effects of policy take time serves as our initial frame of reference. In this model we assess the performance of average inflation targeting and a speed limit policy relative to optimal policy.

The motivation behind choosing a simple backward-looking framework is twofold. First, our objective is to provide a tractable analysis of the performance of the two monetary policy strategies in a modeling framework distinctly different from the forward-looking framework. After all, a given monetary policy strategy can only be considered superior to another if it performs well across the board. The need to assess the performance of competing strategies in a variety of models arises from our limited understanding of how the true economy works.[6] Second, the backward-looking model captures the observed dynamic behavior of the target variables and introduces realistic lags into the transmission process of monetary policy. A change in monetary policy affects the output gap with a one-period lag and the rate of inflation with a two-period lag. Given the inherent dynamics and the distinct lag structure of the backward-looking model, the current paper thus goes beyond merely embedding a dynamic monetary policy strategy into a framework where monetary policy effects occur contemporaneously, which is the norm in forward-looking New Keynesian models. Forward-looking expectations of inflation and the output gap –the quintessential features of the New Keynesian model - are added to the model after the initial analysis to examine their importance in evaluating the relative attractiveness of average inflation targeting and a speed limit policy.

The paper's two major findings can be briefly summarized here. One finding concerns the specification of the target rule in the purely backward-looking model. The other finding addresses the performance of the two competing strategies in, first, a purely backward-looking model and then in a hybrid model with forward-looking expectations.

The target rule underlying average inflation targeting and a speed limit policy, respectively, must conform to the policy lag structure imposed by the purely backward-looking model. Choosing the appropriate target horizon for expected

[6] This is the view of McCallum (1988) and Rudebusch and Svensson (1999).

inflation in the target rule is absolutely essential for ensuring that both strategies live up to their potential to stabilize the economy and control inflation. This finding for *target rules* in the backward-looking model complements the observation by Batini and Haldane (1999), according to whom correctly specified *instrument* rules reflect the existence of transmission lags.

After assessing and comparing the performance of the two strategies for monetary policy, we are of the view that average inflation targeting *is* and a speed limit policy *can* be a sound strategy of monetary policy in the purely backward-looking model. The undesirable consequences associated with a conventional speed limit policy – huge swings in the output gap and the rate of inflation - can be avoided if the target rule underlying a speed limit policy adheres to the two period lag between the policy instrument and the rate of inflation. Average inflation targeting does exceedingly well compared to a speed limit policy if society values stability of output. As society's aversion to inflation variability increases, however, average inflation targeting becomes less attractive. The exact opposite result holds for a speed limit policy. Thus society's relative aversion to inflation variability is a critical element in determining the performance of the two strategies of monetary policy. Surprisingly, introducing forward-looking expectations into a stylized model that features a slightly different transmission mechanism overturns this result. In the amended model average inflation targeting is unambiguously superior to a speed limit policy.

The remainder of the paper is as follows. Section 2 introduces a simple backward-looking model and discusses society's preferences. Section 3 analyzes optimal policy from society's perspective. Section 4 analyzes average inflation targeting and Section 5 discusses in detail a speed limit policy in the backward-looking model. The relative performance of either strategy vis-à-vis optimal policy in the backward-looking model is evaluated in Section 6. Section 7 re-examines the performance of both strategies in a slightly adapted framework where forward-looking expectations of the output gap and inflation are present. Section 8 concludes.

2. The Backward-Looking Model and the Policymaker's Preferences

The purely backward-looking model consists of two equations that describe the dynamic behavior of the output gap and the rate of inflation:

$$y_t = -\beta r_{t-1} + \lambda y_{t-1} + \varepsilon_t \quad (1)$$

$$\pi_t = \gamma \pi_{t-1} + \alpha y_{t-1} + \eta_t \quad (2)$$

$$\beta > 0,\ 0 \le \lambda < 1 \qquad \alpha > 0,\ 0 \le \gamma \le 1$$

y = the output gap (the difference between real output and its potential)

r = the real rate of interest

π = the rate of inflation

ε and η = white noise shocks with constant variances σ_ε^2 and σ_η^2, respectively.

The backward-looking IS relation of equation (1) has two prominent features. First, the output gap exhibits persistence, with λ measuring the degree of persistence. Second, the output gap responds to a change in the real rate of interest with a one-period lag. Persistence and a lagged response are also critical elements in the Phillips curve. According to equation (2), the current rate of inflation depends on the previous period's rate and reacts to the output gap with a one period lag.[7]

This lag structure of the model gives rise to the following relationship between the policy instrument and the two key variables of the model:

$$r_t \rightarrow y_{t+1} \rightarrow \pi_{t+2}$$

The effect of policy on the economy and the rate of inflation takes time, with policy affecting the output gap sooner than the rate of inflation. Time is measured in years. This distinctive lag structure provides the model with a dose of realism. As pointed out in the introduction, there is general agreement in policy circles that the effects of changes in monetary policy are not instantaneous but set in with delay. The existence of transmission lags in turn has profound implications for the efficient operation of monetary policy strategies such as average inflation targeting and a speed limit policy. Not only must the policymaker adopt a forward-looking approach in the design of the target rules that underlie both strategies but these target rules must also conform to the notion that policy affects inflation with a two-period lag.

Society is concerned about the variability of the output gap and the rate of inflation in period *t*:

[7] In Ball (1999) the parameter $\gamma = 1$. As shown in the appendix, $\gamma = 1$ results in an accelerating Phillips Curve which in turn accounts for the instability of nominal income growth targeting

$$E\left[L_t\right] = V(y_t) + \mu V(\pi_t) \qquad (3)$$

The objective is to minimize the above expected loss function where μ is society's aversion to inflation relative to output gap variability. Equation (3) represents society's loss function, which the policymaker minimizes under optimal policy.

3. Optimal Policy

Given the quadratic objective function, the policymaker follows a linear policy rule in the conduct of policy. This policy rule provides for a systematic relationship between the two targets of monetary policy. As policy works with lags, in the current period the policymaker chooses the expected output gap next period and treats the rate of inflation as predetermined:[8]

$$\theta E_t y_{t+1} + E_t \pi_{t+1} = 0 \qquad (4)$$

The target rule embodied by equation (4) assumes that the target value for the output gap and the rate of inflation is zero, respectively. The parameter θ represents the weight that the policymaker places on the output gap relative to the rate of inflation when setting policy.

Combining the target rule in (4) with the IS and PC relations yields the policymaker's reaction function:

$$r_t = \frac{\gamma}{\theta\beta}\pi_t + \left(\frac{\alpha}{\theta\beta} + \frac{\lambda}{\beta}\right) y_t \qquad (5)$$

The reaction function specifies how the policymaker adjusts the policy instrument, the real rate of interest, in the wake of deviations of the output gap and the rate of inflation from target.[9] In the event of a one-percentage point rise (fall) in the rate of inflation, the policymaker raises (lowers) the interest rate by $\gamma/\theta\beta$ percentage points. Similarly, following a one percentage rise (fall) in the output gap, the policymaker responds by raising (lowering) the interest rate by $\alpha/\theta\beta+\lambda/\beta$ percentage points.

[8] Svensson (1997) invokes dynamic programming to derive an equivalent linear target rule.

[9] As Ball (1999) we assume that the policymaker has complete control over the real rate of interest and responds to the true output gap. The former assumption is necessary to implement a given policy strategy successfully. Distinguishing between the nominal and real rate of interest will not provide any additional insights in the current paper. The latter assumption is invoked to allow us to test the performance of average inflation targeting and a speed limit policy vis-à-vis optimal policy under the same informational conditions as found in Nessén and Vestin (2005), Söderström (2005), and Walsh (2003). Notice the absence from (5) of monetary policy shocks which are the key drivers of movements in the target variables in Christiano et al (1999, 2005).

Computing the variances of the target variables requires a few steps. First, backdate equation (5) by one period and insert it into the IS equation to obtain the reduced form equation of the output gap:

$$y_t = -\frac{\gamma}{\theta}\pi_{t-1} - \frac{\alpha}{\theta} y_{t-1} + \varepsilon_t \tag{6}$$

Along with the Phillips curve, equation (6) can be set up in matrix form to calculate the variances of the rate of inflation and the output gap:[10]

$$V\left(y_t\right) = \frac{\left(2\alpha\gamma\theta + \theta^2(1-\gamma^2)\right)\sigma_\varepsilon^2 + \gamma^2\sigma_\eta^2}{\theta^2 - (\gamma\theta - \alpha)^2} \tag{7}$$

$$V\left(\pi_t\right) = \frac{\alpha^2\theta^2\sigma_\varepsilon^2 - \left(\alpha^2 - \theta(2\alpha\gamma + \theta)\right)\sigma_\eta^2}{\theta^2 - (\gamma\theta - \alpha)^2} \tag{8}$$

The variability of the output gap and the rate of inflation, respectively, does not depend exclusively on the extent of uncertainty on the supply side of the economy. Due to the one-period transmission lag between the policy instrument and output, the variance of the IS disturbance also affects the variance of both target variables. Inserting (7) and (8) into (3) and minimizing the loss function with respect to θ yields the optimal value of the policy parameter, the weight on the output gap in the target rule:

$$\theta^* = \frac{1-\gamma^2+\alpha^2\mu+\sqrt{4\alpha^2\gamma^2\mu+\left(-1+\gamma^2-\alpha^2\mu\right)^2}}{2\alpha\gamma\mu} \tag{9}$$

The optimal policy parameter θ^* depends on two parameters from the Phillips curve: α (the responsiveness of inflation to the lagged output gap) and γ (the degree of persistence of inflation). θ^* also depends on society's preference parameter μ. The higher the aversion to inflation variability, the lower θ^* is.[11]

By choosing values for $0 \le \mu \le \infty$ and picking representative values for α and γ as well as the variances of the shocks, we can trace out the optimal policy frontier which depicts the trade-off between the variance of inflation and the variance of the

[10] For further details see Hendry (1995, pp.111-112), Ball (1999) or Chapter 12 of Froyen and Guender (2007).

[11] As $\mu \to \infty, \theta \to \alpha / \gamma$. Thus even if the policymaker cares only about the variability of inflation, he still puts some positive weight on the output gap in setting policy. The weight on the output gap remains positive because the policymaker can affect the output gap, which affects the rate of inflation, sooner than inflation proper. In the opposite case where the policymaker cares only about output variability the weight on the output gap in the target rule becomes infinitely large: $\theta \to \infty$ as $\mu \to 0$.

output gap.[12] In Figure 1 the solid line depicts the tradeoff between output and inflation variability in the purely backward-looking model.[13]

In the following two sections, we examine the properties of average inflation targeting and a speed limit policy. Both are considered plausible strategies for policymaking in practice. Our objective is twofold. First, we wish to assess whether a policy that focuses in part on past inflation is superior to a policy that focuses in part on the lagged output gap. Second, we want to determine how either policy compares to optimal policy. Specifically, we want to measure the efficiency loss that average inflation targeting and a speed limit policy impose on society.

4. Average Inflation Targeting

This section examines the performance of average inflation targeting (AIT). Average inflation $\overline{\pi}_t$ is defined over two periods as in Néssen and Vestin (2005):

$$\overline{\pi}_t = \tfrac{1}{2}\left(\pi_t + \pi_{t-1}\right) \qquad (10)$$

We follow their example of choosing a short horizon for calculating the average rate of inflation because we wish to obtain analytical solutions for the key policy parameter and the variances of the target variables.[14]

The policymaker's objective is to minimize the unconditional variances of both average inflation and the output gap, with μ^{AIT} denoting the policymaker's relative aversion to average inflation variability:

$$E\left[L_t\right]^{AIT} = V(y_t) + \mu^{AIT} V(\overline{\pi}_t) \qquad (11)$$

Letting θ^{AIT} denote the relative weight on the expected output gap, we specify the target rule underlying average inflation targeting as:

$$\theta^{AIT} E_t y_{t+1} + E_t \overline{\pi}_{t+2} = 0 \qquad (12)$$

---

[12] The parameter values $\alpha = 0.4$, $\sigma_\varepsilon^2 = 1$ and $\sigma_\eta^2 = 1$ are taken from Ball (1999). In addition, we choose $\gamma = 0.9$.

[13] The policy frontier is based on values of the preference parameter μ that range from *0.01* to *60.*

[14] Néssen and Vestin (2005) briefly consider longer horizons for calculating the average rate of inflation but concede that the evaluation of the performance of average inflation becomes rather difficult as the optimal horizon depends critically on the policymaker's relative aversion to output gap variability in his objective function. Extending the definition of average inflation to three and more periods rules out the derivation of analytical results.

Given the two-period lag between the interest rate in period *t* and the rate of inflation, the target for average inflation is expected average inflation in period *t+2* rather than period *t+1.* Grounding policy on expected average inflation two periods into the future avoids the inclusion of the current rate of inflation in the target rule. As a rule, the inclusion of contemporaneous information in the form of current inflation in the target rule (or the current output gap in the case of a speed limit policy) complicates the determination of the optimal policy parameter.[15]

Combining the above rule with the IS and Phillips curve equations and making use of equation (10) yields the response of the policy instrument to the rate of inflation and the output gap under average inflation targeting:

$$r_t = \frac{\gamma(1+\gamma)}{(\alpha+2\theta^{AIT})\beta}\pi_t + \left(\frac{\alpha(1+\gamma)}{(\alpha+2\theta^{AIT})\beta} + \frac{\lambda}{\beta}\right)y_t \tag{13}$$

Substituting (13) back into the IS relation and following the aforementioned solution procedure yields the variances of the output gap, the rate of inflation and average inflation:[16]

$$V(y_t^{AIT}) = (1 + \frac{\alpha^2(1+\gamma)}{4\theta^{AIT}(\alpha+\theta^{AIT}(1-\gamma))})\sigma_\varepsilon^2 + \frac{\gamma^2(1+\gamma)}{4\theta^{AIT}(\alpha+\theta^{AIT}(1-\gamma))}\sigma_\eta^2 \tag{14}$$

$$V(\pi_t^{AIT}) = \frac{\alpha^2(\alpha^2+4\theta^{AIT}(\alpha+\theta^{AIT}))\sigma_\varepsilon^2 + (\alpha^2\gamma^2+4\theta^{AIT}(\alpha(1+\gamma(1+\gamma))+\theta^{AIT}))\sigma_\eta^2}{4(1+\gamma)\theta^{AIT}(\alpha+\theta^{AIT}(1-\gamma))} \tag{15}$$

$$V(\bar{\pi}_t^{AIT}) = \frac{\alpha^2(\alpha+2\theta^{AIT})}{4(\alpha+\theta^{AIT}(1-\gamma))}\sigma_\varepsilon^2 + \frac{2(\alpha(1+\gamma)+\theta^{AIT})+\alpha\gamma^2}{4(\alpha+\theta^{AIT}(1-\gamma))}\sigma_\eta^2 \tag{16}$$

Inserting (14) and (16) into (11) and minimizing the loss function with respect to θ^{AIT} results in the optimal policy parameter under average inflation targeting:

$$\theta^{AIT} = \frac{1-\gamma+\sqrt{(1-\gamma)^2+\alpha^2\mu^{AIT}}}{\alpha\mu^{AIT}} \tag{17}$$

---

[15] The target rules presented in this and the following section target the inflation rate at time *t+2* because they dominate alternative specifications of the target rules from a welfare maximizing perspective. The inferior target rules, which contain the expected rate of inflation in period *t+1*, are briefly discussed in the appendix. The appendix also shows that the evolution of expected inflation becomes more complex if contemporaneous information enters the target rule. The added complexity makes the derivation of the optimal policy parameter more difficult.

[16] The acronym AIT is added as a superscript to emphasize that equations (14) – (16) represent the variances of the output gap, the rate of inflation, and average inflation under average inflation targeting.

Given the solution for θ^{AIT} , we can trace out the policy frontier ground out by average inflation targeting. How this policy frontier compares to the policy frontier under optimal policy or a speed limit policy will be discussed further in Section 6.

5. Speed Limit Policy

As stated in the introduction, the speed limit is shorthand for the change in the output gap, and is defined as $y_t - y_{t-1}$. Accordingly, under a speed limit policy (SL) the policymaker's objective is to minimize the weighted sum of the unconditional variances of the change in the output gap and the rate of inflation:

$$E\left[L_t\right]^{SL} = V(y_t - y_{t-1}) + \mu^{SL} V(\pi_t) \tag{18}$$

μ^{SL} = the weight the policymaker accords to the variance of inflation relative to the output gap in the objective function.

Again taking proper account of the transmission lag, we specify the target rule for a speed limit policy as:

$$\theta^{SL}\left[E_t y_{t+1} - y_t\right] + E_t \pi_{t+2} = 0 \tag{19}$$

Combining the IS and Phillips curve equations with the target rule (19) yields the response of the policy instrument to the rate of inflation and the output gap under a speed limit policy:

$$r_t = \frac{\gamma^2}{\left(\alpha + \theta^{SL}\right)\beta}\pi_t + \left(\frac{\alpha\gamma - \theta^{SL}}{\left(\alpha + \theta^{SL}\right)\beta} + \frac{\lambda}{\beta}\right)y_t \tag{20}$$

Comparing equation (20) with equation (5), the reaction function under optimal policy, we find that the coefficients on π_t and y_t in (20) are most likely smaller.[17] Indeed the coefficient on the output gap in (20) is not unambiguously positive. Its sign depends on the size of the model parameters α and γ and the policy parameter θ^{SL}.

Backdating equation (20) by one period and inserting it into the IS equation (1) reveals the behavior of the output gap under a speed limit policy:

$$y_t = -\frac{\gamma^2}{\alpha + \theta^{SL}}\pi_{t-1} - \frac{(\alpha\gamma - \theta^{SL})}{\alpha + \theta^{SL}} y_{t-1} + \varepsilon_t \tag{21}$$

[17] A direct comparison is not possible as $\theta^* \neq \theta^{SL}$. A definitive answer can be given if numerical values are assigned to the parameters and variances of the shocks. This is done later in the paper.

Combining this equation with the Phillips curve (2), we can calculate the variances of the output gap (y_t), inflation (π_t), and the change in the output gap $(y_t - y_{t-1})$:

$$V\left(y_t^{SL}\right)=\frac{(\alpha+\theta^{SL})^2[\alpha(1+\gamma^2)+\theta^{SL}(1-\gamma(1+\gamma-\gamma^2))]\sigma_\varepsilon^2+\gamma^4(\theta^{SL}(1+\gamma)+\alpha)\sigma_\eta^2}{\alpha D} \quad (22)$$

$$V\left(\pi_t^{SL}\right)=\frac{\alpha(\theta^{SL}+\alpha)^2(\alpha+\theta^{SL}(1+\gamma))\sigma_\varepsilon^2+[(1+\gamma^2)(\alpha(\alpha+\theta^{SL}(3+\gamma)))+2\theta^{SL^2}]\sigma_\eta^2}{D} \quad (23)$$

$$V\left((y_t-y_{t-1})^{SL}\right)=[1+\frac{\alpha(\alpha(1+2\gamma(1+\gamma))+\theta^{SL}(1+\gamma))}{D}]\sigma_\varepsilon^2+\frac{2\gamma^4\sigma_\eta^2}{D} \quad (24)$$

$$D=(\alpha+\theta^{SL}(1-\gamma))(\alpha+2\theta^{SL}(1+\gamma))$$

Inserting (23) and (24) into (18) and minimizing the expected loss function with respect to θ^{SL} yields the optimal value of the policy parameter under a speed limit policy. The analytical solution turns out to be rather complex and unwieldy. However, its general form is as follows:

$$\underset{(-)\,(?)\,(-)\;(-)\;(+)}{\theta^{SL}=f(\alpha,\ \gamma,\ \mu^{SL},\sigma_\varepsilon^2,\sigma_\eta^2).} \quad (25)$$

The (-) (+) sign denotes the effect of an increase in the size of the parameter or variance on the size of the policy parameter. A question mark implies that the effect cannot be signed unambiguously.[18]

The distinctive feature of θ^{SL} is that it depends on the variances of demand and cost-push shocks (σ_ε^2 and σ_η^2) while θ^* and θ^{AIT} do not. Thus, under a speed limit policy the origin of the disturbance influences the setting of the policy parameter. Additionally, the policy-setting process becomes more complicated because not only does a speed limit policy depend on contemporaneous information (as the current output gap appears in the definition of the speed limit in (19)) but the policymaker also relies on less information: the expected *change* in the output gap – and not the

[18] To assess the effect of a change in a given parameter or variance on the size of θ^{SL}, we varied the size of the parameter or variance in question but left all other parameters or variances unchanged. The values of the parameters and variances for the benchmark case are: $\alpha = 0.4, \gamma = 0.9, \mu = 1, \sigma_\varepsilon^2 = 1$ and $\sigma_\eta^2 = 1$.

expected output gap proper - guides a speed limit policy. This point is discussed further in the next section.

6. Performance of Strategies: An Assessment

Of central concern in this section is the attractiveness of average inflation targeting and a speed limit policy, respectively, vis-à-vis optimal policy. More precisely, to what extent does a policymaker who implements a monetary policy rule that differs from the one society finds most desirable achieve optimal results?[19]

We approach this question from two different angles, using graphical and quantitative performance measures. The first performance criterion is the output-inflation variability tradeoff while the second is a quantitative evaluation of society's welfare losses under average inflation targeting and a speed limit policy, respectively, relative to optimal policy.

Figure 2 clearly establishes the fact that average inflation targeting dominates a speed limit policy in the backward-looking framework over a sizeable range of the preference parameters.[20] A speed limit policy does very poorly in case the policymaker cares relatively little about inflation stability. For rather low values of the preference parameter $($e.g. $\mu^{SL} = 1)$, a speed limit policy keeps fluctuations in the *change* of the output gap at bay but only at the expense of large fluctuations in both the output gap and the rate of inflation. In sharp contrast average inflation targeting is far more efficient for low relative weights on the variance of average inflation $($e.g. $\mu^{AIT} = 1)$.[21] The policy frontier traced out by average inflation targeting lies below the policy frontier under a speed limit policy unless the policymaker places almost exclusive emphasis on minimizing inflation variability. Indeed, the policy

---

[19] Rogoff's (1985) observation that a central bank's objectives may differ from society's is relevant here.

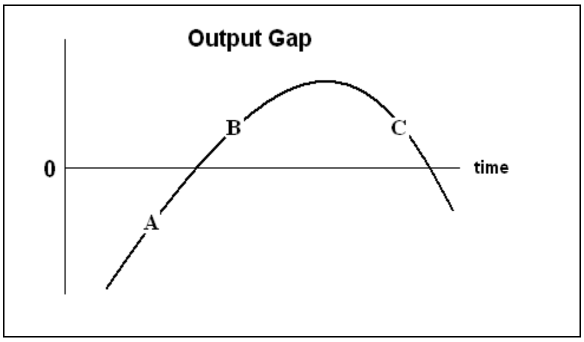
[20] The parameters μ^{AIT}, μ^{SL}, and μ vary from a low of *0.01* to a high of *1000* to cover the extreme cases of low and high relative aversion to inflation variability. For each strategy, the output-variability tradeoff is measured in terms of the variances of the (single-period) rate of inflation and the output gap. The box-, triangle-, and diamond-shaped icons indicate the loci of the variances of the rate of inflation and the output gap on the respective policy frontier for three different values of the preference parameters: *1*, *10*, and *50*.

[21] The inefficiency of a speed limit policy relative to average inflation targeting becomes even more pronounced if the policymaker has almost no concern for inflation stability. To be concrete, for $\mu^{AIT} = \mu^{SL} = 0.1$, $V(\pi_t^{AIT}) = 4.36$ and $V(y_t^{AIT}) = 1.06$ while $V(\pi_t^{SL}) = 6.21$ and $V(y_t^{SL}) = 3.88$. Interestingly, for $0 < \mu^{SL} < 1$ a speed limit policy leads to continuously decreasing variances of both the output gap and inflation but increasing variances of the *change* in the output gap as μ^{SL} increases.

frontier under average inflation targeting is virtually identical to the optimal policy frontier over a plausible range for the preference parameter μ^{AIT} ($0 < \mu^{AIT} < 25$). Notice, however, that unlike the policy frontiers associated with optimal policy and a speed limit policy, the policy frontier under average inflation targeting bends upward eventually. For values of $\mu^{AIT} > 25$, average inflation targeting produces hugely inefficient outcomes relative to optimal policy and even a speed limit policy.

Intuitively, the reason for the inferior performance of a speed limit policy for plausible values of the preference parameter is that the target rule of this policy strategy relies on less information than average inflation targeting. Some important information (whether real output is above or below potential) is lost. It is this information that is essential for providing an effective response to future inflation, because inflation in the next period depends on the current output gap. This is illustrated graphically in Figure 3. At points A and B, the output gap is negative and positive respectively, while at both points the speed limit is positive. By taking account of the speed limit only, the policymaker responds in a similar manner at both points (tightening monetary policy), even though at point A the economy is below potential.[22]

Figure 3: The Output Gap



That a speed limit policy produces an inferior policy response can also be seen by comparing the reaction of the policymaker at points B and C. At both points the output gap is the same and positive so that a tightening of monetary policy is called

[22] Based on the parameter values in Ball (1999), a comparison of the reaction functions associated with optimal policy (equation (5)) and a speed limit policy (equation (20)) reveals that the coefficient on the output gap is much smaller under a speed limit policy than under optimal policy. This indicates that the response of the policy instrument to the output gap is too weak under a speed limit policy compared to optimal policy.

for under optimal policy. The change in the output gap is positive at B so that policy tightens under a speed limit policy. However, at point C, the change in the output gap is negative which prompts the policymaker to ease the stance of policy even though a tightening is warranted.

The results of a quantitative evaluation of the performance of average inflation targeting and a speed limit polity are set out in Tables 1 and 2. A simple two-stage procedure is followed. Initially, we determine what values of μ society must have for average inflation targeting to generate the same welfare losses for both the policymaker and society. We then compare society's welfare losses under average inflation targeting relative to optimal policy for those given values of μ. This two-stage procedure is repeated for a speed limit policy.

The last column of Table1 shows that average inflation targeting is almost as efficient as optimal policy, especially for rather low values of μ. For $\mu < 4$ average inflation targeting is only slightly less efficient than optimal policy, with the efficiency loss well below one percent. For $\mu < 8$ the relative welfare loss associated with average inflation targeting still amounts to less than two percent. The performance of average inflation targeting relative to optimal policy steadily worsens as the size of μ increases with the efficiency loss approaching *12* percent for μ around *60*.

In contrast, a speed limit policy is vastly inferior to optimal policy if society is as much concerned about output gap variability as it is about inflation variability. The efficiency loss associated with a speed limit policy declines as the size of μ increases but still hovers around *6.5* percent for $\mu = 10.58$. The relative welfare loss under a speed limit policy ranges from a maximum of approximately *62* percent when society cares less about inflation than output gap variability ($\mu \approx 0.67$) to a minimum of nearly zero when society is far more concerned about inflation variability than output gap variability ($\mu \approx 103$).

Taken altogether, if society values output stability, then average inflation targeting is superior to a speed limit policy in the simple backward-looking framework. However, if society shows far greater, i.e. almost exclusive concern for inflation stability, then a speed limit policy dominates average inflation targeting. In such a scenario, a welfare-maximizing policymaker all but ignores the expected change in the output gap when setting policy and pays almost exclusive attention to the expected rate of inflation. The target rule is essentially the same as under strict inflation targeting which is an extreme form of optimal policy and hence efficient.

7. Check for Robustness

At this stage a natural question arises: how robust are the results discussed so far to a change in model specification? The purely backward-looking case can be criticized on the grounds that expectations are formed adaptively. To examine how the introduction of forward-looking rational expectations causes the stabilization performance of average inflation targeting and a speed limit policy to change, we propose the following amended model:[23]

$$y_t = (1-w)E_{t-1}y_{t+1} + w\lambda y_{t-1} - \beta r_{t-1} + \varepsilon_t \quad (26)$$

$$\pi_t = (1-w)E_{t-2}\pi_{t+1} + w\gamma\pi_{t-1} + \alpha E_{t-2}y_{t-1} + \eta_t \quad (27)$$

$$0 \le w \le 1$$

The new parameter *w* indicates the extent to which agents in the model economy are backward-looking. The closer *w* is to one, the more backward-looking agents are.[24]

The conduct of monetary policy in the amended model is more complex than in the purely backward-looking model. The precise way in which the policy instrument reacts to the pre-determined feedback variables depends on the strategy pursued:

$$r_t = g_1 z_{1t} + g_2 z_{2t} + g_3 E_t \pi_{t+1} \quad (28)$$

z_{1t} and z_{2t} are the target variables of a particular policy strategy at time *t*. Under the optimized instrument rule, the target variables are the rate of inflation and the output gap; under average inflation targeting, the target variables are average inflation (defined over two periods) and the output gap while under a speed limit policy the target variables are the rate of inflation and the change in the output gap. Following Svensson (2000), we include only pre-determined variables in the reaction function.

---

[23] We are thankful to a referee for suggesting this model. It is a simplified version of the open-economy model by Svensson (2000).

[24] While we do not propose to delve into the microeconomic underpinnings of the amended model, we wish to point out that it is consistent with a scenario where firms set their prices two periods in advance and households make their consumption decisions one period in advance. Notice that the amended model does not correspond exactly to the model proposed in section 2 even if *w=1*, i.e. agents are completely backward-looking. The appearance of $E_{t-2}y_{t-1}$ in the amended model makes it impossible for an IS shock in period *t-1* to affect inflation in period *t*.

As a result, the expectation of inflation in period $t+1$ appears in the reaction function.[25] The parameters g_i, $i=1, 2, 3$ are chosen so that the loss function associated with the strategy is minimized.[26]

Figure *4* underscores that w is a key parameter in shaping the inflation-output variability trade-off under the optimized simple rule (OSR). As agents become more forward-looking, i.e. the size of w decreases, the inflation-output variability trade-off improves. At the same time, as agents become more forward-looking, the inflation-output variability trade-off becomes more and more compact. When $w=0.2$, there is no discernible trade-off between inflation and output gap variability as the inherent dynamics of the model have effectively been shut off. In addition, the expectations channel of the transmission process of monetary policy cannot be activated through a change in the setting of the policy instrument (in period $t-1$) as the expectation of the rate of inflation in period $t+1$ is formed in period $t-2$.

The sensitivity of the performance of the three monetary policy strategies to varying degrees of "backward-lookingness" ($w=1, 0.8, 0.5$) is brought out in Figures *5* and *6*. Inspection of the two figures reveals, first, that average inflation targeting dominates a speed limit policy in the amended model irrespective of the size of w.[27] Second, the policy frontier traced out by average inflation targeting tracks its counterpart under the optimized simple rule very closely except for rather high values of preference parameter in the loss function (approx. $\mu^{AIT} \geq 20$). According to Figure *5*, the dominance of average inflation targeting over a speed limit policy is most pronounced when all agents are backward-looking ($w=1$). Even if inflation expectations are just as important as inflation persistence ($w=0.5$), as shown in Figure *6*, the policy frontier associated with a speed limit policy clearly lies above the one ground out by average inflation targeting. Indeed, what is remarkable about Figure *6* is that average inflation targeting can produce a marginally better inflation-output

[25] Not being pre-determined, $E_t\pi_{t+2}$ is ruled out as a feedback variable in the reaction function. If it were to appear on the right-hand side of the reaction function, then the policy instrument would respond to an endogenous variable, a variable the policy instrument directly affects. Rudebusch and Svensson (1999) and Svensson (1999) label such rules *implicit reaction* functions, i.e. equilibrium conditions due to the endogeneity involved.

[26] We used the optimal simple rule procedure in DYNARE to determine the parameters in question.

[27] Even if the problematic nature of *implicit* reaction functions were to be ignored and $E_t\pi_{t+2}$ appeared in place of $E_t\pi_{t+1}$ in equation (28), average inflation targeting remains superior to a speed limit policy in the amended model unless society places very little value on output stability irrespective of the size of w. These results, which evoke those reported for the purely backward-looking model, are not reported here but are available upon request by the authors.

variability trade-off than the optimized simple rule. This result is a direct consequence of the fact that the optimized simple rule is not a globally optimal reaction function. Should both society and the policymaker care enough about output stability, then average inflation targeting rather than single-period inflation targeting under the optimized instrument rule is the superior policy strategy.[28] The additional inflation persistence introduced by targeting inflation over a two-period interval is responsible for this result.

8. Summary and Conclusion

The objective of the Reserve Banks of Australia and New Zealand is to control average inflation over the medium term or cycle. Federal Reserve policy has recently been interpreted as an attempt to follow a speed limit policy. Both strategies of monetary policy have been analyzed from a theoretical perspective in forward-looking New Keynesian models where the effect of policy on the target variables occurs within the same period. A speed limit policy tends to dominate average inflation targeting in such forward-looking models. How robust is the superior performance of a speed limit policy to a change in the modeling framework?

This paper evaluates the performance of average inflation targeting and a speed limit policy in an alternative framework. The study is motivated by the observance that sound policymaking in practice requires central bankers to take account of lags in the effect of monetary policy on the target variables. It is widely accepted that a change in monetary policy has a delayed effect on output and inflation, with the impact on output occurring sooner than on inflation. A purely backward-looking model featuring this lag pattern serves as the initial framework within which the efficiency losses of average inflation targeting and a speed limit policy vis-à-vis optimal policy are investigated. Particular attention is paid to the specification of the target rules that underlie both strategies of monetary policy in light of Ball's (1999)

[28] The policy frontier under AIT lies below the frontier under OSR for $\mu^{AIT} = 0.5, 1, 2$, represented by the first three markers closest to the vertical axis. For instance, when $w=0.5$ and $\mu = \mu^{AIT} = 1$, the response of the policy instrument to current average inflation is positive *(0.133)* but negative and weak to expected inflation *(-0.012)*. Just the opposite is the case under the optimized simple rule where the policymaker all but fails to respond to current inflation *(-0.027)* but reacts strongly to expected inflation *(0.311)*.

claim that nominal income growth targeting, which is a special case of a speed limit policy, has undesirable properties.

A central finding of this paper is that policymakers must pay heed to the transmission lags in monetary policy in designing target rules for average inflation targeting and a speed limit policy in the purely backward-looking model. A speed limit policy can be very inefficient if the target rule is not properly specified in the sense that the target horizon for expected inflation does not conform to the two-period lag imposed by the structure of the model. The choice of a target horizon for expected average inflation is less crucial under average inflation targeting but choosing a shorter target horizon results in some welfare loss for society.

Our findings suggest further that average inflation targeting imposes little cost on society relative to optimal policy if output stability is an important macroeconomic goal. In contrast, a properly specified speed limit policy is considerably less efficient if output stability is deemed important. As a rule, average inflation targeting becomes less attractive while a speed limit policy becomes more attractive as the concern about inflation variability relative to output gap variability increases. In case there is no concern about output fluctuations, a speed limit policy dominates average inflation targeting. Under these circumstances, a speed limit policy approaches strict inflation targeting which is a special case of optimal policy.

The contrast between the competing strategies is even starker if forward-looking expectations are added to the model. Although the presence of forward-looking expectations causes the transmission mechanism of monetary policy to change slightly, the comparison of the two strategies in the amended model produces an unambiguous result: average inflation targeting is clearly superior to a speed limit policy.

To sum up, there are at least two key elements that shape the performance of monetary policy strategies in any simple macro model: the extent to which agents are backward/forward-looking and the time pattern of the monetary policy transmission mechanism. This paper shows that the dominance of a speed limit policy over average inflation targeting in the New Keynesian model where policy effects are contemporaneous does not carry over to a model economy where monetary policy works with lags. The existence of lags in the effects of policy actions on output and inflation is hardly a matter of dispute in policy circles. A speed limit policy may thus prove to be an inferior monetary policy strategy even if agents are partially forward-looking. Given that the attractiveness of the two policy strategies is largely model-specific, the question of which policy is superior remains unsettled.[29] Further analysis of the performance of average inflation targeting and a speed limit policy across a wider spectrum of modeling frameworks is warranted.

[29] There are of course many other issues and problems that policymakers face in the real world that this paper does not address. For instance, Orphanides and Williams (2002), Orphanides and van Norden (2002) and Orphanides (2003) describe the setting of policy under imperfect information. The fact that there is a measurement problem associated with potential output is often cited in support of a speed limit policy.

References

Ball, L. (1999). Efficient Rules for Monetary Policy. *International Finance*, 2(1), 63-83.

Batini, N. and Haldane A. (1999). Forward-Looking Rules for Monetary Policy. In *Monetary Policy Rules*. Edited by J.B. Taylor. University of Chicago Press, Chicago.

Batini, N. and Nelson E. (2001). The Lag from Monetary Policy Actions to Inflation: Friedman Revisited. *International Finance* 4(3), 381-400.

Blinder, A. (1997). What Central Bankers Could Learn from Academics – and Vice Versa. *Journal of Economic Perspectives*, 11, 3-19.

Christiano, L. J., Eichenbaum M., and Evans C. L. (1999). Monetary Policy Shocks: What Have We Learned and to What End? In *Handbook of Macroeconomics,* vol 1A, edited by J. B. Taylor and M. Woodford. North-Holland, Amsterdam.

______ (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113, 1-45.

Culbertson, J. M. (1960). Friedman on the Lag in the Effect of Monetary Policy. *Journal of Political Economy*, 68, 617-621.

Friedman, M. (1961). The Lag in the Effect of Monetary Policy. *Journal of Political Economy*, 69, 447-466.

Froyen, R.T. and Guender, A.V. (2007). *Optimal Monetary Policy under Uncertainty*. Edward Elgar, Cheltenham, UK.

Galí, J. (2008). *Monetary Policy, Inflation and the Business Cycle: An Introduction of to the New Keynesian Framework*. Princeton University Press, Princeton and Oxford.

Goodhart, C.A.E. (2001). Monetary Transmission Lags and the Formulation of the Policy Decisions on Interest Rates. *Federal Reserve Bank of St. Louis Review*, 69(4), 165-182.

Hendry, D.F. (1995). *Dynamic Econometrics*. Oxford University Press, Oxford, UK.

Issing, O., Gaspar V., Angeloni I., and Oreste T. (2001). *Monetary Policy in the Euro Area: Strategy and Decision Making at the European Central Bank*. Cambridge University Press, Cambridge, UK.

McCallum, B.T. (1988). Robustness Properties of a Rule for Monetary Policy. *Carnegie-Rochester Conference Series*, 29, 173-203.

Nessén, M. and Vestin, D. (2005). Average Inflation Targeting. *Journal of Money, Credit, and Banking*, 37 (5), 837-864.

Orphanides, A. (2003). Monetary Policy Evaluation with Noisy Information. *Journal of Monetary Economics*, 50, 605-631.

Orphanides, A. and van Norden, S. (2002). The Unreliability of Output-Gap Estimates in Real Time. *The Review of Economics and Statistics*, 84 (4), 569-583.

Orphanides, A. and Williams, J. (2002). Robust Monetary Policy Rules with Unknown Natural Rates. *Brooking Papers on Economic Activity*, 2, 63-118.

Rogoff, K. (1985). The Optimal Degree of Commitment to an Intermediate Monetary Target. *The Quarterly Journal of Economics*, 100, 1169-1189.

Rudebusch, G. and Svensson L.E.O. (1999). Policy Rules for Inflation Targeting. In *Monetary Policy Rules*. Edited by J.B. Taylor. University of Chicago Press, Chicago.

Söderström, U. (2005).Targeting Inflation with a Role for Money. *Economica*, 72, 577-596.

Svensson, L.E.O. (1997). Inflation Forecast Targeting: Implementing and Monitoring Inflation. *European Economic Review*, 41, 381-399.

______ (1999). Inflation Targeting as a Monetary Policy Rule, *Journal of Monetary Economics*, 43, 607-654.

______ (2000). Open-Economy Inflation Targeting. *Journal of International Economics*, 50, 155-183.

Taylor, J.B. (1994). The Inflation / Output Variability Trade-off Revisited, in Fuhrer, J.C. (ed.), *Goals, Guidelines, and Constraints Facing Monetary Policymakers*, Federal Reserve Bank of Boston.

Walsh, C. (2003a). Speed Limit Policies: The Output Gap and Optimal Monetary Policy. *American Economic Review*, 93 (1), 265-278.

_____, (2003b). *Monetary Theory and Policy*. MIT Press, Cambridge, USA.

Woodford M. (2003). *Interest and Prices. Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton and Oxford.

Figure 1: The Optimal Policy Frontier

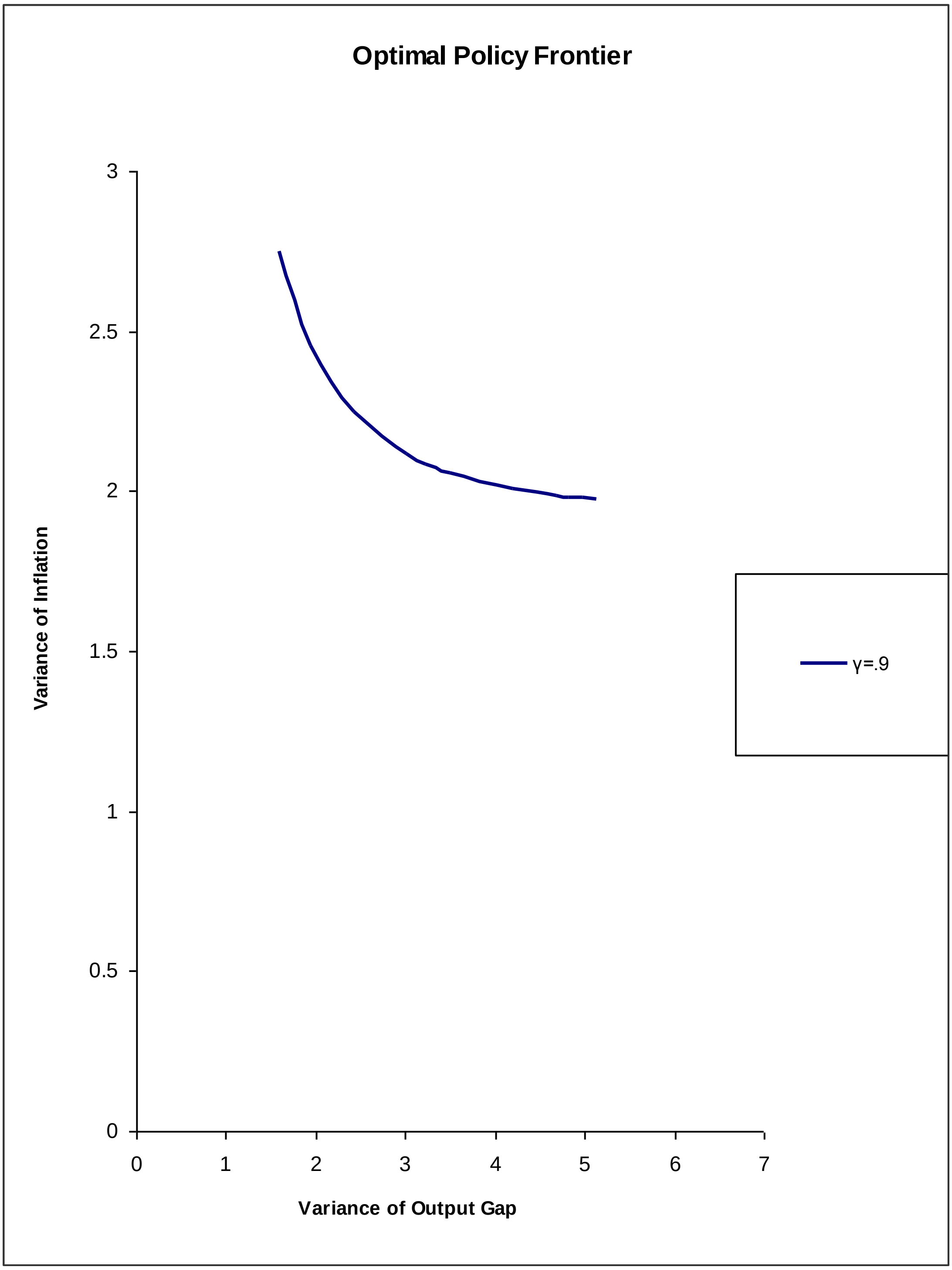
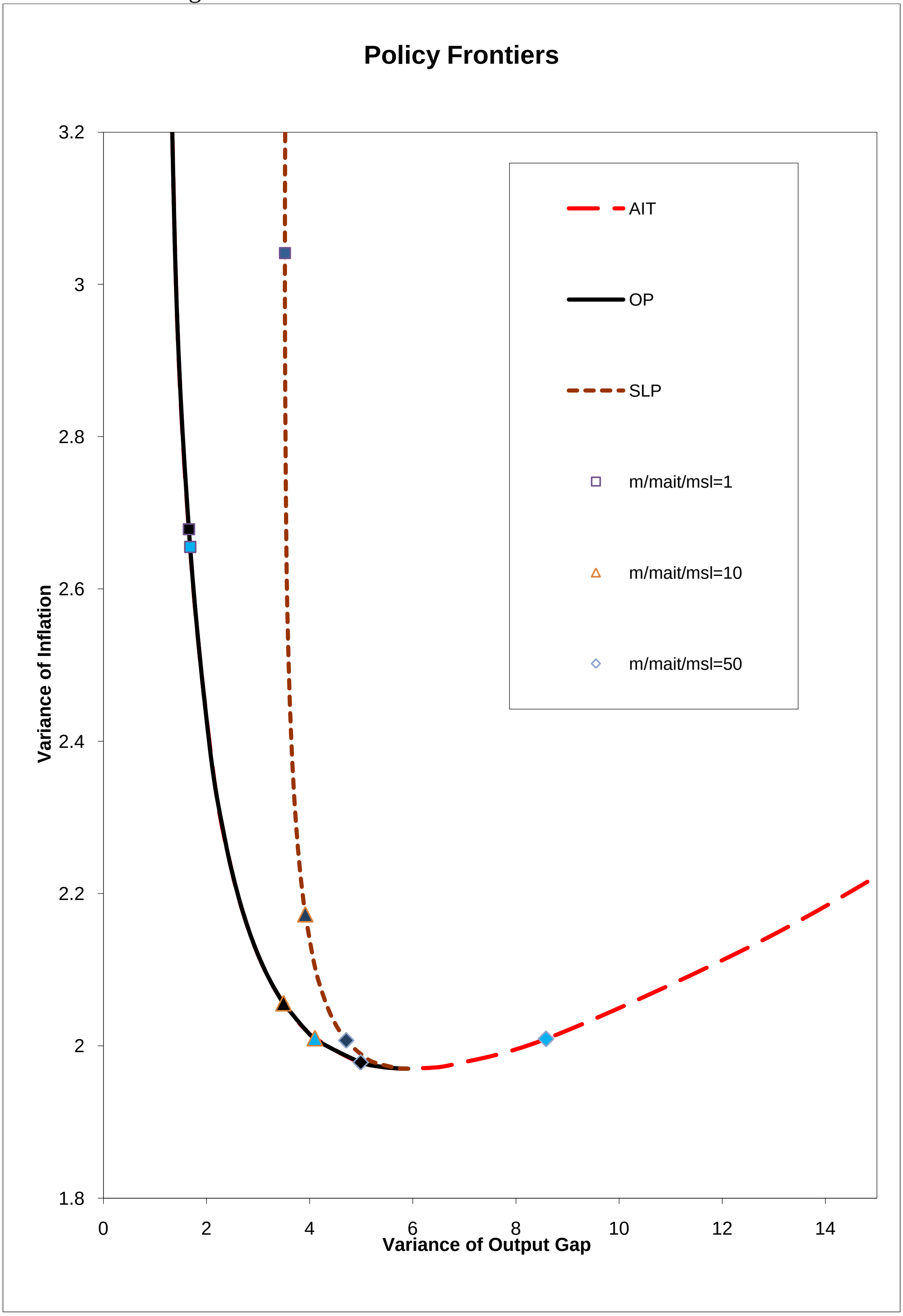


Figure 2: A Comparison of the Output-Inflation Variability Trade-Off in the Backward-Looking Model



Note: m= μ ; mait= μ^{AIT} ; msl= μ^{SL}

Figure 4: A Portfolio of Policy Frontiers Based on the Optimized Instrument Rule

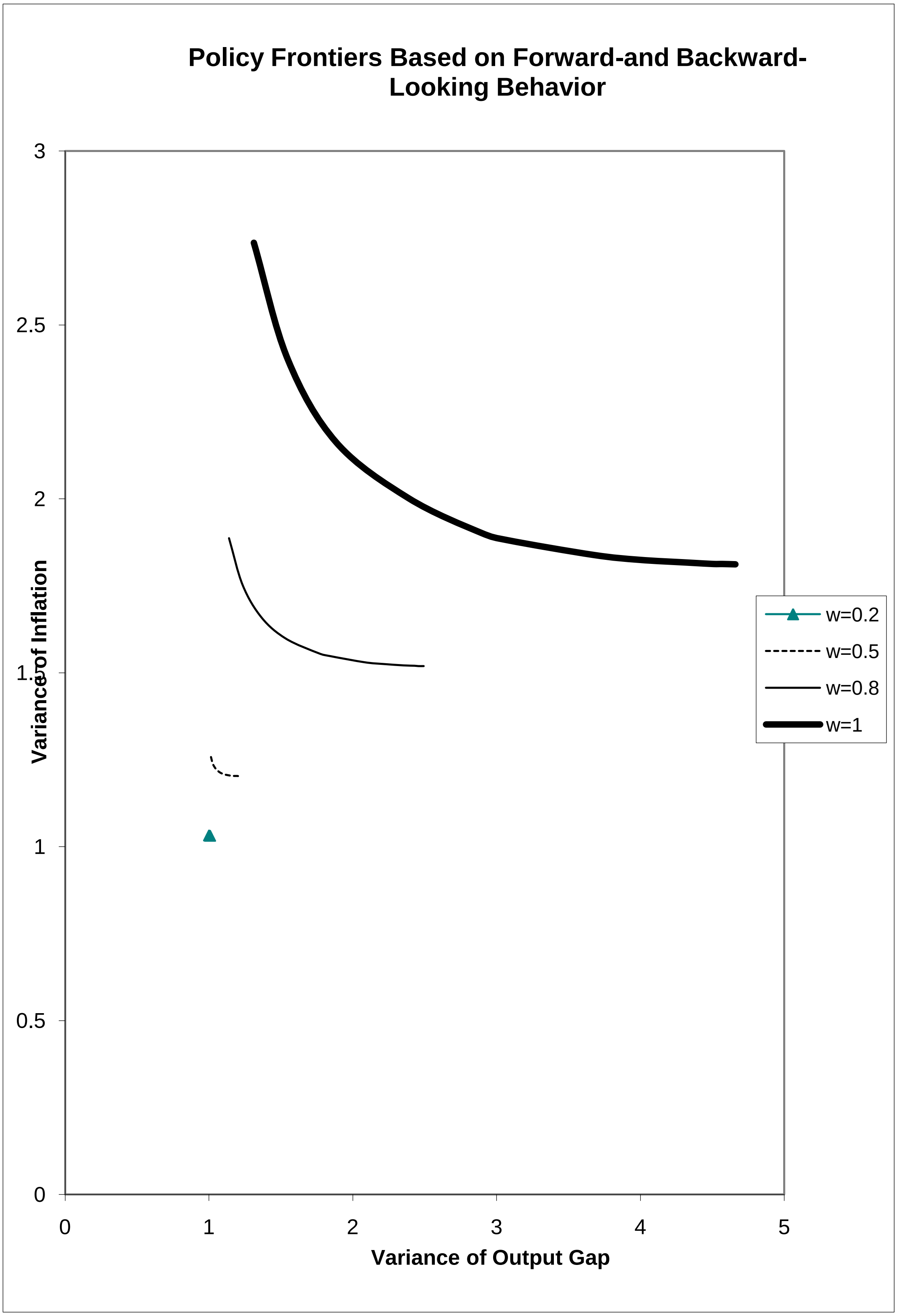
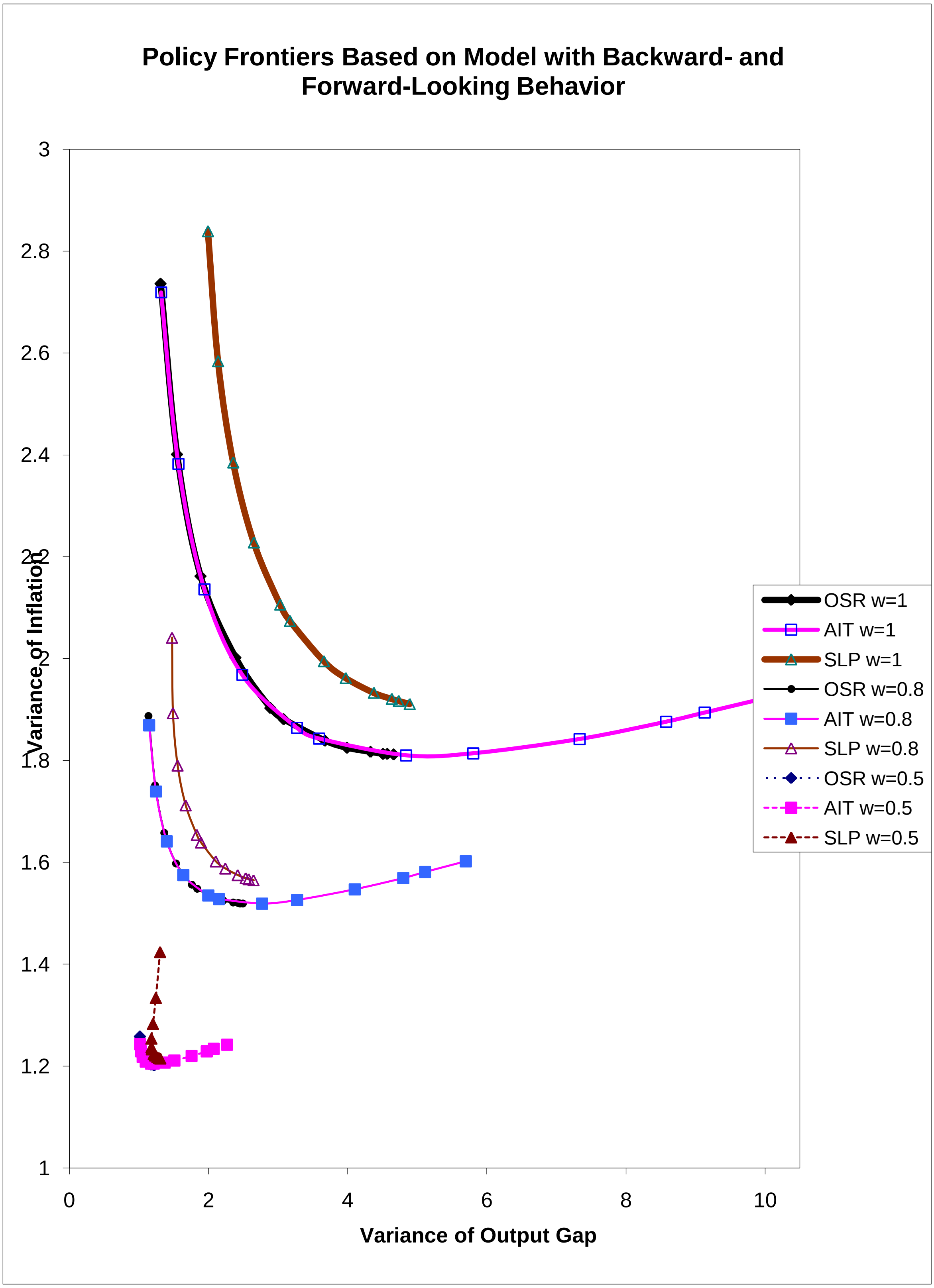
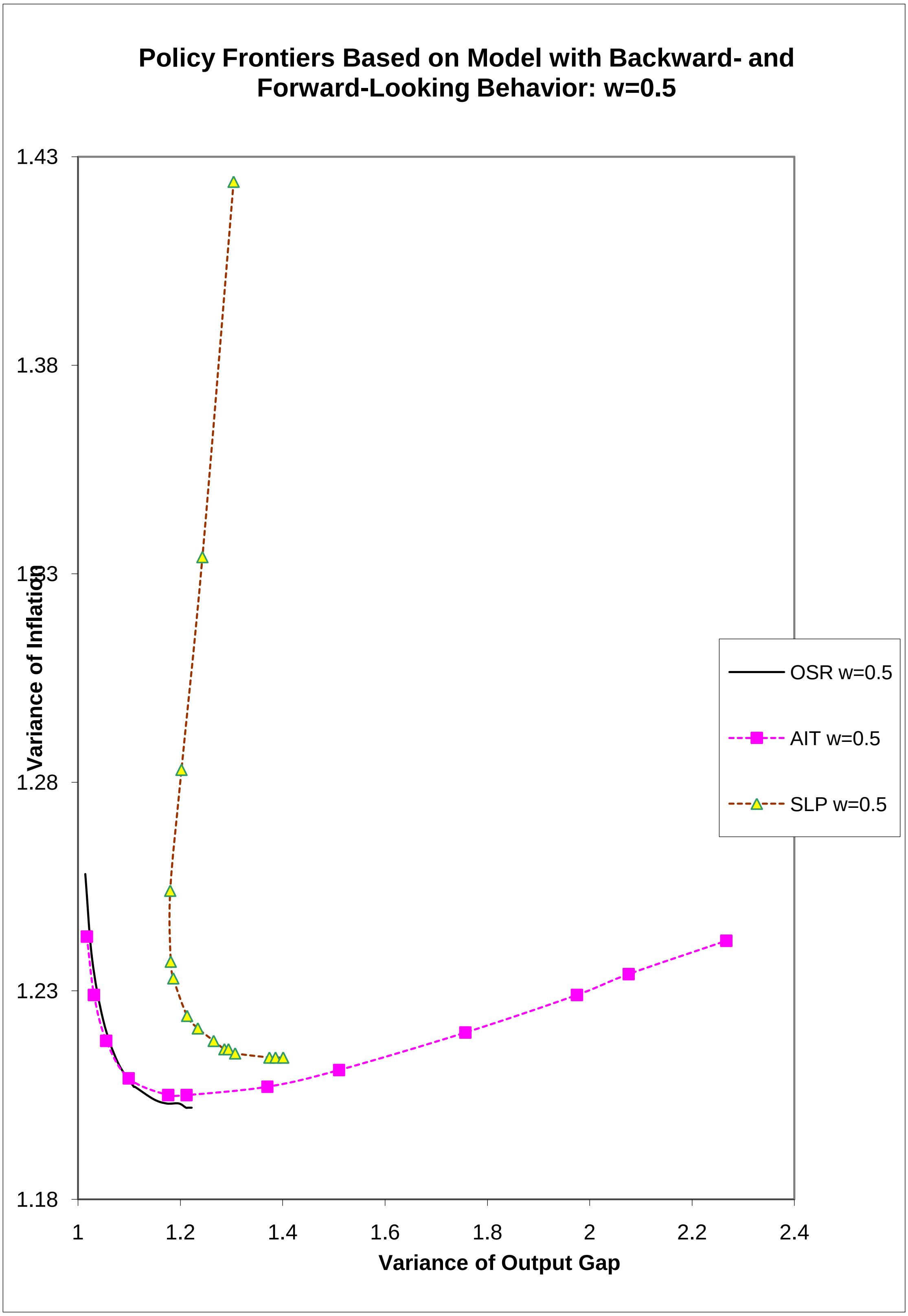


Figure 5: A Comparison of the Output-Inflation Variability Trade-Off in the Amended Model



Note: The markers represent values of the preference parameter that appears in the respective loss function. These values range from *0.5* to *100*. The individual values are: *0.5, 1, 2, 4, 8, 10, 20, 30, 50, 70, 80, 100.*

Figure 6: Equal Balance of Backward-and Forward-Looking Behavior in the Amended Model



Note: The policy frontier under a speed limit policy never reaches the AIT policy frontier, not even for extremely large values of μ^{SLP}. See note attached to Figure 5 for interpretation of markers.

Table 1: Society's Welfare under Average Inflation Targeting Relative to Optimal Policy

Policymaker's aversion to inflation variability	Society's aversion to inflation variability	Society's welfare loss under AIT	Society's welfare loss under optimal policy	Relative Loss: loss under AIT relative to loss under optimal policy
μ^{AIT}	μ	$E\left[L_t\right]_{AIT} = V(y_t^{AIT}) + \mu V(\pi_t^{AIT})$	$E\left[L_t\right] = V(y_t) + \mu V(\pi_t)$	$\frac{E[L_t]_{AIT} - E[L_t]}{E[L_t]}$
1	0.870	3.991	3.983	0.002
2	1.690	6.117	6.095	0.003
4	3.267	9.837	9.774	0.006
8	6.280	16.505	16.311	0.011
10	7.739	19.656	19.373	0.014
50	33.730	76.349	71.667	0.065
100	61.949	142.718	127.529	0.119

Note: μ^{AIT} is the weight the policymaker places on average inflation in his objective function. The first column lists plausible values for μ^{AIT}. The second column reports the values for μ that society would have to have for average inflation targeting to generate the same welfare losses for both the policymaker and society: $E\left[L_t\right]^{AIT} = E\left[L_t\right]_{AIT}$ where $E\left[L_t\right]^{AIT} = V(y_t^{AIT}) + \mu^{AIT} V(\bar{\pi}_t^{AIT})$ and $E\left[L_t\right]_{AIT} = V(y_t^{AIT}) + \mu V(\pi_t^{AIT})$.

Table 2: Society's Welfare under a Speed Limit Policy Relative to Optimal Policy

Policymaker's aversion to inflation variability	Society's aversion to inflation variability	Society's welfare loss under speed limit	Society's welfare loss under optimal policy	Relative Loss: loss under SL relative to loss under optimal policy
μ^{SL}	μ	$E\left[L_t\right]_{SL} = V(y_t^{SL}) + \mu V(\pi_t^{SL})$	$E\left[L_t\right] = V(y_t) + \mu V(\pi_t)$	$Rel.\ Loss = \frac{E[L_t]_{SL} - E[L_t]}{E[L_t]}$
1	0.676	5.577	3.439	0.621
2	1.824	8.390	6.424	0.305
4	4.072	13.388	11.562	0.157
8	8.441	22.535	20.835	0.081
10	10.588	26.921	25.261	0.065
50	51.972	109.019	107.806	0.011
100	102.597	208.661	207.758	0.004

Note: μ^{SL} is the weight the policymaker places on the rate of inflation in his objective function. The first column lists plausible values for μ^{SL}. The second column reports the values for μ that society would have to have for a speed limit policy to generate the same welfare losses for both the policymaker and society: $E\left[L_t\right]^{SL} = E\left[L_t\right]_{SL}$ where $E\left[L_t\right]^{SL} = V((y_t - y_{t-1})^{SL}) + \mu^{SL} V(\pi_t^{SL})$ and $E\left[L_t\right]_{SL} = V(y_t^{SL}) + \mu V(\pi_t^{SL})$.

Appendix:

The alternative specification of the target rule for a speed limit policy in the purely backward-looking model takes the following form:

$$\theta^{SL}\left[E_t y_{t+1} - y_t\right] + E_t \pi_{t+1} = 0 \tag{A1}$$

Because of the two-period transmission lag expected inflation next period is predetermined. This target rule is equivalent to nominal income growth targeting (where the target growth rate has been normalized to zero) if $\theta^{SL} = 1$.

Combining the above target rule with the model equations determines the output gap under a speed limit policy:

$$y_t = -\frac{\gamma}{\theta^{SL}}\pi_{t-1} + \frac{\theta^{SL} - \alpha}{\theta^{SL}} y_{t-1} + \varepsilon_t \tag{A2}$$

Along with the Phillips curve, equation (A2) gives rise to the following variances of the output gap (y_t), inflation (π_t), and the change in the output gap $(y_t - y_{t-1})$:

$$V\left(y_t^{SL}\right) = \frac{\theta^{SL}[2\alpha\gamma + \theta^{SL}(1-\gamma(1+\gamma-\gamma^2))]\sigma_\varepsilon^2 + \gamma^2(1+\gamma)\sigma_\eta^2}{(1-\gamma)\alpha(2\theta^{SL}(1+\gamma)-\alpha)} \tag{A3}$$

$$V\left(\pi_t^{SL}\right) = \frac{\alpha(\theta^{SL})^2(1+\gamma)\sigma_\varepsilon^2 + \left(2\theta^{SL} - \alpha(1-\gamma)\right)\sigma_\eta^2}{(1-\gamma)(2\theta^{SL}(1+\gamma)-\alpha)} \tag{A4}$$

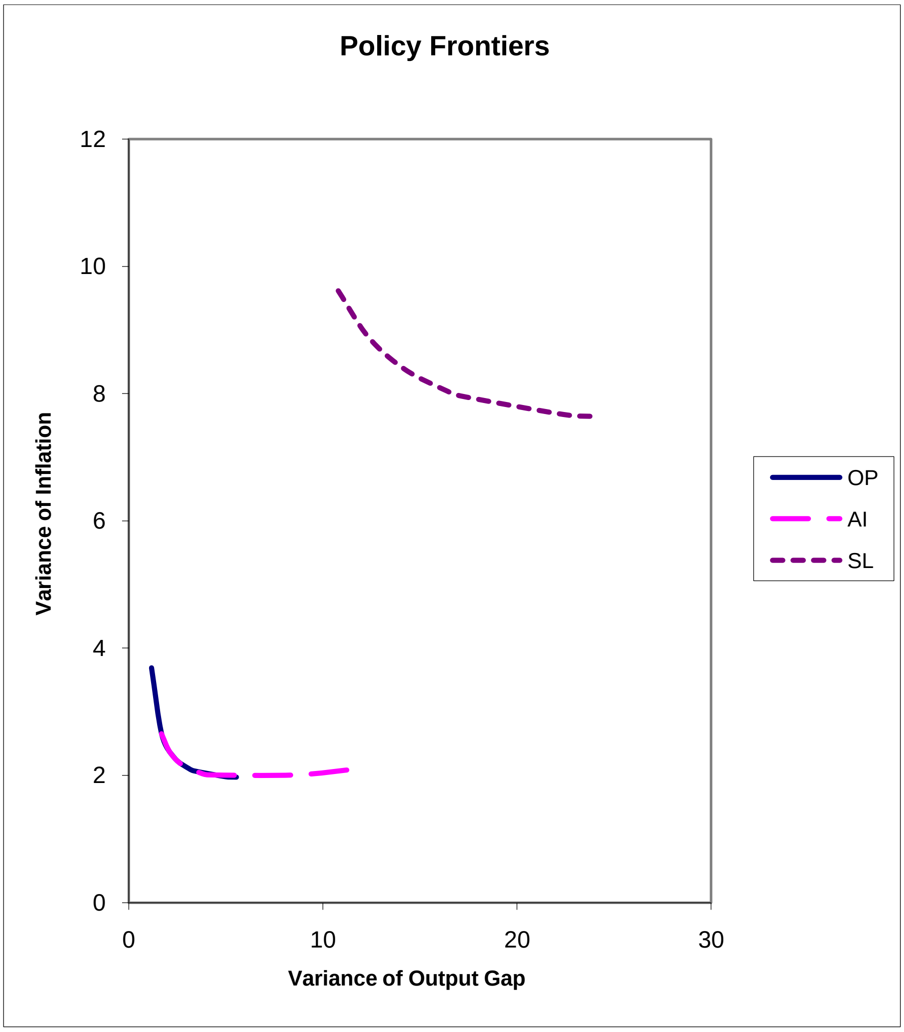
$$V\left((y_t - y_{t-1})^{SL}\right) = \frac{2[\theta^{SL}(\alpha\gamma + \theta^{SL}(1-\gamma^2))\sigma_\varepsilon^2 + \gamma^2\sigma_\eta^2]}{(1-\gamma)\theta^{SL}(2\theta^{SL}(1+\gamma)-\alpha)} \tag{A5}$$

Here we see that all three variances are well defined provided that $\gamma \neq 1$, i.e. that the Phillips Curve is not of the accelerating type.

Inserting (A4) and (A5) into (18) and minimizing the expected loss function with respect to θ^{SL} yields the optimal value of the policy parameter under a speed limit policy. The optimal policy parameter is again a function of the parameters of the Phillips Curve, the preference parameter of the policymaker, and the variances of the shocks of the model.

Figure A1 shows that the alternative specification of the target rule under a speed limit policy leads to average inflation targeting strictly dominating a speed limit policy. The policy frontier traced out by average inflation targeting lies below the policy frontier ground out by a speed limit policy. Average inflation targeting generates a more favorable output-inflation variability trade-off than a speed limit policy.

Figure A1: The Output-Inflation Variability Trade-Off under the Alternative Specification of a Speed Limit Policy



The target rule underlying average inflation targeting could also be specified as:

$$\theta^{AIT} E_t y_{t+1} + E_t \overline{\pi_{t+1}} = 0 \qquad \text{(A6)}$$

Basing policy on this target rule causes the variability of the output gap to be lower but the variability of average inflation and single-period inflation to increase compared to the target rule of Section 4. Expected losses under the above target rule exceed those under the target rule discussed in the paper. In addition, there is no simple closed-form solution for the optimal policy parameter θ^{AIT} .

Evolution of Expected Inflation:

This part of the appendix discusses the factors that drive expected inflation in period *t+2*. In each of the four cases considered, the target rule is combined with the Phillips Curve to determine the inflation forecast.

1. Target Rule under Average Inflation: $\theta^{AIT} E_t y_{t+1} + E_t \overline{\pi_{t+2}} = 0$

   The resulting rate of inflation expected in period *t+2* evolves gradually and is tied only to the expected rate of inflation in period *t+1:*

$$E_t \pi_{t+2} = \frac{2\theta^{AIT}\gamma - \alpha}{2\theta^{AIT} + \alpha} E_t \pi_{t+1} \qquad \text{(A7)}$$

2. Target Rule under a Speed Limit Policy: $\theta^{SL}\left[E_t y_{t+1} - y_t\right] + E_t \pi_{t+2} = 0$

   Under a speed limit policy, contemporaneous information enters the target rule. This complicates the policy-setting process. The evolution of expected inflation depends not only on expected inflation in period *t+1* but also on the current output gap:

$$E_t \pi_{t+2} = \frac{\theta^{SL}}{\theta^{SL} + \alpha}(\gamma E_t \pi_{t+1} + \alpha y_t) \qquad \text{(A8)}$$

The two remaining target rules describe less efficient policy outcomes under average inflation targeting and a speed limit policy.

3. Target Rule under Average Inflation: $\theta^{AIT} E_t y_{t+1} + E_t \bar{\pi}_{t+1} = 0$

   Notice that the current rate of inflation appears in the definition of $E_t \bar{\pi}_{t+1}$. As a result, expected inflation in period *t+2* depends on expected inflation in period *t+1* and the current rate of inflation:

$$E_t \pi_{t+2} = \frac{1}{2\theta^{AIT}}((2\theta^{AIT}\gamma - \alpha)E_t \pi_{t+1} - \alpha \pi_t) \qquad \text{(A8)}$$

   Compared to the first case, the policy-setting process becomes more complicated.[30]

4. Target Rule under a Speed Limit Policy: $\theta^{SL}\left[E_t y_{t+1} - y_t\right] + E_t \pi_{t+1} = 0$

   Expected inflation two periods into the future depends on both expected inflation in period *t+1* and the current output gap:

$$E_t \pi_{t+2} = \frac{\gamma\theta^{SL} - \alpha}{\theta^{SL}} E_t \pi_{t+1} + \alpha y_t \qquad \text{(A9)}$$

[30] Equation (A8) is akin to a second-order difference equation.

The current output gap carries a greater weight in determining expected inflation in period *t+2* compared to the other specification of a speed limit policy.