6. (Introspection and Cognitive Hierarchy Models) Insert initial beliefs of 1/13 in column D of the spreadsheet, using $R = 10$ and $R = 50$ and an error rate of 10. Think of these initial beliefs as level 0 beliefs that give equal probability to each decision. The noisy level 1 beliefs are then those in column K, and the expected claim for these beliefs is found in cell L21. In this manner, find the level 2 beliefs and the average claim that results, and similarly for several higher levels. At what level does the expected claim stop changing by more than 10 percent?

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chapter 26

Coordination Games

A major role of management is to coordinate decisions so that better outcomes can be achieved. The game considered in this chapter highlights the need for such external management, since otherwise players may get stuck in a situation where nobody exerts much effort because others are not expected to work hard either. In these games, the productivity of each person's effort depends on that of others, and there can be multiple Nash equilibria, each at a different common effort level. Behavior is sensitive to factors like changes in the effort cost or the size of the group, even though these have no effect on the set of Nash equilibria. The experiment can be run with the Veconlab Coordination Game. Alternatively, the matrix game with seven possible effort decisions discussed below is implemented by the default parameters for the large matrix game program, the $N \times N$ Matrix Game program. The instructions in this chapter's Effort Game in the Appendix are for the matrix-game version with seven decisions.

26.1 "The Minimum Effort Game? That's One I Can Play!"

Most productive processes involve specialized activities, where distinct individuals or teams assemble separate components that are later combined into a final product. If this product requires one of each of the components, then the number of units finally sold is sensitive to bottlenecks in production. For example, a marine products company that produces 100 hulls and 80 engines will only be able to market 80 boats. Thus, there is a bottleneck caused by the division with the lowest output. Students (and professors too) have little trouble understanding the incentives of this type of minimum-effort game, as one student's comment in the section title indicates.

The minimum-effort game was originally discussed by Rousseau in the context of a stag hunt, where a group of hunters form a large circle and wait for the stag to try to escape. The chances of killing the stag depend on the watchfulness and effort exerted by the encircling hunters. If the stag observes a hunter to be napping or hunting for smaller game instead, then the stag will attempt an escape through that sector. If the stag is able to judge the weakest link in the circle, then the chances of escape depend on the minimum of the hunters' efforts. The other aspect of the payoff is that effort is individually costly, e.g., in terms of giving up the chance of bagging a hare or taking a rest.

The game in Table 26.1 is a 2×2 version of a minimum effort game. First, consider the lower-left corner where each person has a low effort, and payoffs are 70 each. If the Row player increases effort, while the Column player maintains low efforts, then the relevant Row payoff is reduced to 60, as can be seen from the top-left box. Think of it this way: the unilateral increase in effort will not increase the minimum, but the extra cost reduces Row's payoff from 70 to 60, so the cost of this extra unit of effort is 10.

Now suppose that the initial situation is a high effort for Row and a low effort for Column, as in the upper-left box. An increase in Column's effort will raise the minimum, which raises Row's payoff from 60 to 80. Thus, a unit increase in the minimum effort will raise payoffs by 20, holding one's own effort constant. This one-unit increase in effort did not raise the Column player's payoff by 20 because effort is costly, and the fact that the Column player's payoff only went up by 10 suggests that the cost of the extra unit of effort is 10. These observations can be used to devise a mathematical formula for payoffs that will be useful in devising more complex games. Let the low effort be 1 and the high effort be 2, as indicated by the numbers in parentheses next to the Row and Column labels in the payoff table. Then the payoffs in this table are determined by the formula: 60 + (20 times the minimum effort) – (10 times one's own effort). Let M denote the minimum effort and E denote one's own effort, so that this formula, shown in Equation (26.1) is

$$\text{Own payoff} = 60 + 20\overset{\text{min effort}}{M} - 10E \tag{26.1}$$

For example, the upper-right payoffs are determined by noting that both efforts are 2, so the minimum (M) is 2, and the formula yields: 60 + (20)*2 – (10)*2 = 80.

The formula in Equation (26.1) was used to construct the payoffs in Table 26.2, for the case of efforts that can range from 1 to 7. Notice that the four numbers in the bottom-left corner of the table correspond to the Row payoffs in Table 26.1. With a larger number of possible effort levels, we see the dramatic nature of the potential gains from coordination on high-effort outcomes. At a common effort of 7, each person earns 130, which is almost twice the amount earned at the lowest effort level. Movements along the diagonal from the lower-left to the upper-right corner show the benefits from coordinated increases in effort; a one-unit increase in the minimum effort raises payoff by 20, minus the cost of 10 for the increased effort, so each diagonal payoff is 10 larger than the one lower on the diagonal.

Table 26.1 A Minimum Effort Game (Row's Payoff, Column's Payoff)

	Column Player	
Row Player	Low Effort (1)	High Effort (2)
High Effort (2)	60, 70 ⇓	⇒ 80, 80
Low Effort (1)	70, 70 ⇐	⇑ 70, 60

Table 26.2 The Van Huyck et al. (1990) Minimum Effort Game with Row Player's Payoffs Determined by the Minimum of Others' Efforts

	Column's Effort (or Minimum of Other Efforts)						
Row's Effort	1	2	3	4	5	6	7
7	10	30	50	70	90	110	**130**
6	20	40	60	80	100	**120**	120
5	30	50	70	90	**110**	110	110
4	40	60	80	**100**	100	100	100
3	50	70	**90**	90	90	90	90
2	60	**80**	80	80	80	80	80
1	**70**	70	70	70	70	70	70

Besides the gains from coordination, there is an additional feature of Table 26.2 that is related to risk. When the Row player chooses the lowest effort (in the bottom row), the payoff is 70 for sure, but the highest effort may yield payoffs that range from 10 to 130, depending on the Column player's choice. This is the strategic dilemma in this coordination game: there is a large incentive to coordinate on high efforts, but the higher effort decisions are risky.

An additional element of risk is introduced when more than two people are involved. Suppose that payoffs are still determined by the formula in Equation (26.1), where M is the minimum of all players' efforts. The result is still the payoff shown in Table 26.2, where the payoff numbers pertain to the Row player as before, but where the columns correspond to the minimum of the other players' efforts. For example, the payoffs are all 70 in the bottom row because Row's effort of 1 is the minimum regardless of which column is determined by the minimum of the others' efforts. Notice that the incorporation of larger numbers of players into this minimum-effort game does not alter the essential strategic dilemma, i.e., that high efforts involve high potential gains but more risk. At a deeper level, however, there is more risk with more players, since the minimum of a large number of independently selected efforts is likely to be small when there is some variation from one person to another. This is analogous to having a large number of hunters spread out in a large circle, which gives the stag more of a chance to find a sector where one of the hunters is absent or napping.

26.2 Nash Equilibria, Numbers Effects, and Experimental Evidence

These intuitive considerations (the gains from coordination and the risks of unmatched high efforts) are not factors in the structure of the Nash equilibria in this game. Consider Table 26.1, for example. If the other person is going to exert a low effort, the best response is a low effort that saves on effort cost. Thus, the lower-left outcome, low efforts for each, is a Nash equilibrium. But if the other player is expected to choose a high effort, then the best response is a high effort, since the gain of 20 for the increased minimum exceeds the cost of 10 for the additional unit of effort. Thus, the high-effort outcome in the upper-right corner of Table 26.1 is also a Nash equilibrium. This is an important feature of the coordination game: there are multiple equilibria, with one that is preferred to the other(s).

The presence of multiple equilibria is a feature that differentiates this game from a prisoner's dilemma, where all players may prefer the high-payoff outcome that results from cooperative behavior. This high-payoff outcome is not a Nash equilbrium in a prisoner's dilemma since each person has a unilateral incentive to defect. The equilibrium structure for the coordination game in Table 26.1 is not affected by adding additional players, each choosing between efforts of 1 and 2, and with the Row player's payoffs determined by the column that corresponds to the minimum of the others' efforts. In this case, adding more players does not alter the fact that there are two equilibria (in non-random strategies): all choose low efforts or all choose high efforts. Just restricting attention to the Nash equilibria would mean ignoring the intuition that adding more players would seem to make the choice of a high effort riskier, since it is more likely that one of the others will choose low effort and pull the minimum down.

The problem of multiple equilibria is more dramatic with more possible efforts, since any common effort is a Nash equilibrium, as shown in Table 26.2. To see this, pick any column and notice that the Row player's payoffs are highest on the diagonal of payoffs in bold. As before, this structure is independent of changes in the number of players, since such changes do not alter the payoff table. These considerations were the basis of an experiment conducted by Van Huyck et al. (1990), who used the payoffs from Table 26.2 for small groups (size 2) and large groups (sizes 14–16). The large groups played the same game 10 consecutive times with the same group, and the lowest effort was announced after each round to enable all to calculate their payoffs (in pennies). Even though a majority of individuals selected high efforts of 6 or 7 in the first round, the minimum effort was no higher than 4 in the first round for any large group. With a minimum of 4, higher efforts were wasted, and effort reductions followed in subsequent rounds. The minimum fell to the lowest level of 1 in all of the large groups, and almost all decisions in the final round were at the lowest level.

This experiment is important since previously it had been a common practice in theoretical analysis to assume that individuals could coordinate on the best Nash equilibrium when there was general agreement about which one is best, as is the case for Table 26.2. In contrast, the subjects in the experiment managed to end up in the equilibrium that is worst for all concerned. With groups of size 2, individuals were able to coordinate on the highest effort, except when pairings were randomly reconfigured in each round. With random matching, the outcomes were variable, with average efforts in the middle range. In any case, it is clear that group size had a large impact on the outcomes, even though changes in the numbers of players had no effect on the set of equilibria.

The coordination failures for large groups captured the attention of macroeconomists, who had long speculated about the possibility that whole economies could become mired in low-productivity states, where people do not engage in high levels of market activity because no one else does. The macroeconomic implications of coordination games are discussed in Bryant (1983), Cooper and John (1998), and Romer (1996), for example.

26.3 Effort-Cost Effects

Next, consider what happens when the cost of effort is altered. For example, suppose that the effort cost of 10 used to construct Table 26.1 is raised to 19, so that the payoffs (for an effort of E and a minimum effort of M) will be determined by: $60 + 20M - 19E$. In this case, a one-unit increase in each person's effort raises payoffs by 20 minus the cost of 19, so the payoffs in the upper-right box of Table 26.3 are only 1 unit higher than the payoffs in the lower-left box. Notice that this increase in effort cost did not change the fact that there are two Nash equilibria, and that both players prefer the high-effort equilibrium. However, simple intuition suggests that effort levels in this game may be affected by effort costs. From the Row player's perspective, the top row offers a possible gain of only 1 and a possible loss of 19, as compared with the bottom row.

Goeree and Holt (2005a) report experiments in which the cost of effort is varied between treatments, using the payoff formula shown in Equation (26.2):

$$\text{Own payoff} = M - cE \tag{26.2}$$

where M is the minimum of the efforts, E is one's own effort, and c is a cost parameter that is varied between treatments. Effort decisions were restricted to be any amount between

Table 26.3 A Minimum-Effort Game with High Effort Cost (Row's Payoff, Column's Payoff)

	Column Player	
Row Player	Low Effort (1)	High Effort (2)
High Effort (2)	42, 61	62, 62
Low Effort (1)	61, 61	61, 42

(and including) $1.10 and $1.70. As before, any common effort is a Nash equilibrium in this game, as long as the cost parameter, *c*, is between 0 and 1, because a unilateral decrease in effort by one unit will reduce the minimum by 1, but it will only reduce the cost by an amount that is less than 1. Therefore, a unit decrease in effort will reduce one's payoff by $1 - c$. Conversely, a unilateral increase in effort by one unit above some common level will not raise the minimum, but the payoff will fall by *c*. Even though deviations from a common effort are unprofitable when *c* is greater than 0 and less than 1, the magnitude of *c* determines the relative cost of "errors" in either direction. A large value of *c*, say 0.9, makes increases in effort more costly, and a small value of *c* makes decreases more costly.

Figure 26.1 shows the results for sessions that consisted of 10–12 subjects who were randomly paired for a series of 10 rounds. There were three sessions with a low effort cost parameter of 0.25; the averages by round for these sessions are plotted as thin dashed lines. The thick dashed line is the average over all three sessions of this treatment. Similarly, the thin solid lines plot round-by-round averages for the three sessions with a high effort cost parameter of 0.75; the thick solid line shows the average for this treatment.

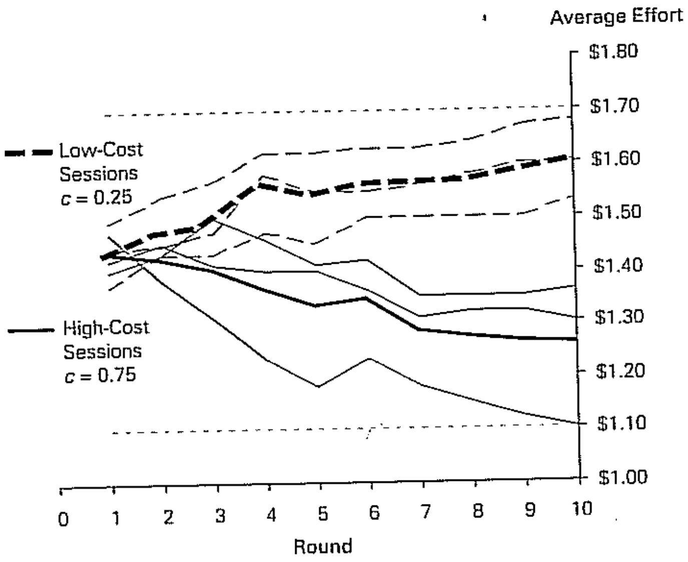

Figure 26.1 Average Efforts for the Goeree and Holt (2005a) Coordination Experiment: Thick Lines Are Averages over Three Sessions for Each Cost Treatment

Efforts in the first round are in the range from $1.35 to $1.50, with no separation between treatments. Such separation arises after several rounds, and average efforts in the final round are $1.60 for the low-cost treatment, versus $1.25 for the high-cost treatment. Thus, we see a strong cost effect, even though any common effort is a Nash equilibrium.

One session in each treatment seemed to approach the boundary, which raises the issue of whether behavior will lock on one of the extremes. This pattern was observed in a Veconlab classroom experiment in which efforts went to $1.70 by the 10th period. This kind of extreme behavior is not universal, however. Goeree and Holt (2005a) report a pair of sessions that were run for 20 rounds. With an effort cost of $.25, the decisions converged to about $1.55, and with an effort cost of $.75 the decisions leveled off at about $1.38. Both of these outcomes seem to fit the pattern seen in Figure 26.1.

26.4 Equilibrium with Noisy Behavior

The spreadsheet-based analysis of noisy behavior for the travelers' dilemma, presented in the Class Experiments section in the Appendix to Chapter 25, can be adapted for the coordination game. The steps for constructing this new spreadsheet are provided later in this chapter (An Analysis of Noisy Behavior in the Coordination Game). The purpose of this analysis is to show how some randomness in individual decisions can result in data patterns (for numbers and effort-cost effects) that are roughly consistent with those observed in the experiments, even though these data patterns are not predicted on the basis of an analysis of the Nash equilibria for the game.

As before, we begin by considering successive levels of iterated strategic thinking. A level 0 person chooses each decision with equal probability, so the average decision is in the middle of the range, at 140, as shown in the Level 0 column on the left side of Table 26.4. A level 1 person believes that each decision is equally likely, and that person makes a noisy best response calculated with the logit probabilistic choice function (ratio of exponential functions). The resulting average effort increases to 147 for the low-cost treatment and decreases to 132 for the high-cost treatment, as can be seen from the Level 1 column. It is apparent from Table 26.4 that the average claims converge by about the fifth round, to levels that are reasonably close to the data averages shown in the far-right column of the table.

As explained in Chapter 25, the process of copying blocks of the spreadsheet cells to adjacent locations causes the choice probabilities for one level to become the initial beliefs of a person at the next highest level of iterated rationality. At successively higher levels of iterated rationality, the belief and choice distributions get closer and closer. When the belief and choice distributions converge, the result is a quantal response equilibrium for the particular error parameter used ($\mu = 10$). The series of choice distributions for the first five levels of iterated rationality are graphed in Figure 26.2, beginning with a flat dashed line (level 0). For each treatment, the equilibrium is reached by about level 5 (the dark line with the QRE label).