PROB17.7

	B	C	D	E	F
3		state 1	state 2	state 3	state 4
4	Total	$100,000.00	$200,000.00	$300,000.00	$400,000.00
6	John	$25,000.00	$50,000.00	$75,000.00	$100,000.00
7	Paul	$25,000.00	$50,000.00	$75,000.00	$100,000.00
8	George	$25,000.00	$50,000.00	$75,000.00	$100,000.00
9	Ringo	$25,000.00	$50,000.00	$75,000.00	$100,000.00
11	probs	0.4	0.3	0.2	0.1
13	John's utility	158.113883	223.6067977	273.8612788	316.227766
14	Paul's utility	353.5533906	387.2983346	418.3300133	447.2135955
15	George's utility	29.2303089	36.82703058	42.15585666	46.398079
16	Ringo's utility	-0.778800783	-0.60653066	-0.472366553	-0.367879441
18		EU	CE		
19	John	216.7226249	$46,968.70		
20	Paul	385.9982188	$48,994.62		
21	George	35.81121196	$45,925.83		
22	Ringo	-0.624740766	$47,041.85		

Sheet1 / Sheet2 / Sheet3

Figure 17.1. Problem 17.7. Calculating the CE's of the Fab Four, for the equal-division sharing rule. This spreadsheet calculates expected utilities and certainty equivalents for John, Paul, George, and Ringo, assuming they split the proceeds from their venture in equal shares.

This does not seem efficient to Ringo, who wonders if there is not some better way to split the proceeds from the venture. Ringo wishes to know, assuming we give John, Paul, and George payoff shares that leave them, respectively, at the certainty equivalent levels $46,968.70, $48,994.62, and $45,925.83, how high can we push Ringo's certainty equivalent? Answer this question for Ringo. (Warning: I had some problems getting Solver to work on this problem. Everything worked once I asked for *Automatic Scaling* under the *Options* menu.)

(b) Suppose Ringo is risk neutral and John, Paul, and George are all as in part a of the problem. This would mean that Ringo's CE from the equal division shares would be not $47,041.85, but instead (0.4)(25,000) + (0.3)(50,000) + (0.2)(75,000) + (0.1)(100,000) = $50,000. It seems unfair that Ringo has such a large CE while John, Paul, and George all have CEs under $49,000, so the Fab Four are looking for a scheme that gives John, Paul, and George CE's of at least $50,000, while making Ringo as well off as possible. What sharing rule scheme accomplishes this (and how well off can we make Ringo)?

18. Hidden Information, Signaling, and Screening

This chapter concerns economic transactions in which some parties have access to important information that other parties lack. This sort of *hidden information* can lead to substantial problems, including a breakdown of markets. The antidote to such difficulties is for the hidden information to be revealed, but when the information is revealed strategically, its content must be carefully considered. Among the applications discussed is *the winner's curse*, a phenomenon in competitive auctions in which winning bidders systematically find that they have bid more than the object at auction is worth to them.

Imagine an auction for the rights to extract minerals such as fossil fuels from a tract of land. Suppose the auction is conducted as a sealed tender auction: Bidders submit sealed bids, the bids are opened simultaneously, and whoever bids the highest amount wins the extraction rights, paying this bid. The winning bidder (he) can regret his bid on two different grounds.

1. Suppose that the winning bidder wins with a bid of $100 million, the tract turns out to be worth $120 million, and the second highest bidder bid $60 million. Even though the winning bidder winds up ahead $20 million, he suffers *winner's remorse* or leaving money on the table, because he bid more for the rights than was necessary to win the auction.

2. Suppose instead that the winning bid is $100 million, the next highest bid is $95 million, and the rights turn out to be worth only $60 million. Not much money was left on the table, but the winner is out $40 million. This is not necessarily the result of a bad decision. The amount of fossil fuel under the ground at a particular site is usually unknown at the time of the auction, and while the rights turn out to be worth only $60 million, at the time of the auction it might have been reasonable to assess that the rights would be worth $150 million on average. But suppose someone who bids repeatedly in such auctions finds the following "when winning" phenomenon: Averaging across *all* the auctions in which he participates, his bids on average are less than the objects being auctioned eventually prove to be worth. But averaging across those auctions *that he wins*, his bids exceed on average what the items turn out to be worth. This bidder is suffering from the *winner's curse*.

The winner's curse is much more than a hypothetical possibility. It is a common affliction among winners of auctions in which the bidders have access to different information about the value of the object being auctioned. This chapter is in part about the winner's curse, why it happens (and when it does), and what bidders must do to avoid it. More broadly, this chapter concerns economic transactions in which some parties to the transaction have information that other parties lack. Since the winner's curse is a relatively complex phenomenon, we begin with simpler situations with a single buyer who lacks information possessed by the seller of some object.

18.1. Hidden Information and Adverse Selection

Suppose you are a venture capitalist (VC). An entrepreneur (she) approaches you with a business plan and asks you to take an equity stake in the venture. In most cases of venture capital and entrepreneurs, the entrepreneur needs cash to fund her initial operations. Suppose, however, that this entrepreneur wants your participation only for risk sharing, as in the last chapter.

Could it ever make sense for you, as the VC, to be willing to pay $10 million for a 50% stake of the venture but be unwilling to pay even $5 million for a 90% stake? Assume that the entrepreneur retains whatever you do not buy.

The answer to this question is Yes; in fact, there are several reasons that the answer might be Yes. The VC (you) might be risk averse: We saw last chapter how a risk-averse person might have a negative certainty equivalent for a whole gamble but a positive CE for half of it. Something like that might be at work here, but to avoid this possibility, assume that you are so wealthy you are risk neutral.

The answer might also be Yes because the entrepreneur's efforts are crucial to the success of this venture, and you worry that an entrepreneur who retains only a 10% share has a greatly reduced incentive to put in the effort necessary. This is an issue of incentives and moral hazard, the topic of the next chapter.

A third reason the answer might be Yes, the reason that fits in this chapter, concerns things that the entrepreneur might know and you do not. To be very specific, suppose the venture's eventual economic success depends on whether a particular unproven technology can be made to work. If this technology works, the venture will be worth a lot of money, say, $100 million. But if it cannot be made to work, the venture has a net value of $0, counting in expenses.

Whether you would be willing to pay $10 million or even $5 million for an appreciable share of the venture depends on how likely it is that this

technology can be made to work. Suppose you assess probability $\frac{1}{4}$ for this. As a VC, you are good at these sorts of assessments. Since we assume you are risk neutral, this makes the expected monetary value of the venture $25 million, and half is certainly worth $10 million.

Then why is 90% not worth $5 million? Imagine that the entrepreneur, who after all has been thinking about this project for a lot longer than you, had the opportunity to run some tests that would substantially resolve whether the new technology works. Specifically, if the entrepreneur had positive test results, then the odds that the technology works are 0.5, while if the test results were negative, the technology will not work. You know that the entrepreneur ran the test; based on the test results, her assessment that the technology works is either 0.5 or 0. But because you lack the information she has, you don't know which assessment she holds. You assess probability 0.5 that she has positive information and 0.5 that her information is negative (which means the marginal probability that the technology will work is 0.25, consistent with the previous paragraph).

And now for the point of all this: *If* the entrepreneur has negative information, she presumably would be happy to sell whatever share she can unload for any amount of money. But *if* her information is positive, she has a pretty good venture on her hands. Would she sell 90% of this for $5 million? Would she sell 50% for $10 million?

The answers to these questions are that it depends. Specifically, it depends on how risk averse is the entrepreneur. If she is very risk averse, she might well sell a 90% share for $5 million, even though the expected monetary value of the entire venture, if she has essentially positive information, is $50 million. (Would you give up a coin flip gamble with prizes $0 or $90 million for a payment of $5 million?) On the other hand, if she is risk neutral, she would want a lot more than $10 million for 50% of the venture.

But there are intermediate levels of risk aversion that would lead her to be willing to sell 50% for $10 million but not sell 90% for $5 million. Suppose you decide that this is her level of risk aversion. Then,

- If the test results she saw were negative, she would sell on any terms.
- If her test results were positive, she would not sell 90% for $5 million.

So a willingness to sell 90% for $5 million confirms that she must have negative information. In this case, the 90% you are about to buy is worthless. But a willingness to sell 50% for $10 million is consistent with her having either sort of information. Not knowing her information, this means you are back to assessing probability 0.25 that the technology works. In this case you would be willing to pay $10 million for 50% of the venture.

Adverse Selection

This is a contrived example, but it starkly illustrates the basic phenomenon of hidden information: In many transactions, one party has information that the second party lacks and that is relevant to the second party's evaluation of the transaction.

The simplest examples involve goods of different quality levels, where the seller has information about the quality of the good she offers for sale. The prototypical example concerns the sale of a used car, where the seller, having owned the car, knows things about the quality of the car that the buyer does not. A far more significant example concerns financial transactions—for instance, when a firm issues debt or equity to outside investors. The investors try to learn as much as they can about the prospects of the enterprise, but it is easy to imagine that insiders to the venture (the entrepreneur in a small venture, or management in a large, publicly traded firm) know a lot more than outsiders.

In these cases and in others, the good being bought and sold can be of high quality or low or somewhere between. If the seller, who knows the item's quality, is willing to sell, that willingness to sell is an indication that the item in question is more likely to be of lesser quality. This is not a sure indication, but on the general principle that owners of lesser-quality goods are more likely to want to get rid of their stuff, the indication is still there. Therefore, a buyer must figure that the item for sale is more likely to be of lesser quality or, in other words, an adverse selection of the entire population.

What is more, a vicious cycle often takes over. Buyers, not knowing the quality of the particular item they are buying, are willing to pay a price for the average quality in the population. But is this an average of the entire population? It is not, if the equilibrium price for an average draw from the full population is not high enough to induce holders of higher-quality goods to sell. This is the adverse selection. Now watch it get worse.

The price must be lowered to reflect this adverse selection. But, as the price lowers, owners of intermediate-quality goods begin to withhold their items, making the adverse selection worse. The price falls again, and the adverse selection gets even more adverse.

The Market for Lemons: A Stylized Example

A stylized example illustrates the vicious cycle.[1] Imagine that a variety of used cars is "out there," some good, some okay, some real lemons. Specifically, there are used cars worth every price from \$1000 to \$3000 to new owners, with each price equally likely. Suppose as well that a car worth $\$x$ to a new owner is worth $\$x - 200$ to its current owner. And suppose that a finite supply of cars can be found at each value level, while a much larger number of people stand ready to buy.

If the quality of a car were apparent to both buyer and seller, then the logic of supply equals demand would imply that cars worth $\$x$ to buyers would sell for precisely that price; competition among buyers would bid the price of these cars up to that level. Every seller of a used car would be willing to sell and pocket the \$200 in surplus value thus created.

But the quality of a used car is rarely known precisely to buyers, while sellers have a very good idea what their cars are worth. Since this is a stylized example, I assume that buyers are incapable of seeing anything about the quality of a car, and they are willing to pay for the average quality of cars put on the market.

Since all the cars are sold if both buyers and sellers know the quality levels, we begin with the guess that all cars continue to be sold. That is, cars of value from \$1000 to \$3000 are put on the market by their owners. Then the average value of cars in the market is \$2000, and that is the price the market establishes. Now adverse selection kicks in. If someone has a used car worth more than \$2200 to buyers—hence worth more than \$2000 to the seller—this seller takes that car off the market. Why sell a car for \$2000 that is worth more than this amount to the seller?

Therefore, the only cars that would come on the market, if the market price is \$2000, are those which are worth between \$1000 and \$2200 to buyers. But the average value of this set of cars is \$1600, so that is the price. And adverse selection bites again: Owners of cars worth more than \$1800 to buyers—hence, worth more than \$1600 to sellers—withdraw from the market if the price is \$1600. Now the only cars that come to market are those with values between \$1000 and \$1800.

So the market price drops to \$1400. Now cars of value above \$1600 drop out of the market, so the market consists of cars with values between \$1000 and \$1600, which means that the price drops to \$1300, and more cars are withdrawn from the market, and the price drops again.

Where does all this end? What is a market equilibrium in this stylized model? Suppose that the market price is \$1200. This means that cars worth \$1200 or less to their current owners—therefore, worth \$1400 or less to buyers—are brought to the market. Then the average value (to buyers) of cars on offer is \$1200. This is the market price, and the market is in equilibrium.

What a crummy equilibrium. A scant 20% of the used cars in existence

[1] This example is roughly drawn from the classic paper by G. Akerlof, "The Market for Lemons: Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics* 89, 1970, 488–500, which began the economic literature on this general subject and for which Akerlof shared the Nobel Prize in 2001.

go onto the market, even though every used car is worth $200 more to a new buyer than to its current owner. Furthermore, this is not a random 20% but the 20% of worst cars. This is truly a lemons market.

Of course, reality is not this bad. Some buyers of fine used cars are forced to sell their cars, which improves the distribution of cars in the market, raising the price and bringing more cars into the market. Moreover, buyers can tell something of the quality of individual cars and can learn more by having the car inspected by a competent mechanic. As we'll see in a bit, the buyer or seller can do still more to signal or discern the quality of individual cars. The stark adverse selection problem of this stylized example is not found in reality. But the effect is there in real life, and it is particularly strong when (1) sellers have a hard time learning the value of the good they are buying and (2) the value of the object to prospective buyers is close to its value to prospective sellers.

18.2. Hidden Information on the Buyer's Side: Insurance and Contract Fulfillment

In the examples given so far, the seller knows more about the quality of the good than the buyer. But cases in which the buyer holds hidden information also arise, typically involving a service that the seller provides the buyer: The cost of providing the service is uncertain, and the buyer knows more about the cost than the potential service provider.

Life and health insurance are classic examples. In fact, the term *adverse selection* comes first from the literature of actuarial science. The cost of providing life or health insurance depends on how likely it is that the client will get sick or die. At any premium level, it is precisely the sick and nearly dead, assuming they know these sad facts, who are most anxious to buy insurance. Therefore, in terms of expected payouts on the policy, the insurance company faces an adverse selection of the population as a whole. This can lead to the sort of vicious cycle we saw before: Premium rates must be high, to compensate for the adverse selection problem. This leads the fairly healthy to go without insurance, and the selection of folks signing up for insurance becomes more adverse, raising premiums, worsening the adverse selection, and so forth.

This sort of problem arises as well in the context of outsourcing and service contracts. Suppose Corporation X requires a specific service at one of its facilities. It has the in-house capability to perform the service but has asked Contractor Y to take on the project, offering (say) $140,000. Contractor Y is unsure what it will cost to provide the required service, thinking that its cost will be somewhere between $100,000 and $170,000.

Contractor Y must ask itself, Why is Corporation X not using its in-house capability to provide this service? A good answer is, Because Y, being a specialist in this sort of service, can provide it more cheaply. In particular, suppose Y believes that it would cost Corporation X $10,000 more to do this project in-house than it would cost Y. But, *if Y believes that management at X knows what this project would cost X and X is offering $140,000, then the cost to X of the project is more than $140,000, which means the cost to Y is more than $130,000.* In this case, the relevant range of costs for Y is not $100,000 to $170,000 but instead $130,000 to $170,000. So perhaps Y should insist on a price of $155,000. But if X agrees to $155,000 as the price, then the cost to X must be more than $155,000, which means a cost of $145,000 or more to Y. And so forth, just as in the case of the used car market.

18.3. Problems of Average Selection

Imagine you are setting up a new manufacturing plant in a rural locale. Relative to other businesses in the area, your business involves a lot of on-the-job and off-the-job training of workers; your manufacturing technology demands skills that the local workforce lacks, and you will have to provide those skills to new employees. The training is costly, and to get a good return on your investment in training your employees, you want employees who will remain with your firm for a long period of time. Indeed, that is one reason you chose a rural location for your plant; people living in rural locations tend to be relatively less mobile than folks who live in urban or suburban settings. Also, you face less competition in the labor market for the people you train than you would in a more urban setting.

The problem is not that you get, among job applicants, an adverse selection of the population in terms of the quality you want, longevity on the job; it is that you get an average selection and you want to do better. Assuming potential employees have information that bears on their probable job longevity, you want to find a way to encourage them to "volunteer" that information, so you can select for employment and training the better-than-average selection of the whole population you desire.

18.4. Signals and Screens

The obvious response to situations where one party to a transaction has information that the second party lacks is for the second party to do what it can to get some or all the relevant information. Relevant information can come in many forms, among which are the following.

Freely Available Relevant Information

Relevant information is sometimes freely available, if the uninformed party knows where to look. Demographic information is often used in this manner. For instance, age and sex are important statistical indicators of mortality rates, useful to someone selling life insurance. If you are hiring for a rurally located plant, you might avoid young women, in the belief that they are more apt to get married or have children and then quit. (Please finish reading this section before deciding I am a male chauvinist.) Banks that sell mortgages sometimes engage in redlining (refusing to make loans on houses within certain geographical districts) because the default rate for homes inside the red line are historically high.

The use of demographic information in this manner is widespread, so much so that a caution is in order, on at least three grounds.

1. The basic statistical hypothesis can be incorrect, and the nature of the decision precludes learning this. For instance, professional partnerships sometimes discriminate against younger women in both initial hiring and promotion decisions, citing higher quit rates for family reasons. But careful empirical investigation does not support this hypothesis. Organizations that discriminate against women do not learn this, however, because too few women are hired or promoted to test the hypothesis. Compounding this are standard cognitive biases associated with making inferences from data: An organization that promotes only a few women and then sees one quit for family reasons might well overprocess this one piece of data, relative to its true statistical value.
2. Such hypotheses can be self-generating. Take a professional partnership that chooses not to promote young women to partner because of a perception that young women are more prone to drop out for family reasons. The partnership makes this choice because, it claims, it sees this pattern of behavior among its young woman associates. But this can be a vicious cycle: Young woman associates may be more likely to drop out precisely because they (correctly) perceive that this organization is less likely to promote them. Or in the case of redlining, if mortgage seekers inside the redlined area are forced to take loans with higher interest rates because of the perception that they are more likely to default, they may actually default more frequently because of the higher interest rates.
3. This form of discrimination raises legal and ethical questions. Is it fair to judge individuals based on membership in a demographic class that they can't control? Also, in response to its pernicious social effects, laws banning this form of discrimination are often enacted.

Legally Mandated Information

Direct and relevant information is sometimes available by legal or legislative mandate. For instance, good-faith disclosure of relevant information is required in the sale of real property: The owners of houses, who presumably know a lot about the hidden defects of the property, must disclose all known defects. An extremely important example is the legal requirement for publicly traded firms to disclose relevant financial information to protect potential investors. Of course, the recent spectacular case of Enron shows that legal requirements to provide information do not guarantee that the information will be revealed.

Information Required by an Independent Authority

Informed parties sometimes provide direct and relevant information to gain the certification of an independent authority. For instance, to be listed for trading on the New York Stock Exchange, firms must "voluntarily" reveal information beyond that required by government regulation. I put *voluntarily* in quotes here because the voluntary action is the decision by the firm to seek a listing on the NYSE; once that decision is made, the firm must reveal information about itself.

Information Provided Voluntarily

The final category is information provided voluntarily. For instance, when screening prospective employees, the employer might decide that possession of a high school diploma is a good signal of the employee's longevity prospects. That is, high school dropouts are more likely to be less accepting of authority on average, more likely to give up in the face of difficulty, and so forth. You might argue that the possession of a high school diploma is similar to demographic information—indeed, education level is often classified as demographics—but while an individual cannot change race or gender, the decision to stay or drop out of high school is at least partly voluntary.

Voluntarily provided information is a big category, covering a huge variety of cases and types. We define this category broadly, to include

- *Honestly volunteered information.* Many people, asked about problems they had with a used car or a house, will volunteer the truth, simply because they are honest. Of course, to trust such information, you have to be able to judge who is honest and who might be deceptive. But some people are pretty good at sizing up another person's character, especially when it comes to nonprofessional transactions, such as the sale of a used car from one individual to another. Social norms, supported by social structure, such as a small and tight-knit community, and backed by the threat of sanctions against those discovered to have broken with the community's norms, often guarantee honesty and openness. In some instances, individuals cultivate and then have the motivation to protect a reputation for honesty; we see how reputation works in Chapter 23.

- *Providing the uninformed party with an opportunity to procure information.* For instance, in the U.S. used-car market, it is standard for prospective buyers to take the car to their own mechanic for a checkup, with the prospective buyer paying the cost of the inspection. This is possible only if the seller permits the prospective buyer to borrow the car. Even more prosaically, sellers of used cars "permit" prospective buyers to look under the hood and take the car out for a spin.

18.5. Equilibrium Signaling

Of course, the value of a particular piece of information depends on how it correlates with the characteristic of interest to the uninformed party. For instance, if you are interested in finding new employees who are unlikely to quit over, say, a 5-year horizon and you screen on the basis of a high school diploma, the value of that screen depends on the strength of the statistical association between the diploma and longevity of employment.

When it comes to signals sent voluntarily, the information content of the signal depends on whether the informed party recognizes how the signal will be used. For instance, suppose one employer finds that employees in a local labor market are much more likely to last at least 5 years on the job if they have a high school diploma than if they do not. Accordingly, this employer begins to screen prospective employees in a local labor market based on whether they have a high school diploma. If this is one employer among many, not representing a significant fraction or a particularly desirable segment of employment opportunities in the region, students in high school are unlikely to change their dropout decisions. Hence the information quality of the signal (the statistical association between possession of a high school diploma and longevity of employment) will not change because the signal is being used by this one employer. But suppose this is a significant employer in the local labor market (it represents a significant fraction of local job opportunities, as in a rural mill town, or the "cream" of job opportunities) or suppose other employers in the area begin to apply the same screen. Then high school students in the area, or their parents, to the extent that the parents have any influence, may recognize how important a high school diploma is to future job prospects. This, in turn, may lead students who would otherwise drop out of school to remain, and thus it may weaken

the informational content of a high school diploma. Please note that, in this case, the informational content of *no* diploma is likely to strengthen.

In equilibrium models of signaling, it is assumed that people who might send the signal recognize how the signal will be used and respond in whatever manner is best for them. Take the used-car market, for instance, and to make matters really simple, imagine that there are only two types of used cars: lemons, worth $2000 to buyers and $1800 to sellers; and cream puffs, worth $3000 to buyers and $2800 to sellers. Suppose as well that in the population of all used cars, three-quarters of the cars are cream puffs and that buyers learn nothing about a car's quality by driving or inspecting it.

In this model, adverse selection is a killer. Suppose all cars were brought to market. Then any single car has a 0.25 chance of being a lemon and a 0.75 chance of being a cream puff. The expected value to buyers of this "lottery" is a car worth, on average, $2750. Competition among buyers, assuming there are a lot of them, forces the price up to $2750. But then the owners of cream puffs would withdraw their cars from the market. The only equilibrium without signals is for the lemons only to be bought and sold for $2000.

Now suppose that sellers of used cars can, if they choose, voluntarily send a signal about the value of their car. Specifically, they can offer a limited warranty. Imagine that, for the owner of a cream puff, warranties cost $60 per month in expected value, for warranties up to 12 months. And warranties on cream puffs are worth $50 per month to buyers. But for lemons, warranties are worth $100 per month to buyers and cost sellers $200 for the first month, and $300 more per month, for every additional month the warranty runs.

With these assumptions, a signaling equilibrium exists on the following terms: used cars without warranties sell for $2000 apiece; a used car offered with a warranty of 5 months (or more) sells for $3250. In this equilibrium, all the lemons are sold for $2000 without a warranty and all the cream puffs for $3250, with a 5-month warranty.

Why is this an equilibrium?

- Imagine you own a cream puff. If you offer your cream puff for sale without a warranty, you net $2000. It is worth $2800 to you if you retain it. And if you sell it with a 5-month warranty, you gross $3250, from which the warranty cost of $60 × 5 = $300 must be subtracted, for a net of $2950. So you sell with a 5-month warranty.
- Imagine you own a lemon. If you offer your car for sale without a warranty, you net $2000. If you hold onto it, you have a car worth $1800 to you. And if you sell with a 5-month warranty, you gross $3250, against which the cost to you of the warranty ($200 + $300 + $300 + $300 + $300 = $1400) must be subtracted, for a net of $1850. So you do best to sell

without a warranty.

- Imagine you are a buyer. Because of what we just saw, if a car is offered to you without a warranty, you know it is a lemon, and you would be willing to pay $2000 for it. If it is offered with a 5-month warranty, you know it must be a cream puff, worth $3000 plus 5 months of warranty, at $50 apiece, or $3250 in total. Assuming lots of buyers and a limited supply of cars for sale, supply equals demand leads to prices of $2000 and $3250, respectively, which is the asserted equilibrium.

There are two keys to this sort of signaling equilibrium. First, the signal sent must be more costly for low-quality car owners than for high-quality car owners, more costly enough that low-quality car owners are unwilling to send the signal, even knowing that to do so would elicit the price appropriate for high-quality cars. Second, the signal cannot be so expensive, relative to its value to the buyers of cars, that owners of high-quality cars are unwilling to send it.

For another example, go back to the case of hiring screens used by an employer. When high school students realize that possession of a diploma is used to screen prospective job applicants, the students may choose to stay in school, when previously they did not. If this happens, it is because students who would otherwise drop out are willing to pay the cost of staying in school (subjecting themselves to the arbitrary discipline of shop teachers and vice principals) to land a job. What the employer needs, then, is to find a signal that is very expensive for the prospective employees that are not desired but not very expensive for those desired and that, ideally, generates some positive payoff for the employer. Something like a tour of duty in the armed forces might fit the bill.

In the example of car warranties, there are only two types of cars, and the equilibrium signals *separate* the two types. In other signaling equilibria, called *pooling* or *partially pooling*, a single signal is sent by more than one type or quality level.

Signaling versus Screening

Imagine the buyer and seller of a used car meet, in a world with a 5-month warranty equilibrium. Does the seller take the initiative and say, "I want $3250 for my car, and I'm offering it with a 5-month warranty." Or does the buyer take the initiative and say, "I'll pay you $2000 for your car as it is, but I'll increase the price to $3250 if you give me a five-month warranty along with it. Take your pick." It would not seem to matter very much, but economists distinguish between the two: A *signaling* equilibrium is a situation in which the informed party takes the initiative. When the uninformed party offers the

informed party a menu of choices, where the choice from the menu becomes the informative signal, economists say that we have a *screening equilibrium*.

Screening is especially prevalent in cases where the uninformed party provides a service for the informed party. For instance, in health, life, and casualty insurance, screens that are frequently employed include the size of the deductible, the percentage of the total loss insured, and reduced benefits for some period of time.

Economists see subtle distinctions between signaling and screening, but these distinctions are too subtle for most practitioners to see or care about. The leading reason you see one or the other is the cost of processing the required information. Take insurance, for instance. Insurance companies collect reams of data from many clients and use those data to assess fairly well how, say, the rate of payout on a health insurance policy varies with the policy's deductible. An individual, out to buy health insurance, is going to have a lot harder time fine-tuning the details of the contract offered to the insurance company. And, unless the client comes up with a standard proposal, the insurance company may be unwilling to evaluate the offer.

Multiple Equilibria and the Problem of Unexpected Signals

In the equilibrium involving car warranties, it takes a 5-month warranty to get a premium price. The reason for 5 months, instead of 4 or less, is that a 4-month warranty would not separate lemons from cream puffs, if the lemon owners catch on to the buyer's logic. Let me do the calculation: A 4-month warranty on a cream puff is worth $3200 (in total) to a buyer. A 4-month warranty on a lemon costs the old owner $200+$300+$300+$300 = $1100, so the owner of a lemon who can sell the car with a 4-month warranty for $3200 nets $2100. This is more than the owner gets ($2000) if the car is recognized as a lemon.

So we need at least 5 months of warranty. But longer warranties are used in other equilibria. In particular, consider the following terms: Cars without warranties sell for $2000 and cars offered with warranties of at least 6 months sell for $3300. You can do the calculations to check that this is an equilibrium in which all lemons sell for $2000 and all cream puffs for $3300: Owners of lemons want no part of a 6-month warranty on these terms; cream puff owners are happy to offer the 6-month warranty, if that is what it takes to sell their car at a decent price; and buyers are paying for each car what that car is worth.

While this is an equilibrium, it is worse for cream puff owners than the equilibrium with 5-month warranties; cream puff owners get $3250 less $300 or $2950 in net value in the 5-month warranty equilibrium, whereas in this new equilibrium, they net $3300 less $360 or $2940. And buyers and lemon owners are no better off in this second equilibrium. Buyers come away with zero surplus in either case, and lemon owners do just as well as before.

Why do cream puff owners settle for this worse equilibrium? Why does some enterprising cream puff owner (she) not offer her car with a 5-month warranty, expecting to get a price of $3250? Maybe this would work. But we cannot be sure. The key question is, What would buyers infer about a car offered with a 5-month warranty, in a world where 6-month warranties are the norm? Would they decide that the car must be a cream puff? Would they think, "I cannot tell, so I assess probability 0.25 that the car is a lemon, that being the overall population proportion"? Since the 5-month warranty signal is not being sent in the 6-month warranty equilibrium, we cannot tell what buyers will infer, so we cannot be sure that such an offer would be accepted.

Yet another signaling equilibrium, but without any signaling, has cars selling for $2000 no matter how long a warranty is offered. In this case, only lemons are sold; owners of cream puffs retain their cars. This is an equilibrium, and a particularly bad one for cream puff owners, because if the equilibrium doesn't involve warranties, buyers don't know what to make of them, and cream puff owners can't predict how buyers would react if offered a warranty. (Some economists argue that buyers should be able to figure out that owners of lemons would never offer a 5-month warranty if they can sell their cars for $2000, and so a 5-month warranty *must* mean the car is a cream puff. Therefore, equilibria that require more than 5 months of warranty or that do not involve warranties at all can be ruled out. This requires that buyers are quite sophisticated, however. Advanced treatments of market signaling will discuss this issue; look for the term *refinement of out-of-equilibrium beliefs.*)

As fanciful as this discussion may seem, it does attend to real-life phenomena. In the real-life used car market in the United States, when a car is sold privately by its owner, the practice is for prospective buyers to take the car to their mechanics for a diagnostic checkup. But in Israel, for instance, the seller of a used car gets a diagnostic report from one of a few well-known reputable sources for such reports, then hands out copies. It is pretty clear the U.S. system is relatively inefficient, because it involves the cost of multiple diagnostic checkups.

What would happen if the owner of a used car in the United States had her car inspected, had a report written up, and when buyers show up, handed out copies, pointing out how this saves prospective buyers the cost of an inspection, which ought to be reflected in the final price? I suspect that prospective buyers, unused to this way of doing things, would be unsure of what to make of the proffered report. At least, my inclination as a buyer

would be to suspect that something is amiss. Why does the owner not want me to have *my* mechanic check out the car? How much did the owner pay this garage, to falsify this report? Well, perhaps I would not be that suspicious. But any suspicions along these lines would kill unilateral attempts to move from the less-efficient U.S.-style equilibrium to the more-efficient Israeli-style equilibrium. This is precisely the problem observed in the 6-month-warranty and the no-warranty equilibria. When a signal normally is not sent, receivers do not know what to make of it.

18.6. The Winner's Curse

We conclude with the *winner's curse.* I explore this phenomenon in the case of a sealed tender auction of some single object, whose value to the different bidders is unknown to them. Think, if you wish, of firms bidding for mineral rights or individuals bidding to purchase a week's vacation in Hawaii.

While no bidder can be certain about the value to himself or herself of the prize, each bidder forms an assessment of this value in the form of a probability distribution. For instance, imagine that Bidder A, based on her analysis and information, assesses that the expected value to her of the object being auctioned is $45 million.

Suppose that Bidder A learned the assessments of her rivals. Suppose, for instance, she learned that Bidders B, C, and D assessed expected values of $30 million, $38 million, and $25 million, as the expected values of the worth of the object to each of them, respectively. *Would this information affect Bidder A's assessment of the value to her?* Would she, for instance, on learning that her assessment is the most optimistic, have reason to temper her optimism?

The answer to this question depends on the context:

- If the object is a week's vacation in Hawaii, the value any bidder puts on the object is, probably, purely personal. Bidder A knows as much as anyone about what this vacation would be worth to her. She might be uncertain about the value (will it rain?) but the fact that Bidders B, C, and D attach different values to the vacation does not affect the value she attaches to it. This is not to say that she wouldn't like to know their valuations, because if she knew those, she might be better able to predict what they will bid and thus avoid leaving money on the table. But the question is not whether she wants to know their valuations. The question is, Does knowledge of their valuations change her own? In this case of *private values*, the answer is No.
- If the object is the right to extract minerals from a particular tract of land, the answer may be quite different. The value of the prize depends on

exactly how much oil or gas or coal there is to be found and on how hard it will be to extract those resources. There is more to it than that (one bidder may be better equipped to exploit the resources than another) but there is a lot of *common value* in the object being auctioned. And different bidders probably have access to different information about the tract of land. They presumably did different geological surveys of the site. They may have run different seismic tests. They may have different experiences with similar, nearby tracts of land. For a variety of reasons, it is not hard to believe that Bidder A, based on her own private information, assesses an expected value of $45 million for the rights, *but were she told that all three rivals value the rights at less than $40 million, she would lower her expected value.* This is not definite: She might believe that her information is simply superior to theirs—she would not spend a dime to learn things that they know and she does not when it comes to determining her assessment of value of the rights at auction—and she believes her geologists are simply superior to theirs. But this is pretty extreme; probably, she would move her assessment in the direction of theirs, if she knew what assessments they were making.

We find the winner's curse in the second situation. Suppose that on the basis of an assessed value of $45 million, Bidder A bids $40 million. *Bidder A wins the auction when Bidders B, C, and D bid less than $40 million.* And B, C, and D are much more likely to bid less than $40 million when their respective assessments are values such as $30 million, $38 million, and $25 million than when their assessments are values on the order of $50 million, $75 million, and $65 million. *Bidder A, in consequence, wins the auction precisely in those instances where her colleagues are relatively more pessimistic. But in those instances, Bidder A should be more pessimistic than her own information indicates as well.*

The winner's curse occurs when unsophisticated bidders in an auction face this sort of statistical structure of information and valuation, forming their bids without taking into account that the winner of the auction is apt to be a party with relatively optimistic assessments. The major oil companies are quite sophisticated about this effect; they have come to understand the winner's curse through bitter experience and nowadays formulate their bids bearing in mind that the winner of an auction is going to be relatively optimistic and often the most optimistic of all the bidders. But less sophisticated or inexperienced bidders, who fail to understand this phenomenon, can find themselves cursed by winning.

How should you bid in a context where a possible winner's curse lurks? This is not a question that can be answered easily. Perhaps the only simple takeaway is that if you are involved in an auction that may possess a winner's curse and your rival bidders are unlikely to be sophisticated about this fact, find another place to do business: Unsophisticated bidders in this sort of situation are likely to overbid, and so the only way you can win the auction is by bidding more than parties who, on average, overbid. This is unlikely to lead to generous profits.

But beyond this simple advice, questions remain. What if your rivals are reasonably sophisticated? What if the object at auction is likely to be worth more to you than to other bidders; in other words, what if the auction mixes private value and common value features, so that even if your rivals are overbidding, you can still make a profit bidding more than they do? How do you find the optimal strategy for bidding in such cases, where the simple rule *Don't even try* is no longer valid? A detailed analysis of the auction is necessary, concerning the value of the object at auction to you, the values and assessments of other bidders conditional on the value to you, and often crucially, the bidding strategies employed by the other bidders. Optimal bidding strategies are rarely easy to find, and this is a place where a knowledgeable consultant can probably help a lot.

Executive Summary

- Problems of *hidden information* appear when one party to a transaction has access to information that other parties cannot access, if the information is relevant to the transaction.

- When items offered for sale are disproportionately drawn from the lower-quality end of the spectrum, because owners of higher-quality items are more likely to want to hold on to their items and where buyers cannot discern the quality, a problem of *adverse selection* exists.

- It is also a problem of adverse selection when, in markets for "services" such as insurance or contract fulfillment, high-cost-of-provision buyers (assumed to be better informed than the service providers about the cost of providing the service) are disproportionately represented in the population of buyers.

- In both types of adverse-selection problem, adverse selection can be a vicious cycle: Lower prices are paid because the average quality sold is low (or higher prices charged because high-cost-of-provision buyers are disproportionate in the population of buyers), which drives from the market goods of intermediate quality or clients of intermediate costs of provision, further driving the price of the goods down or the services up.

- Hidden information problems extend beyond adverse selection. For instance, companies seeking to hire workers would like to get a better-than-average selection of the population applying for jobs. And the *winner's curse* refers to a problem of hidden information that occurs in auctions of various types.

- Information revelation cures problems of hidden information. Very generally, this involves information freely available that is correlated with characteristics of interest, such as demographic information; information whose provision by the informed party is compelled by law; and information voluntarily provided by the informed party or elicited voluntarily from the informed party.
- To understand correctly information that is voluntarily provided, you must consider whether informed parties understand the uses to which the information will be put. In signaling and screening equilibria, it is assumed that informed parties fully appreciate the uses to which the information will be put and that they respond in a manner that is optimal for them.
- Signaling and screening equilibria can be *separating*, where differently informed parties take distinct actions, or *pooling*, where differently informed parties send the same signal.
- When information is voluntarily provided at the initiative of the informed party, economists call it a *signal*. When the uninformed party finds a means for eliciting this information, it is called a *screen*.
- In signaling equilibria, the presumed interpretation of signals that are not sent in the equilibrium can be crucial.
- The winner's curse refers to auctions in which different bidders have access to different information about the value of the item at auction to all bidders. The curse occurs when bidders do not take into account that in such circumstances, the winner of the auction is apt to be someone with relatively optimistic information, information that must in consequence be discounted.

Problems

18.1 At the Famous East Coast Business School (FECBUS), all MBA students want summer jobs working for investment banks in New York City. They want this sort of job so much that, if they are not offered a position of this sort, they refuse out of pride to do any other work. A summer job with an investment bank carries a stipend of $50,000. Therefore, a student at FECBUS who has probability p of landing such a job owns a lottery with payoffs $50,000 or $0, with probabilities p and $1-p$. Beyond the prestige of having such a job or the shame of not having one, FECBUS students use this money to help pay for their second year of school, and being risk averse, they consider the possibility of buying insurance against the contingency of no job. Specifically, Beantown Casualty, a local insurance company, has always provided catastrophic summer income insurance to FECBUS students. Beantown offers simple policies in which the student pays a premium P and Beantown reimburses the student Q if the student fails to land a job. If a student buys this policy, the outcomes are $\$50{,}000 - P$ with probability p and $Q - P$ with probability $1-p$.

(a) Drake Duck is a typical FECBUS first-year student. His probability of landing a summer job at an investment bank is 0.7. His utility function, used for calculating expected utility, certainty equivalents, and the like is

$$U(x) = \sqrt{x + 40{,}000},$$

where x is the proceeds from summer employment, net of any premiums paid to Beantown Casualty and net of any repayments from Beantown if Drake fails to land a job. If the alternative is to go without insurance, would Drake be willing to pay a premium of $P = \$10{,}000$ for insurance that pays back $Q = \$30{,}000$?

(b) Suppose Beantown Casualty is risk neutral. Which policies could Beantown write for Drake, assuming Beantown knows his utility function and probability of finding a job, that (1) give an expected payoff no larger than the premium, (2) give Drake a higher certainty equivalent than he gets with no insurance, and (3) are efficient in terms of risk sharing?

(c) Unhappily for Beantown, it does not know, a priori, the probability with which any individual student at FECBUS will land a summer job. But, we assume, each student knows his or her own probability. Assume that the first-year class of FECBUS consists of 500 students. For 100 students, the probability of landing a summer job is 0.9. For 100, it is 0.8. For 100 more, it is 0.7. For 100 more, it is 0.6. And for the last 100 students, the probability of landing a summer job is $p = 0.5$. To keep the problem as simple as possible, suppose that all 500 students have precisely the same utility function as does Drake.

Suppose Beantown decides to offer a single policy providing full insurance; that is, $Q = \$50{,}000$. Since the "average" student at FECBUS has probability 0.7 of getting a summer job, Beantown decides to charge a premium of $0.3 \times \$50{,}000 = \$15{,}000$. Would Beantown make or lose money with this policy? Why? (Assume that a student who does not buy insurance from Beantown goes without insurance entirely.)

(d) Suppose that Beantown is convinced that offering a single full-insurance policy is the way to go. Is there any premium Beantown could charge for full insurance that would make a positive expected profit for Beantown? Which premiums do this?

(e) In part d, you should find that the only full-insurance policies that would make a positive expected profit for Beantown involve insurance for the 100 students with a 0.5 chance of landing a job. All the rest choose not to purchase insurance. Beantown would like to insure more members of the FECBUS class, even partially, and so Beantown thinks about offering a single policy with a premium of $14,000 and a payout of $30,000. What would be the response to this policy? How would Beantown do in terms of expected profits?

(f) Beantown Casualty is not interested in making a huge profit writing these insurance policies—they are regulated by the insurance commissioner of the Commonwealth of Baystateland, and in any event, the head of Beantown is an old FECBUS grad who would never try to profit at the expense of students from her old school—but it does see a value in selling some insurance to as many students of FECBUS as possible. Beantown is required by law not to offer any policy that has a negative expected profit; cross-subsidization of one policy by another is not permitted. Moreover, it is compelled to offer full insurance at terms that break even. Therefore, it is compelled, in this circumstance, to offer a full insurance policy (one that pays back $50,000) for a premium of $25,000. (See part c.)

Suppose that, in addition to this policy, Beantown offers a $10,000 insurance policy for a premium of $3500, a $2000 policy for a premium of $500, and a $200 policy for a premium of $20. What would happen? In particular, what would be Beantown's expected profit on each policy and altogether?

18.2 In a particular economy, all homeowners own identical homes worth $80,000 apiece. These homes are subject to complete loss via fire, and the Old Reliable Insurance Company (ORIC) offers policies against loss by fire. The chance of a fire at any particular home is a probability p, which is known to the homeowner but not known to ORIC. The values of p run from 0 to as high as 0.4. Homeowners have no control over the value of p; it is simply given. For instance, Peter Reece has a home on the edge of a forest, and for his house, $p = 0.1$. John Yost has a home in the suburbs, and for his house, $p = 0.03$.

ORIC is risk neutral and offers two different policies. The first is a complete insurance policy, which pays the homeowner back $80,000 in the event of a fire. The premium for this insurance policy is $11,600. The second policy offers partial insurance. It has a payback in the event of fire of $58,400, and a premium of $5900. This means that a customer who buys this policy and has no fire is out $5900. With a fire, the customer nets $58,400 – $5900 = $52,500.

All the homeowners in this society are expected utility maximizers and have the same utility function:

$$U(x) = \sqrt{x + 10{,}000},$$

where x is the net of this situation, including the value of the house, if there is no fire. For instance, a consumer who buys the second insurance policy and has no fire has utility $\sqrt{10{,}000 + 80{,}000 - 5900} = \sqrt{84{,}100} = 290$, while with a fire, the utility is $\sqrt{10{,}000 + 58{,}400 - 5900} = \sqrt{62{,}500} = 250$.

(a) Of the three options available (no insurance, full insurance, or partial insurance), what would be the choice of Mr. Reece? What would be the choice of Mr. Yost?

(b) For which values of p would consumers with that value of p choose no insurance? For which values of p would consumers with that value of p choose partial insurance? For which values of p would consumers with that value of p choose full insurance? You should answer so that, for any value of p between 0 and 1, we can tell what the consumer would choose. Don't worry about values of p for which there are ties.

(c) The actuaries at ORIC, based on historical experience, predict that ORIC will sell 100,000 partial insurance policies, with an average profit per policy of $1228. They predict that ORIC will sell 5000 full insurance policies, with an average *loss* per policy of $12,400. Therefore, their net profit from this business is

$$(100{,}000)(\$1228) + (5000)(-\$12{,}400) = \$60.8 \text{ million}.$$

What would be ORIC's net profit from this business if it offers only the partial insurance policy?

18.3 Selling life insurance to senior citizens is a business with an enormously powerful adverse selection problem: People who are ill or in poor health sign up for this sort of insurance. A natural response to this, used extensively by insurance companies, is to require a physical examination before insurance is sold.

You probably have seen TV advertisements for life (or extended care) insurance, marketed for senior citizens, that proudly proclaims that no physical exam is required and "you cannot be turned down." These policies sometimes charge premiums that depend on the individual's age, gender, weight, and so forth. Still, they would seem to be prime candidates for problems with adverse selection. So how do they cope? If you read the fine print, you discover that "benefits are drastically reduced for the first 2 years of the policy." What is the point of this?

18.4 Among the benefits offered by corporations, at least in the United States, is health insurance. One explanation for why corporations offer such insurance is that this benefit is tax favored: Companies can compensate employees with health-insurance benefits that are treated as nontaxable income. Also, health care providers and insurers (such as Blue Shield) have historically offered better rates to employers for their employees than to individuals who ask for the same coverage. Why is this?

Some companies offer so-called flexible benefits to their employees: Employees have a certain number of pretax dollars to spend on benefits and are allowed to choose the portfolio of benefits they wish. In terms of the prices charged employers by health care providers and insurers for their product, provided as benefits, what do you think is the effect of flexible benefit plans?

18.5 In some countries, title to an automobile includes a history of previous owners. And in those countries, the price for a particular car, holding fixed the car model and features, miles driven, and general (discernible) condition, is a decreasing function of the number of previous owners. Why is this, do you think?

18.6 In the 1980s and 1990s, a large number of first-tier firms offered no-layoff employment. These firms promised employees that, perhaps after a probationary period, an employee had a job for life; he or she would never be laid off. Some firms, such as IBM and Eastman Kodak, had made this promise for decades; in the 1980s and 1990s, this employment practice, as a piece of so-called high-commitment Human Resource management, became increasingly prevalent.

Such policies have substantial benefits when times are good, but they do not come for free. In particular, a firm that offers such promises and that faces economic difficulties must choose between breaking these promises and desirable reorganization. For instance, before the return of Steve Jobs to Apple Computer, Apple, which had prided itself on its no-layoff policy, found itself badly hamstrung by the policy. Apple needed to shed excess

employees, but especially given its fairly young workforce, waiting for the natural process of attrition seemed much too slow.

So Apple decided to initiate a program of voluntary layoffs. It offered substantial inducements to employees to quit, including generous severance pay and excellent placement services for those who would consent to go. Apple found, to its dismay, that its program of voluntary layoffs had some bad repercussions. In particular, Apple top management, because of the voluntary layoff program, learned a lot about adverse selection. Precisely what sort of adverse selection problem did they learn about? And can you think of any way to put in place a voluntary layoff program that negates (or, at least, ameliorates) this problem of adverse selection?

18.7 In the U.S. real estate brokerage industry, brokers employed by large brokerage firms have traditionally worked for a portion of the commissions they generate. That is, if a house sale generates a 6% commission split equally between the brokers for the two sides (which is standard in many markets in the United States), the broker on one side might personally pocket 1.5% with 1.5% going to the firm for which the broker works. In return for a share in the broker's commissions, the firm provides the broker with an infrastructure, such as clerical support, phone services, and office space, and especially for new brokers, a base wage.

In comparison, RE/MAX, a national brokerage firm, permits its agents to keep 100% of the commissions they earn. The firm provides clerical services, phones, and the like, and charges its agents a fixed monthly fee for those services. RE/MAX is known for having the most aggressive agents in the business on average. If you want to find an aggressive, hardcharging agent, going to RE/MAX is usually a safe bet.

(a) Why does RE/MAX attract more aggressive agents?

(b) RE/MAX charges its agents more for the services it provides than it costs to provide those services. In fact, RE/MAX makes money by marking up the clerical and administrative services it provides to its agents. Why are aggressive agents willing to pay RE/MAX more for these services than it would cost them to procure the services independently?

(c) In addition to joining RE/MAX or a more traditional firm, realtors can go independent. How does this third option affect RE/MAX? How does it affect the more traditional firms? Put another way, would you expect a hardcharging, aggressive agent to remain with RE/MAX for the long haul? Does your answer to the question depend on aspects of the local real estate market; that is, would you give the same answer for, say, the Silicon Valley as you would for a prosperous county seat in South Carolina?

Questions about the winner's curse are generally quite difficult to pose. While very stylized, Problem 18.8 illustrates the phenomenon in about as simple a model as one can find. Problem 18.9 is more realistic, but it is very difficult to tackle and should not be attempted unless you have considerable skills in probability modeling and analysis.

18.8 Three construction firms, Ace, Base, and Case, are considering whether to declare their willingness to undertake a contruction project for the Freedonian government. The cost of fulfilling this construction project is very likely to be \$100,000. But there is a chance that the cost will be \$200,000. This cost does not change depending on the firm; if it is \$100,000 for one firm, it is \$100,000 for the other two.

The Freedonian government declared it will pay precisely \$125,000 for this job. That is, the price paid to the firm doing the work is fixed in advance. The decision facing Ace, Base, and Case is whether to declare that it is willing to undertake this project. If no firm is willing to undertake the project, it will not be done. If only one of the three is willing, that firm will be awarded the job. If two firms are willing, one of the two will be chosen by a coin toss. And if all three firms declare their willingness, one of the three will be chosen at random, with each firm having a $\frac{1}{3}$ chance of getting the job.

The three firms must each decide and then declare simultaneously and independently whether it is willing to take the job. All three are risk neutral. You are advising Ace, and the management of Ace has told you that it is willing to take the job if the expected costs of construction are less than the fixed price of \$125,000.

Each of the three firms initially assessed probability 0.8 that the cost of this project will be \$100,000 and 0.2 that the cost will be \$200,000. But each has a 0.75 probability chance of having learned the true cost. These 0.75 probability chances are independent of one another and independent of the true cost of the facility. That is, conditional on Ace learning the true cost, the probability that Base would learn it is still 0.75, and so on. And if we condition on the true cost being \$100,000, the chance that Ace would learn this is 0.75.

In fact, the management at Ace told you that it did not learn the true cost, so it still assesses 0.8 that the cost will be \$100,000. And it does not know whether Base or Case knows the true cost; it assesses binominal probabilities 0.75^2 that both rivals know the true cost (whatever that is), $2 \times 0.75 \times 0.25$

that one rival knows the true cost and the other does not, and 0.25^2 that neither rival knows the true cost.

Suppose that Base and Case both use the following decision rule to determine whether they should declare their willingness to take on this project:

> If the firm in question is certain that the costs would be \$100,000, then declare that it is willing to take on the contract; otherwise, do not indicate a willingness to take on the contract.

Against two rivals whose decision rules are these, should Ace, which remains uncertain about the true cost, indicate a willingness to take on this project?

18.9 An object being auctioned by sealed tender to one of three bidders has a value V, which is common to all three bidders. The value V is also uncertain, and the three bidders assess, a priori, that $V = e^X$, where X is Normally distributed with mean 15 and variance 1. Each of the three bidders receives a signal about the value of X; the signal received by bidder i (for $i = 1, 2, 3$), denoted s_i, is Normally distributed with mean X and variance 1. Moreover, the signals are conditionally independent of one another, conditional on the true value of X.

Bidder 1 has received the signal $s_1 = 15.3$. Bidder 1 believes that bidder 2 will bid 70% of what bidder 2 believes to be the expected value of V, conditional on s_2, and that bidder 3 will bid 80% of what bidder 3 believes to be the expected value of V, conditional on s_3.

The auction is a first-price auction, meaning the high bidder wins the object, paying his or her bid. Bidder 1 is risk neutral and aims to maximize her net expected value for the auction, which is the probability that she wins times the expected value of V less her bid, conditional on her winning. What is the optimal bid of bidder 1 under these circumstances?

Suppose we change the story, so that bidder 3 learns not s_3 but the true value of X. Bidder 3 therefore bids $.8 \times e^X = .8V$. What is the optimal bid for bidder 1 under these circumstances?