Email pricing

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Abstract

We compare the magnitude of, and welfare generated by, uniform welfare-maximising, Ramsey and monopoly pricing in email networks. Messages are defined by the utility they give to their sender and receiver. Senders tend to pay more than receivers when the average sender utility is higher than the average receiver utility, and vice versa. When message preference distributions are symmetric receivers pay more than senders. Because prices cannot be (too) negative, the interior solutions for all price types hold only when the distributions for sender and receiver utility are similar. The comparative welfare analysis shows that in some situations the use of uniform, Ramsey and zero prices will not generate substantial welfare losses relative to feasible perfectly discriminatory prices. Monopoly prices are unlikely to be efficient.

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1 Introduction

Economists usually trust in the power of prices to influence peoples' behavior yet, to date, only technical and regulatory controls have been used to any extent in email networks. Possibly because pricing in telephone networks is so well understood, there has also been very little consideration given to pricing in email networks in the literature. We contend, however, that there are significant differences between telephone and email networks and that these differences mean that the findings from the telephone network literature are not universally applicable to email networks. Before ISPs or regulators can seriously consider introducing sender and receiver prices they need to have a better understanding of what exactly prices can achieve in email networks, how sender and receiver prices can be best used to achieve economic goals such as maximising welfare, how close to maximum welfare do we get with the status quo of zero prices, and what is the level of the efficient and profit-maximizing prices. We address these matters in this paper.

Communication in a telephone network is a one-stage process that requires two specific actions to occur - communication occurs if and only if a call that is made is also answered. In the literature on telephone networks it is assumed, perhaps incorrectly, that if a call is made but not answered there are no costs imposed on any party. In effect, unless a connection is made between sender and receiver it is as if nothing has happened. Certainly a caller is never required to pay for calls that are unanswered. Moreover, because models of telephone networks assume that everything happens simultaneously there is no loss of generality associated with modeling the value of a message to either party as net of that party's processing cost.

In email networks, on the other hand, communication occurs only after a specific three-stage sequence of actions. In the first stage a message is sent by the sender, is transmitted by the ISP to the receiver and enters into the receiver's mailbox. At this stage the sender incurs a processing cost of drafting the message and pays the sender price (if there is one), the ISP incurs the cost of transmitting the message, and an unavoidable cost of processing the message is imposed on the receiver. The receiver's processing cost is actually incurred in stage two when the receiver processes the message, but the fact that the message has entered into the receiver's mailbox renders this cost unavoidable. In stage two, the receiver chooses to delete or to open the message and a processing cost associated with making and executing this choice is incurred. This processing cost is sunk and so does not influence the receiver's decision to open the message or not. However, the receiver price, paid only if the receiver chooses to open a message, is not sunk. In stage three, the receiver of a message chooses whether or not to read a message that has been opened and if the message is read both the sender and the receiver realize the potential utility of the message. Clearly in email networks it is not reasonable to model the values of a message to the sender and the receiver as net of their processing costs because their costs of processing are incurred regardless of whether the value of the message is realized, that is, regardless of whether the messages is read or not.

The distinction between opening and reading an email message is an important one. The ISP can only observe whether a message is opened and so payment of the receiver price must be based on this action and not on whether or not it has been read. If the receiver price is positive, a receiver will not choose to open a message she does not intend to read. However, if the receiver price is negative the receiver will open all messages and read only those that generate positive utility for her. This means that negative receiver prices cannot achieve anything useful in email networks, which in turn means that perfectly discriminatory prices cannot achieve first-best welfare outcomes if some messages generate negative receiver utility.

Our model of the email network incorporates the three stage process of communication described above into the pioneering analysis of Hermalin and Katz (2004). Hermalin and Katz examine email pricing in a framework where the unit of interest is a message. Each message generates net utility for sender and receiver according to some bivariate distribution. In our viewpoint, one of the main contributions of Hermalin and Katz is the setup that incorporates a very rich preference structure without the need to keep track of the identities of the sender and receiver of each message exchange. These together allow for new insights to be derived about uniform sender and receiver prices (social welfare maximizing, Ramsey and profit maximising prices are considered). While we have seen uniform prices before in the literature for telephone pricing, this was previously accomplished by assuming either that everyone is the same or that everyone is different but from the viewpoint of any network participant everyone else is the same.[1] Thus, demand heterogeneity was one-dimensional, whereas Hermalin and Katz framework allows for investigation of uniform prices with a much richer, two-dimensional heterogeneity. However, the opportunity cost of having the unit of interest be a message rather than a consumer is that the framework cannot be used to analyze access decisions or competition between providers because these require information on the preferences of individual subscribers.

Using a similar approach to Hermalin and Katz, but making the important adjustments

[1] See for example Squire (1973), Littlechild (1975), Dhebar and Oren (1985), Einhorn (1993) and Hahn (2003).

discussed above (sunk receiver processing cost, separating processing costs from utility, and constrained receiver prices), we calculate optimal uniform sender and receiver prices and discuss how and why they differ from those suggested for other networks and from those suggested by others for email networks (for example Hermalin and Katz (2004)). We also calculate uniform Ramsey and monopoly prices. Finally we compare the welfare associated with each pricing regime and with the status quo of zero prices.

Several papers investigate receiver prices in interconnected telephone networks, where the telephone operators also set interconnection charges calls originating in the competitor's network (see for example Jeon et. al. (2004), Kim and Lim (2001) and Doyle and Smith (1998)). The focus of these papers is substantially different from ours because our approach does not lend itself well for discussing issues around competition. MacDonald and Meriluoto (2005) examine efficient perfectly discriminatory sender and receiver pricing and access pricing in telephone networks in the absence of termination charge considerations. Our contribution is investigating uniform pricing as well as incorporating features specific to email networks into the model.

We do not attempt to capture the effects of spam in this paper. In fact, we argue that a model of communication demand with a smooth and continuous distribution of preferences, as most papers in this area assume, is ill suited for addressing spam for a number of reasons. Because spam is not targeted to those receivers who value it, most spam messages generate zero receiver utility. This suggests that one possible way to model spam is to represent it as a mass point within the non-spam preference distribution. A further complication arises, however, because with filtering many spam messages that are sent are not received. One really needs to specifically model the behavior of spammers in the face of email pricing as we do in Eaton, MacDonald and Meriluoto (2008).

There is another potentially serious flaw associated with modeling email networks with a continuous message preference distribution. If senders pay a sufficiently negative sender price, specifically $p^S < -c^S$, there is an absolute incentive for senders to construct phoney messages as a money making activity. This means that there will always be an infinitely large mass point of messages with, or near, zero sender utility. We avoid this problem by imposing a the constraint $p^S \geq -c^S$ to effectively rule out the possibility that this mass point would ever be included in optimally exchanged messages. Because the mass point becomes economically irrelevant in our model we assume it does not exist and instead assume that the message preference distribution is continuous and well-defined.

The first new insight we derive has to do with social welfare maximising uniform prices.

The standard result is that given network effects, the price of a message would be set equal to the cost of the message minus its external effect. We show that this intuition will hold for the sender price and the receiver price in their best response function form. However, when the two prices are solved for simultaneously, the resulting prices will incorporate not just the external effect and the cost of the message but also preferences of the sender/receiver as well. This is true for not only the welfare-maximising prices but also the Ramsey prices and profit-maximising prices, but the weights on the sender's/receiver's own utility and the external effect will be different for the three different prices.

We discuss the circumstances under which it is efficient to use sender pricing alone, receiver pricing alone or sender and receiver prices together. When the maxima in the preference distributions are relatively symmetric, both prices are positive if the maxima and not too large but equal their minima at $p^R = 0$ and $p^S = -c^S$ when the the maxima are large. For asymmetric distributions where senders' utility is larger than receivers' utility, sender price is likely to be positive while the receiver price is zero. For asymmetric distributions where receivers' utility is larger than senders' utility, receiver price is likely to be positive while the receiver price equals its minimum at negative the of sender's processing cost.

We show that the two optimal uniform prices are asymmetric even when the preferences for sending and receiving messages are distributed identically and independently by a uniform distribution. Given such a preference distribution the receiver pays more than the sender and both efficient prices decrease with the maximum in the preference distribution to the point where, if the maximum is sufficiently large, the efficient receiver price is equal to zero and the sender subsidy is equal to the sender's processing cost.

If the maxima in the preference distributions are sufficiently large the sum of the efficient receiver and sender prices does not cover the ISP's costs. We examine the uniform Ramsey prices, i.e. efficient prices given an ISP break-even constraint. The Ramsey prices behave very similarly to the efficient uniform prices with the exception that they reach their minima at a lower level of maxima in the preference distributions and these minima are higher (because they have to cover the ISP cost) than the efficient price minima.

Monopoly prices, however, behave quite differently from efficient uniform and Ramsey prices. They are smaller than efficient prices when the maxima in the message preference distributions are small, but because they increase without bound with the message preference maxima while the efficient prices decrease until they reach their minima, the monopoly prices quickly surpass the efficient prices. The non-negativity constraint for monopoly prices are binding only for very asymmetric distributions.

The main welfare results for identical uniform distributions are as follows. The efficient uniform prices generate welfare that is, perhaps surprisingly, close to the maximum welfare achievable by perfectly discriminatory pricing. Ramsey prices generate welfare that is lower than or equal to the welfare with efficient uniform prices, but this welfare is never too far from the maximum achievable welfare. The status quo of zero prices generates poor welfare results when the message preference maxima are small (because positive prices are required to discourage inefficient message exchange) but the performance of zero prices improves as the message preference maxima increase. Monopoly prices, however, do well only for a small range of parameter values when the monopoly prices equal or are close to the efficient uniform prices. Monopoly welfare can be negative (despite monopoly profit being positive) when the message preference maxima are small. When the maxima are large, the ratio of monopoly welfare to the maximum achievable welfare becomes relatively constant, at $55 - 64\%$ of the maximum achievable welfare in our numerical examples.

The rest of the paper is structured as follows. Section 2 presents the model assumptions on preferences and costs. Section 3 describes the total and private surpluses of messages given some arbitrary sender and receiver prices, and describes what are the constraints set by the email technology on prices. Section 4 describes the general conditions for uniform welfare maximizing prices, Ramsey prices and monopoly prices. Section 5 presents uniform welfare maximizing prices, Ramsey prices and monopoly prices for a uniform distribution. Section 6 presents the comparative welfare analysis. Section 7 concludes.

2 Model set-up

2.1 Preferences

The composition of the network is fixed and every consumer in the network has the ability to both receive and send messages. Consumers choose whether or not to send messages to other consumers, and if they receive a message from another consumer, they choose whether or not to open and read it.

A message from sender s to receiver r is completely described by the pair (σ, ρ), where σ is the benefit the sender gets if the message is read, and ρ is the benefit the receiver gets from reading the message.[2] We place no sign restrictions on these benefits – both σ and ρ can be positive, negative, or zero. Of course, if the message is not read, the realized benefits

[2] When an email turns up in someone's inbox, the heading allows the receiver to identify the sender and the subject of the message. Based on this information, the receiver chooses to either open or delete the message. In effect, the receiver is making an inference about the value of the message to her, and it seems reasonable to suppose that her inference is correct. So we assume that the information in the message heading is sufficient to enable the receiver to correctly estimate ρ.

of both parties are 0.

We describe the potential benefits of the email network by a density function $M(\sigma, \rho)$. The potential benefits are of course realized if and only if the message is sent by the sender, and opened and read by the receiver. We assume that the density function is continuous, which implies that it has no mass points. We assume that σ and ρ are independently distributed by $f(\sigma)$ and $g(\rho)$ with support in $\sigma \in [\sigma_{\min}, \sigma_{\max}]$ and in $\rho \in [\rho_{\min}, \rho_{\max}]$, respectively, so that $M(\sigma, \rho) = f(\sigma)g(\rho)$. The density functions f and g correspond to cumulative density functions F and G, respectively.

2.2 Costs

We assume that for any outgoing message, the sender incurs a constant processing cost c^S.[3] Of course, when the message is sent, the ISP incurs a transmission cost, which we denote by c^U. This cost is incurred regardless of whether or not the receiver actually opens or reads the message.[4] In addition, when the message shows up in the receiver's inbox, the receiver incurs a processing cost c^R, regardless of whether or not she reads the message.[5] This processing cost is the cost incurred when the receiver chooses to either open or delete the message. Finally, if the message is read the receiver incurs an additional reading cost. Since the cost is incurred if and only if the benefit is also received, we include it in the receiver's benefit. That is to say, ρ is the receiver's gross benefit, less the cost of reading the message.

Notice that for every message that is sent, and regardless of whether it is read, there is a constant per message cost $C = c^U + c^S + c^R$. Since two components of this costs are not incurred by the sender, efficiency is problematic.

[3] This assumption is equivalent to that of Loder et. al. and effectively also to that of Hermalin and Katz who assume that the preference parameters are net of any cost associated with sending or opening and reading a message.

[4] This assumption differs from that made by the existing literature of telephone pricing and email pricing. As the cost of a telephone call is realized only if the call is answered and therefore the physical connection is made, it is sensible to assume that the network provider's cost requires both the caller and receiver to act. In email networks, however, a message is transmitted without the receiver's consent, and consumes bandwidth whether or not it is read. However, the literature on email pricing has not previously adopted our assumption. Hermalin and Katz (2004) assume a per message cost m which is incurred only if the sent message is accepted. Loder et. al. (2006) do not include an ISP cost. In perfect information equilibrium, however, both approaches are equivalent because in our model messages are sent only if the sender anticipates them to be read. However, if we introduce asymmetry of information, such as would be reasonable at least if some network participants were spammers who do not know which consumers will respond to their message, the assumption of the ISP cost being incurred regardless or whether or not the receiver reads the message becomes important. In fact, the ISP cost of unwanted spam is one of the major costs of spam.

[5] In contrast, Loder et. al. (and effectively Hermalin and Katz due to the cost being lumped up with the benefit of reading a message) assume that this cost is incurred only if the receiver reads the message. Thus, their assumption is very much in line with the current telephone technology but not with the email technology. This assumption will affect the main results of the model.

2.3 Information

We assume that the benefit density function $M(\sigma, \rho)$, the cost parameters (c^U, c^S, and c^R) and the prices that the ISP charges are common knowledge. We assume that the ISP observes any messages that are sent, the identity of the sender and the receiver, and whether the message is deleted or opened by the receiver, but it does not observe whether the message is or is not read.

3 Preliminary Analysis

In this section we construct the welfare of a message and total network welfare functions. We start by defining the total surplus of a message and deriving the function for first-best network welfare. We then introduce sender and receiver prices and argue that there are lower bounds on the set of feasible prices. We demonstrate the efficiency trade-off of using arbitrary prices (as opposed to zero prices). Last, we argue that first-best network welfare is not achievable through even perfectly discriminatory pricing due to the lower bounds on feasible prices and construct the second-best welfare subject to the constraints on prices. Later in Sections 4 and 5 we discuss three types of uniform prices (welfare-maximizing prices, Ramsey prices and monopoly prices) for general preference distributions and for uniform distributions. Last, in Section 6 we compare the network welfare of the three types of uniform prices to zero prices (status quo) as well as the second best network welfare assuming uniform preference distributions.

3.1 Total surplus of a message and first-best network welfare

The total surplus of a message that is sent and read is

$$\begin{aligned} ss(\sigma, \rho) &= \sigma + \rho - (c^S + c^R + c^U) \qquad (1) \\ &= \sigma + \rho - C \end{aligned}$$

Since the the realized benefits of both parties are 0 when a message is not read, the total surplus of a message that is sent and not read is $-(c^S + c^R + c^U)$. The total surplus of a message that is not sent is, of course, 0.

Cost-benefit optimality requires that a message be sent and read if and only if $ss(\sigma, \rho) \geq 0$, or if $\rho \geq C - \sigma$. It is, of course, never optimal for a message to be sent and not read.

Figure 1 illustrates a possible message space, given our assumptions of network preferences. Messages in the cross hatched area on and above the line $\rho \geq C - \sigma$ are efficient and those in the area below the line are inefficient.

Notice that in this illustration, there exist efficient messages such that either the sender's or the receiver's benefit is negative.[6] In the upper-left portion of the cross hatched area, where $\sigma < 0$, the sender's benefit is negative, and the in the lower-right portion of the crosshatched area, where $\rho < 0$, the receiver's benefit is negative.

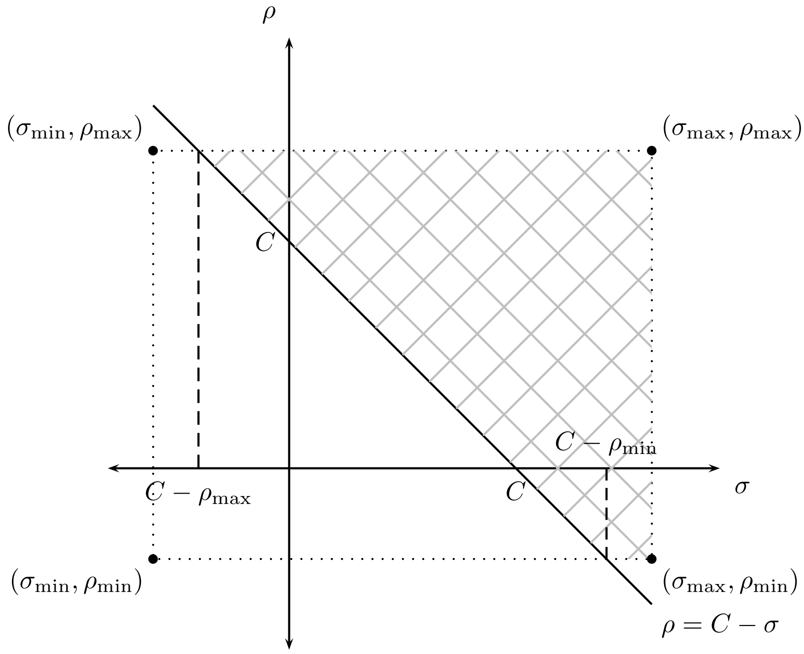

Figure 1: Possible message space. Welfare-improving messages form the cross-hatched area.

The first-best network welfare is given by

$$W_a^{***} = \int_{\sigma=C-\rho_{\max}}^{\sigma_{\max}} \int_{\rho=C-\sigma}^{\rho_{\max}} (\rho + \sigma - C) g(\rho) f(\sigma) d\rho d\sigma \tag{2}$$

if $\sigma_{\min} \leq C - \rho_{\max}$, $\rho_{\min} \leq C - \sigma_{\max}$, $\rho_{\max} > C$ and $\sigma_{\max} > C$ and by

$$\begin{aligned} W_b^{***} &= \int_{\sigma=\sigma_{\min}}^{C-\rho_{\min}} \int_{\rho=C-\sigma}^{\rho_{\max}} (\rho + \sigma - C) g(\rho) f(\sigma) d\rho d\sigma \\ &+ \int_{\sigma=C-\rho_{\min}}^{\sigma_{\max}} \int_{\rho=\rho_{\min}}^{\rho_{\max}} (\rho + \sigma - C) g(\rho) f(\sigma) d\rho d\sigma \end{aligned} \tag{3}$$

if $\sigma_{\min} > C - \rho_{\max}$, $\rho_{\min} > C - \sigma_{\max}$, $\rho_{\max} > C$ and $\sigma_{\max} > C$.[7]

3.2 Prices, choices and network welfare

Our main purpose is to examine the role that prices might play in a network environment. We consider both sender pays and receiver pays prices, which we denote by p^S and p^R. We

[6]This analysis is the same as in Loder et. al. (2006). However, in Hermalin and Katz (2004), the message space is restricted to the positive quadrant.

[7]There are clearly other possible constraint combinations that lead to a slightly different functional form for the first-best network welfare. In Figure 1, $\sigma_{\min} < C - \rho_{\max}$, $\rho_{\min} > C - \sigma_{\max}$, $\rho_{\max} \geq C$ and $\sigma_{\max} \geq C$ and the first-best welfare is $W_c^{***} = \int_{\sigma=C-\rho_{\max}}^{C-\rho_{\min}} \int_{\rho=C-\sigma}^{\rho_{\max}} (\rho + \sigma - C) g(\rho) f(\sigma) d\rho d\sigma + \int_{\sigma=C-\rho_{\min}}^{\sigma_{\max}} \int_{\rho=\rho_{\min}}^{\rho_{\max}} (\rho + \sigma - C) g(\rho) f(\sigma) d\rho d\sigma$.

assume that receiver pays prices are paid if and only if the receiver actually opens the message – to assume otherwise violates the spirit of voluntary exchange. Messages that are sent and read give the sender and receiver the following private surpluses:

$$s^S = \sigma - c^S - p^S \tag{4}$$

and

$$s^R = \rho - c^R - p^R.$$

When the receiver makes a decision to *open* a message at stage 2,[8] c^R is sunk and thus her surplus becomes

$$s^R = \rho - p^R. \tag{5}$$

When at stage 3 the receiver decides whether or not to *read* a message that was opened at stage 2, p^R is sunk as well and the relevant surplus is

$$s^R = \rho. \tag{6}$$

A message is opened *and* read only if both (5) and (6) are non-negative, that is if $\rho \geq \max[p^R, 0]$. Subsequently, negative prices cannot achieve what they are set out to do (that is, induce receivers to read unwanted messages) and therefore all efficient prices satisfy $p^R \geq 0$. Sender's choose to send a message if the surplus in (4) is non-negative, that is if $\sigma \geq c^S + p^S$, and if the sender anticipates the message to be read, that is if $\rho \geq \max(p^R, 0)$. The second condition assures that the potential benefit σ is realized.

Suppose now that the sender's benefit is negative ($\sigma < 0$). In this case, the necessary condition for the efficient sender price is $p^S < -c^S$. However, such a price creates a perverse incentive to manufacture and send phoney messages (that give the sender zero utility) as, in effect, a commercial activity. For this reason we restrict sender pays prices to the set $\{p^S | p^S \geq -c^S\}$.[9]

Given arbitrary uniform prices (p^S, p^R), messages in the following set will be exchanged

$$SR(p^S, p^R) \equiv \{(\sigma, \rho) | \sigma \geq c^S + p^S, \rho \geq \max(p^R, 0)\}. \tag{7}$$

The total network welfare with such prices is given by

$$W = \int_{\sigma = p^S + c^S}^{\sigma_{\max}} \int_{\rho = p^R}^{\rho_{\max}} (\rho + \sigma - C) g(\rho) f(\sigma) d\rho d\sigma, \tag{8}$$

[8] Remember that the ISP can observe if the message is opened but not if it is read, and thus p^R is charged if the message is opened.

[9] As discussed in the introduction, this assumption allows us not to have to worry about the incentive to manufacture phoney messages and thus to have a well-defined message preference space. Prices that satisfy $p^S < -c^S$ could not maximize welfare due to this incentive to create an infinite number of messages that generate at most zero value to the society, and thus this assumption that is made for convenience is not restrictive.

which after integrating by parts can be expressed as

$$\begin{aligned} W &= \left[\rho_{\max} - p^R G(p^R) - \int_{p^R}^{\rho_{\max}} G(\rho)d\rho\right]\left(1 - F\left(p^S + c^S\right)\right) \\ &\quad + \left[\sigma_{\max} - \left(p^S + c^S\right)F\left(p^S + c^S\right) - \int_{p^S+c^S}^{\sigma_{\max}} F(\sigma)d\sigma\right]\left(1 - G\left(p^R\right)\right) \\ &\quad - C\left(1 - F\left(p^S + c^S\right)\right)\left(1 - G\left(p^R\right)\right). \end{aligned} \tag{9}$$

The first line in (9) is the realized surplus of receivers. The first term in the first line is the expected receiver value of the messages that the receivers would willingly read. The second term scales for the fact that only a proportion of the messages are sent. The second line is the realized surplus of senders. The first term on the second line is the expected sender value of messages that the sender would willingly send if they were read, and the second term is the proportion of these messages that are actually read. The last line is the total cost of all messages sent and received. The cross-hatched area in Figure 2 illustrates the messages that are exchanged with arbitrary uniform prices that satisfy our constraints.

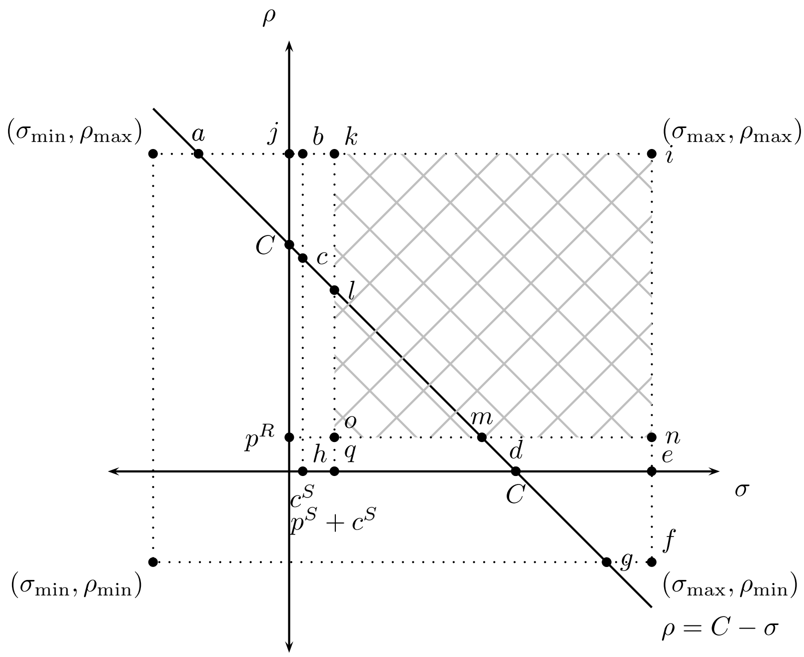

Figure 2: Messages that are sent and read given uniform prices p^S and p^R

The current situation where the uniform prices equal zero implies that the messages that are exchanged are clearly not the efficient set. Figure 3 illustrates the current situation. Messages in abc and in $defg$ are efficient but not exchanged if zero prices are used, while messages in cdh are inefficiently exchanged.

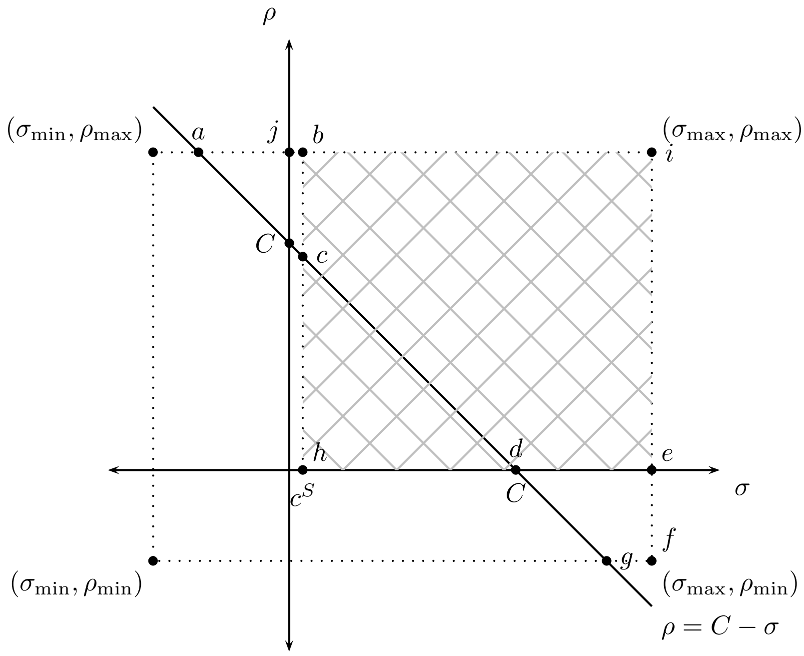

Figure 3: Messages that are sent and read given zero prices

The total welfare of zero pricing is given by

$$\begin{aligned} W^{zero} &= \left[\rho_{\max} - \int_0^{\rho_{\max}} G(\rho)d\rho\right]\left(1 - F\left(c^S\right)\right) \\ &\quad + \left[\sigma_{\max} - c^S F\left(c^S\right) - \int_{c^S}^{\sigma_{\max}} F(\sigma)d\sigma\right](1 - G(0)) \\ &\quad - C\left(1 - F\left(c^S\right)\right)(1 - G(0)). \end{aligned} \tag{10}$$

if $\sigma^{max} > c^S$ and it is zero otherwise. It is not obvious that we are doing as well as we can with the current pricing regime. The efficiency trade-off that arises with arbitrary uniform prices compared to zero uniform prices p^S and p^R is illustrated back in Figure 2. We can see that these prices improve efficiency by eliminating unwanted messages in *clomdqh* but the trade-off is that efficient messages in *bklc* and *mned* are no longer exchanged.

3.3 Feasible perfectly discriminatory pricing and second best network welfare

We have determined earlier that $p^R \geq 0$ because messages with $\rho < 0$ cannot be induced to be read with negative receiver prices and that $p^S \geq -c^S$ because otherwise there would be an incentive to generate phoney messages. Given these restrictions, it is not always possible to achieve efficient message exchange even if perfectly discriminatory prices are contemplated. Figure 4 illustrates the set of efficient message exchanges that could be achieved with perfectly discriminatory prices when these considerations are taken into account.

The welfare that prevails under perfectly discriminatory pricing, which we call second-best

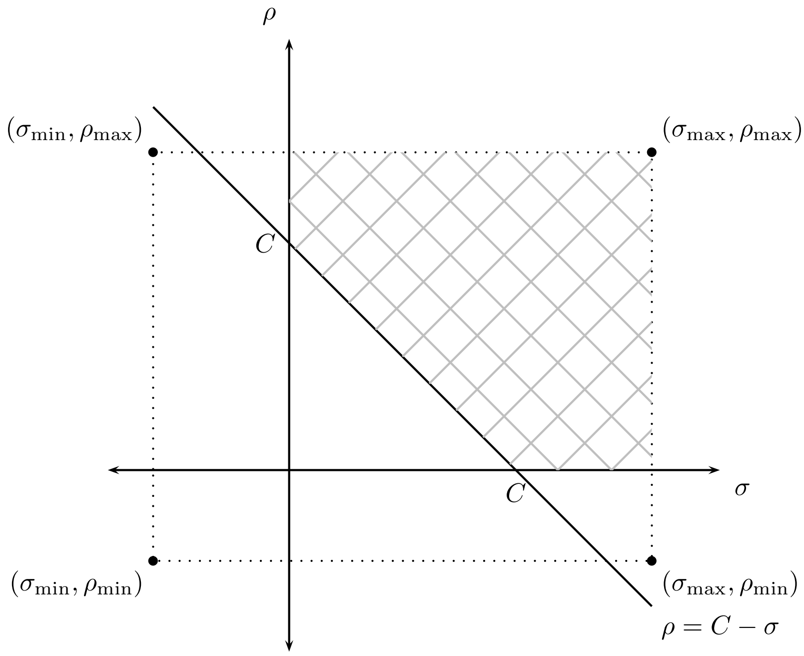

Figure 4: The set of efficient message exchanges that could be achieved with perfectly discriminatory prices

network welfare, is given by

$$\begin{aligned} W_a^{**} &= \int_{\sigma=0}^{C} \int_{\rho=C-\sigma}^{\rho_{\max}} (\rho+\sigma-C) g(\rho) f(\sigma) d\rho d\sigma \\ &+ \int_{\sigma=C}^{\sigma_{\max}} \int_{\rho=0}^{\rho_{\max}} (\rho+\sigma-C) g(\rho) f(\sigma) d\rho d\sigma \end{aligned} \tag{11}$$

if $\sigma_{\max} \geq C$ and $\rho_{\max} \geq C$. If $\sigma_{\max} < C$ and $\rho_{\max} < C$ but $\sigma_{\max} + \rho_{\max} \geq C$, this second best network welfare is given by

$$W_b^{**} = \int_{\sigma=C-\rho_{\max}}^{\sigma_{\max}} \int_{\rho=C-\sigma}^{\rho_{\max}} (\rho+\sigma-C) g(\rho) f(\sigma) d\rho d\sigma. \tag{12}$$

We can see that perfectly discriminatory pricing does better than no pricing in that it would stop the inefficient messages in cdh in Figure 3 from being exchanged as well as induce the efficient messages in $jbcC$ to be exchanged. However, the efficient messages in ajC and in $defg$ cannot be induced to be exchanged even with perfectly discriminatory pricing, given the nature of the message technology in email networks. This important aspect has not been noted in the literature before.

Furthermore, discriminatory pricing requires the ISP to know the utility of communication for both parties in every single potential message. Clearly, this pricing scheme requires too much information to be practical to implement. Uniform pricing, however, only requires the ISP to know the distribution of message preferences and not the preferences of each individual message, a much lighter information requirement than that for perfectly discriminatory prices.

In the following section we derive the conditions for three pairs of uniform prices - those that maximize total welfare, Ramsey prices and those that maximize profits in Sections 4.

4 Optimal uniform prices

In this section we will discuss three types of optimal uniform prices. Each of these price pairs is at best second best in that given these prices some welfare-reducing messages are exchanged and some welfare-enhancing messages are not exchanged.

First we derive and discuss the uniform prices that maximize network welfare. Our non-negativity constraints imply that the interior solution prices are only valid for some regions in the parameter space (roughly speaking relatively symmetric message preference distributions where the preference maxima are not too large) and that there are a number of possible corner solutions as well. These solutions are explored further for a specific uniform distribution in Section 5.1 In some instances (but not always, as we will show) the welfare-maximizing uniform prices lead to ISP losses being made. Therefore, as is the custom in the literature, we also derive the conditions for Ramsey prices, i.e. uniform prices that maximize total welfare subject to ISP break-even constraint. On top of the usual interior solution Ramsey prices our non-negativity constraints lead to us having several corner solutions again. Also, whenever unconstrained (in terms of the ISP break-even constraint) welfare-maximizing prices lead to positive ISP profits Ramsey prices equal them. We ignore both sets of constraints here and only derive the conditions for the interior solution Ramsey prices in this section, but will derive the full set of solutions when preferences are uniformly distributed in Section 5.2. Last, we derive the conditions for interior solution profit-maximizing prices for a monopoly ISP, and all the possible solutions are presented for uniform distribution in Section 5.3.

4.1 Cost-benefit optimal uniform sender and receiver prices

We first investigate the combination of uniform sender and receiver prices that maximize network welfare. These prices are, as discussed before, constrained: $p^S \geq -c^S$ and $p^R \geq 0$. The usual first order conditions for the maximization of (9) w.r.t. p^S and p^R yield the following expressions that we call best-response functions:[10]

$$p^S = C - \frac{\left(\rho_{\max} - p^R G\left(p^R\right) - \int_{p^R}^{\rho_{\max}} G(\rho) d\rho\right)}{1 - G\left(p^R\right)} - c^S \tag{13}$$

[10]These prices are not set by competing firms and therefore this is an unconventional use of the term.

and

$$p^R = C - \frac{\left(\sigma_{\max} - \left(p^S + c^S\right) F\left(p^S + c^S\right) - \int_{p^S + c^S}^{\sigma_{\max}} F(\sigma) d\sigma\right)}{1 - F\left(p^S + c^S\right)}. \tag{14}$$

These best response functions show that the sender's price is set to equal the total cost of the message minus the expected receiver value of the messages minus the sender's processing cost and that the receiver price is set to equal the total cost of messages minus the expected sender value of the messages. Notice the asymmetry in the two prices caused by the fact that the receiver's processing cost is sunk but the sender's processing cost is not.

Equations (13) and (14) jointly define the efficient prices when neither constraint ($p^S \geq -c^S$ and $p^R \geq 0$) on prices is binding.

Other solutions exist when one or both of these constraints bind. These are

$$p^S = -c^S \tag{15}$$

and

$$p^R = C - \frac{\left(\sigma_{\max} - \int_0^{\sigma_{\max}} F(\sigma) d\sigma\right)}{1 - F(0)} \tag{16}$$

when (13) violates its constraint,

$$p^S = C - \frac{\left(\rho_{\max} - \int_0^{\rho_{\max}} G(\rho) d\rho\right)}{1 - G(0)} - c^S \tag{17}$$

and

$$p^R = 0 \tag{18}$$

when (14) violates its constraint, and

$$p^S = -c^S \tag{19}$$

and

$$p^R = 0 \tag{20}$$

when both (13) and (14) violate their constraints. These optimal prices are explored in more detail for a specific uniform distribution in Section 5.

4.2 Ramsey prices

Consider now the welfare-maximizing prices subject to the ISP's break-even constraint, that is, the Ramsey prices. Most research in communication network pricing show that efficient prices do not cover the network provider's cost. We also show for the case of uniform distribution that the efficient prices sum up to less than the total cost of the message. However, as only a part of the total cost of a message is ISP cost, this inequality does not necessarily mean that the ISP would not break even. We will discuss this further in subsection 5.2 where we examine Ramsey prices for a uniform distribution.

Assume for now that the efficient prices do not cover all ISP costs. Given that the break-even constraint will be binding, the Ramsey price can be obtained by substituting the ISP's break-even constraint $p^R + p^S = c^U$ into the welfare in (9):

$$\begin{aligned} W^{Ramsey} &= \\ \left[\rho_{\max} - (c^U - p^S)G(c^U - p^S) - \int_{(c^U-p^S)}^{\rho_{\max}} G(\rho)d\rho\right] &\left(1 - F\left(p^S + c^S\right)\right) \\ + \left[\sigma_{\max} - \left(p^S + c^S\right) F\left(p^S + c^S\right) - \int_{p^S+c^S}^{\sigma_{\max}} F(\sigma)d\sigma\right] &\left(1 - G\left(c^U - p^S\right)\right) \\ - C\left(1 - F\left(p^S + c^S\right)\right)&\left(1 - G\left(c^U - p^S\right)\right). \end{aligned} \tag{21}$$

The FOC that implicitly defines the Ramsey price is:[11]

$$\begin{aligned} &(1 - F(p^S + c^S))g(c^U - p^S)(c^U - p^S) \\ &-f(p^S + c^S)\left[\rho_{\max} - (c^U - p^S)G(c^U - p^S) - \int_{(c^U-p^S)}^{\rho_{\max}} G(\rho)d\rho\right] \\ &-(1 - G(c^U - p^S))f(p^S + c^S)(p^S + c^S) \\ &+g(c^U - p^S)\left[\sigma_{\max} - \left(p^S + c^S\right) F\left(p^S + c^S\right) - \int_{p^S+c^S}^{\sigma_{\max}} F(\sigma)d\sigma\right] \\ &+C\left[(1 - G(c^U - p^S))f(c^S + p^S) - (1 - F(p^S + c^S))g(c^U - p^S)\right] = 0. \end{aligned} \tag{22}$$

Equation (22) can be interpreted as follows. Lines 1 and 2 represents the rate of change (w.r.t. increasing p^S) in the realized surplus of receivers. Line 1 is the rate of change in realized receiver surplus caused by the effective reduction in the receiver price (because the sum of the sender and the receiver price is constant) keeping the number of messages sent constant, and line 2 is the rate of change in receiver surplus caused by the reduction of the number of messages sent keeping the expected receiver surplus per message constant. Lines 3 and 4 represents the rate of change in the realized surplus of senders. Line 3 is the rate of change in realized surplus caused by the rising sender price keeping the number of messages read constant, and line 4 is the rate of change in consumer surplus of sent messages caused by the increase in the number of messages that are read due to the resulting reduction in receiver pays price. Line 5 represents the rate of change in the total cost of the exchanged messages caused by the change in the composition of messages that are both sent and read. The first term is the rate of change in cost caused by the reduction in the messages that are

[11] We are ignoring the constraints on prices in this section but will look at them closely when deriving Ramsey prices for a uniform distribution in subsection 5.2.

sent keeping the messages that are willingly read constant, and the second term is the rate of change in cost caused by the increase in the messages that are read keeping the messages that are sent constant.

Rearranging the FOC (22) yields the following:

$$\begin{aligned} & g(c^U - p^S)(1 - F(p^S + c^S)) \left(C - c^U + p^S - \frac{\sigma_{\max} - (p^S + c^S)F(p^S + c^S) - \int_{p^S+c^S}^{\sigma_{\max}} F(\sigma)d\sigma}{1 - F(p^S + c^S)} \right) \\ = \; & f(p^S + c^S)(1 - G(c^U - p^S)) \left(C - p^S - c^S - \frac{\rho_{\max} - (c^U - p^S)G(c^U - p^S) - \int_{c^U-p^S}^{\rho_{\max}} G(\rho)d\rho}{1 - G(c^U - p^S)} \right) \end{aligned} \tag{23}$$

Thus, the Ramsey sender price (and thus effectively the receiver price) is set such that the rate of change in total welfare caused by the increase in messages that are willingly read equals the rate of change in welfare caused by the reduction in messages that are willingly sent.

4.3 Monopoly prices

The profit of a monopolist ISP is given by

$$\begin{aligned} \pi(p^S, p^R) &= \int_{p^S+c^S}^{\sigma_{\max}} \int_{p^R}^{\rho_{\max}} (p^R + p^S - c^U) g(\rho) f(\sigma) d\rho d\sigma \\ &= [1 - G(p^R)][1 - F(p^S + c^S)](p^R + p^S - c^U). \end{aligned} \tag{24}$$

Maximizing (24) with respect to p^R and p^S yields the following FOCs [12]

$$f(p^S + c^S)(p^R + p^S - c^U) = [1 - F(p^S + c^S)]. \tag{25}$$

and

$$g(p^R)(p^R + p^S - c^U) = [1 - G(p^R)] \tag{26}$$

that jointly define profit-maximising prices for a single firm. As usual, the monopolist must balance the decreased quantity of messages with the increase in revenue on infra-marginal messages.

5 Optimal uniform prices when preferences are distributed uniformly

Let us now consider a specific distribution function for the message preferences. Assume that σ is distributed uniformly in $\sigma \in uni\,[\sigma_{\min}, \sigma_{\max}]$ with density equal to $\frac{1}{(\sigma_{\max} - \sigma_{\min})}$ and ρ is distributed uniformly in $\rho \in uni\,[\rho_{\min}, \rho_{\max}]$ with density equal to $\frac{1}{(\rho_{\max} - \rho_{\min})}$. Notice that

[12] We are again ignoring the constraints on prices but will look at them closely when deriving monopoly prices for a uniform distribution in subsection 5.3.

the density functions are independent but not necessarily identical. Uniform distribution exhibits everywhere increasing hazard rate, that is, is log-concave, and thus is a special case of a group of distributions considered by Hermalin and Katz (2004).

5.1 Cost-benefit optimal uniform prices when preferences are distributed uniformly

Assuming that $p^R \geq 0$ and $p^S \geq -c^S$, all the messages for which $\rho \in [p^R, \rho_{\max}]$ and $\sigma \in [p^S + c^S, \sigma_{\max}]$ will be send and read and the surplus of each such message is $\rho + \sigma - C$. The total surplus, obtained by substituting the specific density functions into (9), is

$$W = \frac{\left(\rho_{\max} - p^R\right)\left(\sigma_{\max} - \left(p^S + c^S\right)\right)}{2(\rho_{\max} - \rho_{\min})(\sigma_{\max} - \sigma_{\min})} \left(\rho_{\max} + p^R + \sigma_{\max} + p^S + c^S - 2C\right). \quad (27)$$

Maximising (27) w.r.t. $p^S + c^S$ and p^R subject to the constraints $0 \leq p^S + c^S \leq \sigma_{\max}$ and $0 \leq p^R \leq \rho_{\max}$ yields the following best-response functions:

$$p^S + c^S = C - \frac{\rho_{\max}}{2} - \frac{p^R}{2} \quad (28)$$

and

$$p^R = C - \frac{\sigma_{\max}}{2} - \frac{p^S + c^S}{2}. \quad (29)$$

There is an intuitive graphical interpretation to these best response functions. Equation (28) can be rewritten as $\rho_{\max} - C - (p^S + c^S) = C - (p^S + c^S) - p^R$ where the left-hand-side is the distance $k - l$ and the right-hand-side is the distance $l - o$ in Figure 2. Similarly, equation (29) can be rewritten as $\sigma_{\max} - C - p^R = C - p^R - (p^S + c^S)$ where the left-hand-side is distance $n - m$ and the right-hand-side is distance $m - o$ in Figure 2. Thus, the first-order conditions require that the prices are set such that at each margin, the number of messages that deteriorate welfare equals the number of messages that improve welfare. Any further increase in sender or receiver price would lead to the elimination of more "good" messages than of "bad" messages and thus overall welfare would go down. These conditions together with the assumption on independent distributions imply that the distribution of the exchanged messages will be a square when prices are chosen optimally and when the constraints for the prices are not violated. [13]

All the solutions to this pricing problem, the resulting welfare and the parameter restrictions for each solution are given in Table 5.1.

[13] The first order condition w.r.t. $p^S + c^S$ can also be expressed as $\frac{\rho_{\max} - C + (p^S + c^S)}{(\rho_{\max} - \rho_{\min})(\sigma_{\max} - \sigma_{\min})} = \frac{C - (p^S + c^S) - p^R}{(\rho_{\max} - \rho_{\min})(\sigma_{\max} - \sigma_{\min})}$ which more clearly shows the general graphical interpretation of the FOCs: the line integral of the message space from n to l must be equal to the line integral from l to m. Similar interpretation can be given to the first order condition w.r.t. p^R.

region	restrictions	p^{S*}	p^{R*}	W^*
a	$\max[-2C + 2\sigma_{\max}, C - \sigma_{\max}] \leq \rho_{\max} \leq C + \frac{\sigma_{\max}}{2}$	$\frac{2C}{3} - \frac{2\rho_{\max}}{3} + \frac{\sigma_{\max}}{3} - c^S$	$\frac{2C}{3} - \frac{2\sigma_{\max}}{3} + \frac{\rho_{\max}}{3}$	$\frac{4(\rho_{\max}+\sigma_{\max}-C)^3}{27(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$
b	$\sigma_{\max} < \min\left[-2C + 2\rho_{\max}, 2C\right]$	$-c^S$	$C - \frac{\sigma_{\max}}{2}$	$\frac{\sigma_{\max}(2\rho_{\max}+\sigma_{\max}-2C)^2}{8(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$
c	$\rho_{\max} < \min\left[-2C + 2\sigma_{\max}, 2C\right]$	$C - \frac{\rho_{\max}}{2} - c^S$	0	$\frac{\rho_{\max}(2\sigma_{\max}+\rho_{\max}-2C)^2}{8(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$
d	$\sigma_{\max} \geq 2C$ and $\rho_{\max} \geq 2C$	$-c^S$	0	$\frac{\rho_{\max}\sigma_{\max}(\sigma_{\max}+\rho_{\max}-2C)}{2(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$
e	$\rho^{\max} < C - \sigma_{\max}$	$> \sigma_{\max} - c^S$	$> \rho_{\max}$	0

Table 1: Different optimal pricing solutions, their parameter restrictions and resulting welfare

Figure 5 illustrates the different solutions as the maxima of the message preference distributions varies.

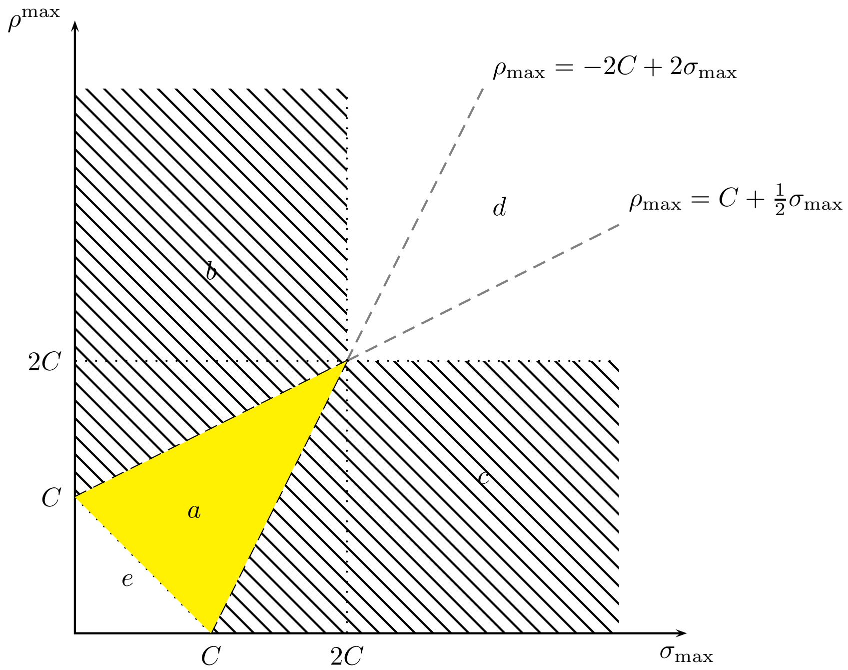

Figure 5: Parameter support for different solutions for welfare-maximizing prices.

When both $\sigma_{\max}$ and $\rho_{\max}$ are larger than $2C$ the exchange of messages should be encouraged by setting prices at their minima because, on average, the value of a message is greater than the cost of the message for both the sender and receiver. Setting prices above their minima would lead to a larger welfare loss due to eliminating "good" messages than a welfare gain due to eliminating "bad" messages. When both $\sigma_{\max}$ and $\rho_{\max}$ are small, there are more "bad" messages than "good" messages at minimum prices, and therefore higher prices increase welfare. When $\rho_{\max} \leq \min[-2C + 2\sigma_{\max}, 2C]$, receivers do not value messages very much compared to the cost of messages but senders do. Since a positive sender price eliminates messages with a small value to the sender and because the expected receiver value $\frac{\rho_{\max}}{2}$ is small in this case, a sender price increases welfare. Receiver price is not used because even if it would eliminate messages with small receiver value, these eliminated messages have a high expected value $\frac{\sigma_{\max}}{2}$ to the sender. Similarly, when $\sigma_{\max} \leq \min[-2C + 2\rho_{\max}, 2C]\rho_{\max}$, senders do not value messages very much compared to the cost of those messages but receivers do. Since a positive receiver price eliminates messages with a small value to the receiver and because the expected sender value $\frac{\sigma_{\max}}{2}$ is small in this case, a receiver price increases welfare. The sender price is set at its minimum because even if a higher price would eliminate messages with small sender value, these eliminated messages have a high expected value $\frac{\rho_{\max}}{2}$

to the receiver.

The two efficient prices sum up to

$$p^{S*} + p^{R*} = \frac{4c^U + c^S + 4c^R - \sigma_{\max} - \rho_{\max}}{3} \tag{30}$$

for the interior solution, and they cover the total cost of a message if

$$\rho_{\max} \leq C - 3c^S - \sigma_{\max}. \tag{31}$$

As the participation constraint of consumers is violated everywhere where (31) holds, the interior solution prices never cover the total cost of messages. It is trivial to see that the prices in the other solutions never cover the cost of the message, either.

Consider now identical distributions for message preferences, that is $\rho_{\max} = \sigma_{\max}$. The efficient prices are now the interior solution prices $p^{S*} = \frac{2C}{3} - \frac{\sigma_{\max}}{3} - c^S$ and $p^{R*} = \frac{2C}{3} - \frac{\sigma_{\max}}{3}$ if $\frac{C}{2} \leq \sigma_{\max} \leq 2C$, and the corner solution prices $p^{S*} = -c^S$ and $p^{R*} = 0$ if $\sigma_{\max} \geq 2C$. Thus, when the interior solution holds, the receiver price is positive and falling linearly in $\sigma_{\max}$ and when $\sigma_{\max} \geq 2C$ we get the corner solution price $p^R = 0$. The sender price is set below the receiver price by the amount of the sender's processing cost. The efficient prices sum up to $p^{S*} + p^{R*} = \frac{4C}{3} - \frac{2\sigma_{\max}}{3} - c^S$ for the interior solution and to $p^{S*} + p^{R*} = -c^S$ for the corner solutions. Given that $\sigma_{\max} \leq 2C$, the efficient prices sum up to less than the full cost of the message. However, it is not clear that they do not cover the ISP cost c^U. In fact, the ISP makes non-negative profit whenever $\sigma_{\max} < \frac{C+3c^R}{2}$. We will later examine Ramsey prices to show that the asymmetry result in efficient prices prevails once we impose a break-even constraint for the ISP.

Let us now determine how the efficient prices are affected when we make a distribution less symmetric. First, assume that $\max[-2C + 2\sigma_{\max}, C - \sigma_{\max}] \leq \rho_{\max} \leq C + \frac{\sigma_{\max}}{2}$, i.e. that we are inside area a in Figure 5 where the interior solution prices found on row a in Table 5.1 hold. Now let $\sigma_{\max}$ increase keeping $\rho_{\max}$ constant - that is, move to the right on a horizontal line from the starting point inside area a. This implies that the average sender value of a message increases but the average receiver value stays constant, and thus that we are stretching the uniform distribution to the right simultaneously reducing the density at any point. This change will increase the optimal sender price and reduce the optimal receiver price until $\sigma_{\max} = C + \frac{\rho_{\max}}{2}$ (the boundary between areas a and c) when the optimal sender price equals $p^{S*} + c^S = C - \frac{\rho_{\max}}{2}$ and the optimal receiver price equals zero. Any increases on $\sigma_{\max}$ thereafter have no impact on the optimal prices. Similarly, starting from an initial point inside area a, let $\rho_{\max}$ increase keeping $\sigma_{\max}$ constant - that is, move up a vertical line from the starting point inside area a. The optimal receiver price increases and the optimal sender price decreases until $\rho_{\max} = C + \frac{\sigma_{\max}}{2}$ when the optimal receiver price equals $p^{R*} = C - \frac{\sigma_{\max}}{2}$ and the optimal sender price equals $p^{S*} = -c^S$, and any increases thereafter have no impact on the optimal prices.

Last, as an example, assume that $\sigma_{\max} = 1.2C$ and $\rho_{\max} = 1.5C$. The optimal prices are now $((p^S + c^S)^*, p^{R*}) = (\frac{0.2C}{3}, \frac{1.1C}{3})$ and they are illustrated in Figure 6. The shaded area includes all sent messages of which the messages in the crosshatched area reduce total surplus. Notice that the prices are set such that the shaded area is a square. Also, for the messages at the left and bottom boundaries of the sent message space (where the senders are indifferent between sending and not sending a message and where the receivers are indifferent between reading and not reading a message, respectively), there is an equal number of good and bad messages. This graphical explanation holds for other distributions as well.

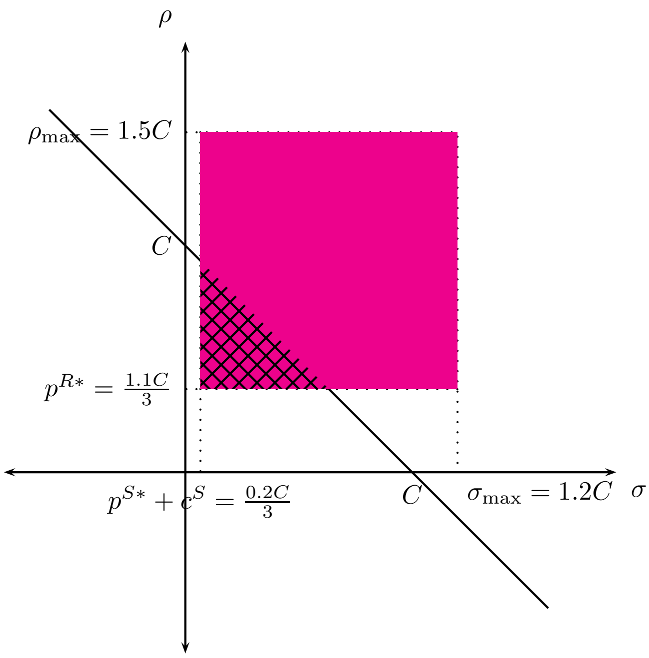

Figure 6: Efficient prices with uniform distribution when $\rho_{\max} = 1.5C$ and $\sigma_{\max} = 1.2C$

The graphical interpretation can also be used to explain why optimal prices are set to their minima in other cases. If the price that would set the number of good messages equal to the number of bad messages at either left or the lower boundary of the sent message space is below its lower bound, the optimal price is set equal to its minimum.

5.2 Ramsey prices when preferences are distributed uniformly

Standard Ramsey prices are found by substituting the ISP break-even constraint $p^R = c^U - p^S$ into network welfare in (27):

$$W^{Ramsey} = \frac{\sigma_{\max} + \rho_{\max} - c^R - C}{2(\rho_{\max} - \rho_{\min})(\sigma_{\max} - \sigma_{\min})} \left((\rho_{\max} - c^U + p^S)(\sigma_{\max} - p^S - c^S) \right) \quad (32)$$

and then maximizing w.r.t. p^S subject to our minima constraints for prices. The Ramsey pricing solutions are presented in Table 5.2.

All the solutions in Table 5.2 are illustrated in Figure 7. The standard Ramsey prices (area f) hold only when $\sigma_{\max}$ and $\rho_{\max}$ are sufficiently symmetric and large. The ISP break-even constraint is binding but prices are constrained because the standard Ramsey prices are too negative in areas g and h. The unconstrained welfare-maximizing prices generate sufficient income to the ISP in area a_R where $\sigma_{\max}$ and $\rho_{\max}$ are relatively symmetric but small. Last, in areas b_R and c_R where tastes are very asymmetric and one of the maxima is small, the corner solution welfare-maximizing prices generate non-negative profit for the ISP.[14]

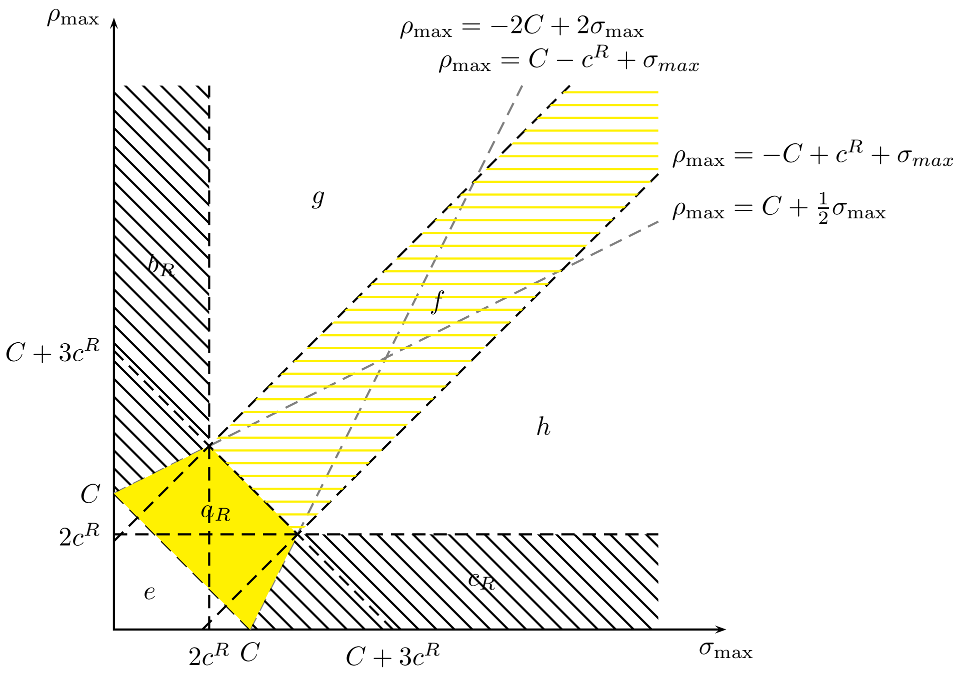

Figure 7: Parameter support for optimal Ramsey prices

If the uniform distributions are identical, such that $\rho_{\max} = \sigma_{\max}$, then the interior solution Ramsey prices in area f in Figure 7 satisfy $p^S_{Ramsey} = \frac{c^U - c^S}{2} < \frac{c^U + c^S}{2} = p^R_{Ramsey}$ and

[14] Note: Areas a_R, b_R and c_R are subsections of areas a, b and c in Figure 5, and area e is the same as in Figure 5.

region	restrictions	$p^{S,Ramsey}$	$p^{R,Ramsey}$	W^{Ramsey}
a_R	$\max[-2C+2\sigma_{\max}, C-\sigma_{\max}] \leq \min[\rho_{\max} \leq C+\frac{\sigma_{\max}}{2}, C+3c^R-\sigma_{\max}]$	$\frac{2C}{3} - \frac{2\rho_{\max}}{3} + \frac{\sigma_{\max}}{3} - c^S$	$\frac{2C}{3} - \frac{2\sigma_{\max}}{3} + \frac{\rho_{\max}}{3}$	$\frac{4(\rho_{\max}+\sigma_{\max}-C)^3}{27(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$
b_R	$\sigma_{\max} < \min\left[-2C+2\rho_{\max}, 2c^R\right]$	$-c^S$	$C - \frac{\sigma_{\max}}{2}$	$\frac{\sigma_{\max}(2\rho_{\max}+\sigma_{\max}-2C)^2}{8(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$
c_R	$\rho_{\max} < \min\left[-2C+2\sigma_{\max}, 2c^R\right]$	$C - \frac{\rho_{\max}}{2} - c^S$	0	$\frac{\rho_{\max}(2\sigma_{\max}+\rho_{\max}-2C)^2}{8(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$
e	$\rho^{\max} < C - \sigma_{\max}$	$> \sigma_{\max} - c^S$	$> \rho_{\max}$	0
f	$\max[C+3c^R-\sigma_{\max}, -C+c^R+\sigma_{\max}] \leq \rho_{\max} \leq C-c^R+\sigma_{\max}$	$\frac{c^U+\sigma_{\max}-\rho_{\max}}{2} - \frac{c^S}{2}$	$\frac{c^U+\rho_{\max}-\sigma_{\max}}{2} + \frac{c^S}{2}$	$\frac{(\sigma_{\max}+\rho_{\max}-C+c^R)^2(\sigma_{\max}+\rho_{\max}-C-c^R)}{8(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$
g	$2c^R < \rho_{\max} < -C+c^R+\sigma_{\max}$	$-c^S$	$c^U + c^S$	$\frac{\sigma_{\max}(\rho_{\max}-c^U-c^S)(\sigma_{\max}+\rho_{\max}-2C+c^S+c^U)}{2(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$
h	$2c^R < \sigma_{\max} < -C+c^R+\rho_{\max}$	c^U	0	$\frac{\rho_{\max}(\sigma_{\max}-c^U-c^S)(\sigma_{\max}+\rho_{\max}-2C+c^S+c^U)}{2(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$

Table 2: Ramsey pricing solutions, their parameter restrictions and resulting welfare

it is clear that the Ramsey prices are asymmetric such that the receivers pay more than the senders even with perfectly symmetric message preference distributions. Furthermore, increasing $\sigma_{\max} = \rho_{\max}$ does not affect the level of the interior solution Ramsey prices.

5.3 Monopoly prices when preferences are distributed uniformly

Given our uniform distributions, the ISP profit in (24) can be written as

$$\pi(p^S, p^R) = \frac{(\sigma_{\max} - (p^S + c^S))(\rho_{\max} - p^R)(p^R + p^S - c^U)}{(\sigma_{\max} - \sigma_{\min})(\rho_{\max} - \rho_{\min})}. \tag{33}$$

The different pricing solutions are given in Table 5.3.

Figure 8 shows the regions in the parameter space where the different solutions hold. The interior solution is highlighted in yellow.

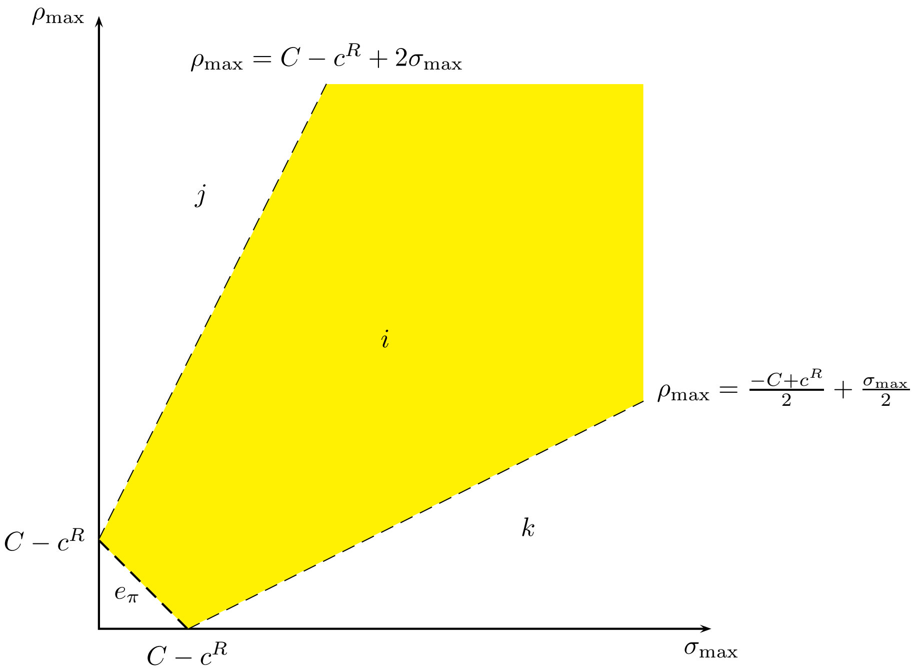

Figure 8: Monopoly prices as functions of $\sigma_{\max}$ and ρ^{max}.

Intuitively, if one maximum in the preference distribution space is very large while the other one is small, it would make sense to subsidize the consumers with small utility to encourage them to send or open messages to allow the consumers with large utility to exchange more messages. However, as feasible prices have a lower bound, this can only be done within the bounds of feasible prices.

As with cost-benefit optimal prices and Ramsey prices, if the message preference distributions are perfectly symmetric ($\sigma^{max} = \rho^{max}$) the uniform monopoly prices are asymmetric

region	restrictions	$p^{S,\pi}$	$p^{R,\pi}$	W^{π}	π
i	$\max[\frac{-C+c^R}{2}+\frac{\sigma_{\max}}{2}, C-c^R] \leq \rho_{\max} \leq C-c^R+2\sigma_{\max}$	$\frac{2\sigma_{\max}-\rho_{\max}}{3}+\frac{c^U-2c^S}{3}$	$\frac{2\rho_{\max}-\sigma_{\max}}{3}+\frac{c^U+c^S}{3}$	$\frac{(\rho_{\max}+\sigma_{\max}-c^S-c^U)^2(2\rho_{\max}+2\sigma_{\max}-3C+c^S+c^U)}{27(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$	$\frac{(\rho_{\max}+\sigma_{\max}-c^U-c^S)^3}{27(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$
j	$\rho_{\max} < C-c^R+s\sigma_{\max}$	$-c^S$	$\frac{\rho_{\max}}{2}+\frac{c^U+c^S}{2}$	$\frac{\sigma_{\max}(\rho_{\max}-c^S-c^U)(2\sigma_{\max}+3\rho_{\max}-4C+c^S+c^U)}{8(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$	$\frac{\sigma_{\max}(\rho_{\max}-c^S-c^U)^2}{4(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$
k	$\rho_{\max} < \frac{-C+c^R}{2}+\frac{\sigma \max}{2}$	$\frac{\sigma_{\max}}{2}+\frac{c^U-c^S}{2}$	0	$\frac{\rho_{\max}(\sigma_{\max}-c^S-c^U)(2\rho_{\max}+3\sigma_{\max}-4C+c^S+c^U)}{8(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$	$\frac{\rho_{\max}(\sigma_{\max}-c^S-c^U)^2}{4(\rho_{\max}-\rho_{\min})(\sigma_{\max}-\sigma_{\min})}$
e_{π}	$\rho^{\max} < C-c^R\sigma_{\max}$	$> \sigma_{\max}-c^S$	$> \rho_{\max}$	0	0

Table 3: Monopoly prices, their parameter restrictions and resulting welfare and profit

such that the sender pays less than the receiver by an amount equal to the sender's processing cost: $p^{S*} = \frac{\sigma_{\max}}{3} + \frac{c^U - 2c^S}{3} < \frac{\sigma_{\max}}{3} + \frac{c^U + c^S}{3} = p^{R*}$. However, while the sum of the Ramsey prices equate by construction to the ISP cost of the message, the monopoly prices are larger. Furthermore, monopoly prices increase in $\sigma_{\max} = \rho_{\max}$ without bound whereas the cost-benefit optimal prices and Ramsey prices decrease in $\sigma_{\max} = \rho_{\max}$ first and then become constant.

Figure 9 combines the constraints of uniform total welfare-maximizing, Ramsey and monopoly pricing. In an attempt to keep the Figure as free of clutter as possible, we have not identified the regions where various pricing solutions hold, so this figure needs to be read in conjunction with Figures 5, 7 and 8.

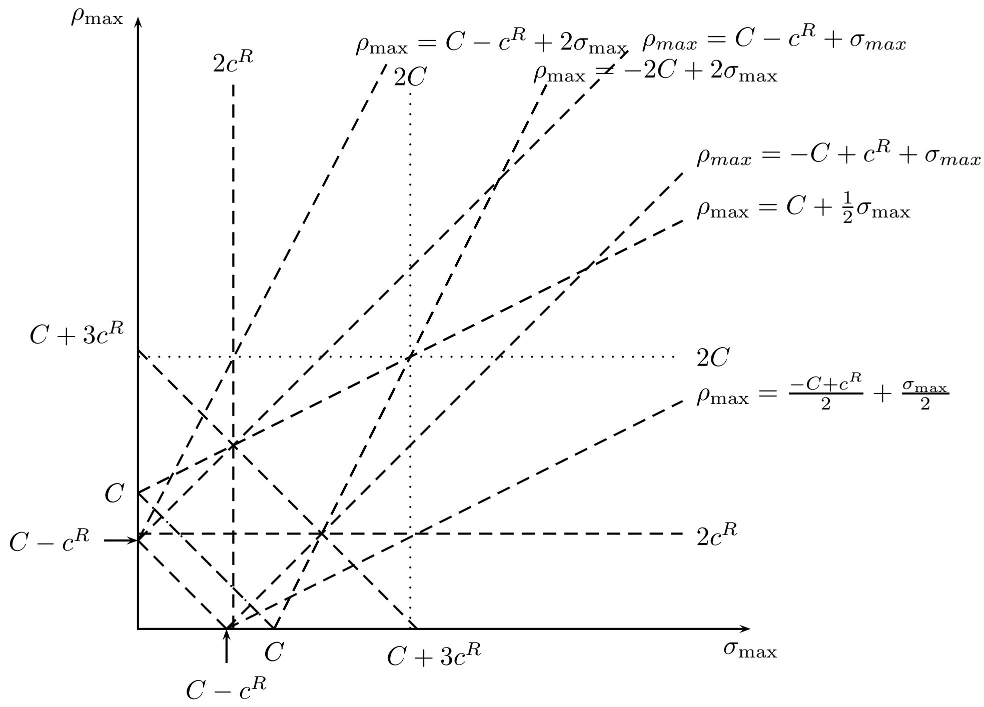

Figure 9: Parameter support for the various solutions for the three types of prices.

6 Welfare comparison of the results for uniform distribution

In this section we explore how the different price types, and resulting welfare, compare with each other and with the status quo (zero prices). For the welfare analysis we compare the ratio of the actual welfare resulting from a particular pricing strategy to the best achievable welfare in the network, that is the second best welfare measure that results from using perfectly discriminatory prices that satisfy the constraints $p^R > 0$ and $p^S > -c^S$. In part the aim of this section is to provide a framework for evaluating whether or not the current regime of using zero prices is likely creating significant welfare losses.

6.1 Symmetric message distributions

Here we look at symmetric distributions where $\sigma_{\max} = \rho_{\max}$ and $\sigma_{\min} = \rho_{\min}$. Given these restrictions and our uniform distribution, we can express the second best welfare in (11) and (12), achievable through perfectly discriminatory prices which are limited to be $p^S \geq -c^S$ and $p^R \geq 0$, by

$$W_1^{**} = \frac{C^3 - 6C\sigma_{\max}^2 + 6\sigma_{\max}^3}{6(\sigma_{\max} - \sigma_{\min})^2} \tag{34}$$

if $\sigma_{\max} \geq C$ and by

$$W_2^{**} = \frac{(2\sigma_{\max} - C)^3}{6(\sigma_{\max} - \sigma_{\min})^2} \tag{35}$$

if $\sigma_{\max} < C$.

The optimal uniform prices are $p_a^{S*} = \frac{2C-\sigma_{\max}}{3} - c^S$ and $p_a^{R*} = \frac{2C-\sigma_{\max}}{3}$ if $\frac{C}{2} \leq \sigma_{\max} \leq 2C$ and $p_d^{S*} = -c^S$ and $p_d^{R*} = 0$ if $\sigma_{\max} > 2C$. The corresponding welfares are $W_a^* = \frac{4(2\sigma_{\max}-C)^3}{27(\sigma_{\max}-\sigma_{\min})^2}$ and $W_d^* = \frac{(\sigma_{\max}-C)\sigma_{\max}^2}{(\sigma_{\max}-\sigma_{\min})^2}$ (from rows a and d in Table 5.1, respectively).

Ramsey pricing involves using the unconstrained optimal uniform prices $p_{a_R}^{S*} = \frac{2C-\sigma_{\max}}{3} - c^S$ and $p_{a_R}^{R*} = \frac{2C-\sigma_{\max}}{3}$ if $\frac{C}{2} \leq \sigma_{max} \leq \frac{C+3c^R}{2}$, and the Ramsey prices $p_f^{S,Ramsey} = \frac{c^U-c^S}{2}$ and $p_f^{R,Ramsey} = \frac{c^U+c^S}{2}$ if $\sigma_{max} > \frac{C+3c^R}{2}$. The corresponding welfares are $W_a^* = \frac{4(2\sigma_{\max}-C)^3}{27(\sigma_{\max}-\sigma_{\min})^2}$ and $W_f^{Ramsey} = \frac{(2\sigma_{\max}-c^U-c^S)^2(2\sigma_{\max}-C-c^R)}{(\sigma_{\max}-\sigma_{\min})^2}$ (from rows a_R and f in Table 5.2, respectively).

The uniform monopoly prices are $p_i^{S,\pi} = \frac{\sigma_{\max}+c^U-2c^S}{3}$ and $p_i^{R,\pi} = \frac{\sigma_{\max}+c^U+c^S}{3}$, which generates welfare equal to $W_i^{\pi} = \frac{(2\sigma_{\max}-c^U-c^S)^2(4\sigma_{\max}-2C-c^R)}{27(\sigma_{\max}-\sigma_{\min})^2}$ (from row i in Table 5.3.

Last, the welfare with zero prices is

$$W^{zero} = \frac{\sigma_{\max}(\sigma_{\max} - c^S)(2\sigma_{\max} - 2C + c^S)}{2(\sigma_{\max} - \sigma_{\min})^2}. \tag{36}$$

Figure 10 shows the three types of price pairs as a function of $\sigma_{max} = \rho_{\max}$. Figure 11 plots the welfare functions from optimal uniform pricing, Ramsey pricing and uniform monopoly pricing as ratios to the second best welfare when $\sigma_{max} = \rho_{\max}$ varies. Both figures assume the same set of cost parameters. Figures 12 and 13 show how the welfare comparisons change when the relative sizes of the cost parameters change. Figures 14 and 15 show the three types of prices and welfare comparisons for a more asymmetric taste structure.

It is evident that optimal uniform prices do fairly well in mimicking second best pricing. For the first set of parameters, these prices achieve at least 89 % of the potential maximum welfare, and this ratio approaches 100% as $\sigma_{\max}$ increases. Ramsey prices are identical to the optimal uniform prices when the break-even constraint is not binding. When the break-even constraint is binding, for $\sigma_{\max} > \frac{C+3c^R}{2}$, the welfare generated using Ramsey prices suffers in relation to the difference in c^S and c^U. When both are small, as is likely to be the case for email networks, there is not a substantial difference between the unconstrained and constrained total welfare-maximizing prices.

The comparison of the welfare generated by uniform monopoly pricing to other pricing regimes is more fascinating. First, when $\sigma_{\max}$ is relatively small (but greater than $\frac{C}{2}$) the monopolist charges small prices and is able to make positive profits even though it is totally optimal to discourage all email activity. As $\sigma_{\max}$ increases in the range $\frac{C}{2} < \sigma_{\max} < \frac{C+c^R}{2}$ the monopoly prices are increasing in magnitude and approaching the optimal and Ramsey prices so there is convergence in these welfare ratios. Uniform monopoly prices are the same as the uniform optimal prices when $\sigma_{\max} = \frac{C+c^R}{2}$. For $\sigma_{\max} > \frac{C+c^R}{2}$, monopoly prices are large compared to uniform optimal prices and so the welfare under monopoly prices falls away again from that generated by other pricing strategies. Thus, a monopoly ISP does rather well when the maxima in the preference distributions are close to $\frac{C+c^R}{2}$ but poorly otherwise and, importantly, worse than zero pricing for $\sigma_{\max} > \frac{C+3c^R}{2}$.

The other interesting comparison is zero pricing (status quo) to second best. When $\sigma_{\max}$ is very small, zero prices result in negative total welfare. As $\sigma_{\max}$ increases the optimal uniform sender and receiver prices approach $-c^S$ and 0 respectively and so the welfare loss associated with zero prices becomes small.

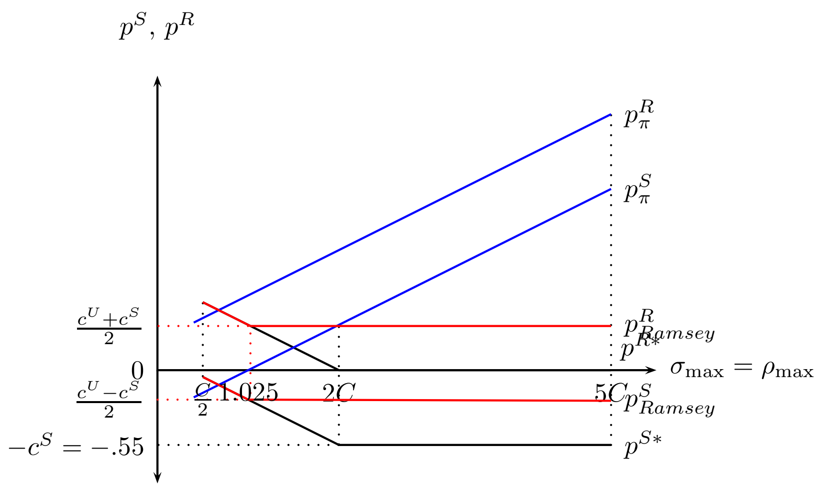

Figure 10: Optimal uniform, Ramsey and monopoly prices as functions of $\sigma_{\max} = \rho_{\max}$. $c^U = 0.1,\ c^S = .55,\ c^R = .35$ $(C = 1)$ and $\sigma_{\min} = \rho_{\min} = -2$

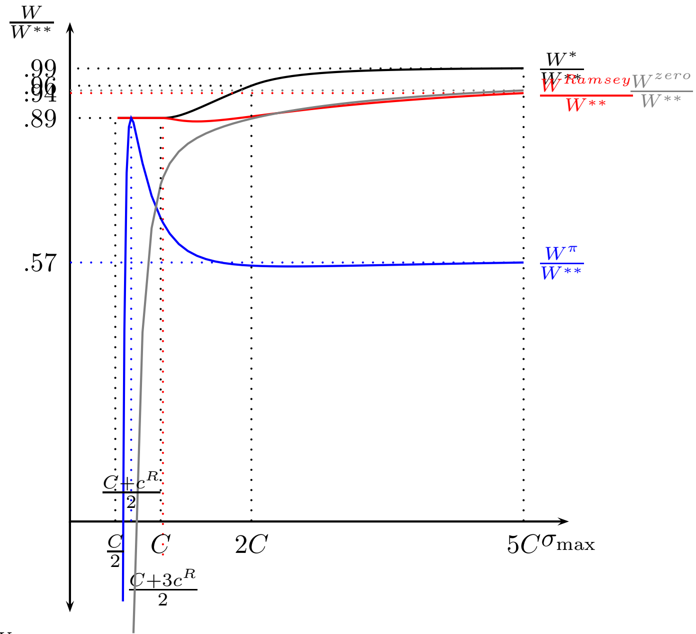

Figure 11: $\frac{W}{W^{**}}$ as a function of $\sigma_{\max} = \rho_{\max}$ for optimal uniform, Ramsey, monopoly and zero prices. $c^U = 0.1,\ c^S = .55,\ c^R = .35$ $(C = 1)$ and $\sigma_{\min} = \rho_{\min} = -2$

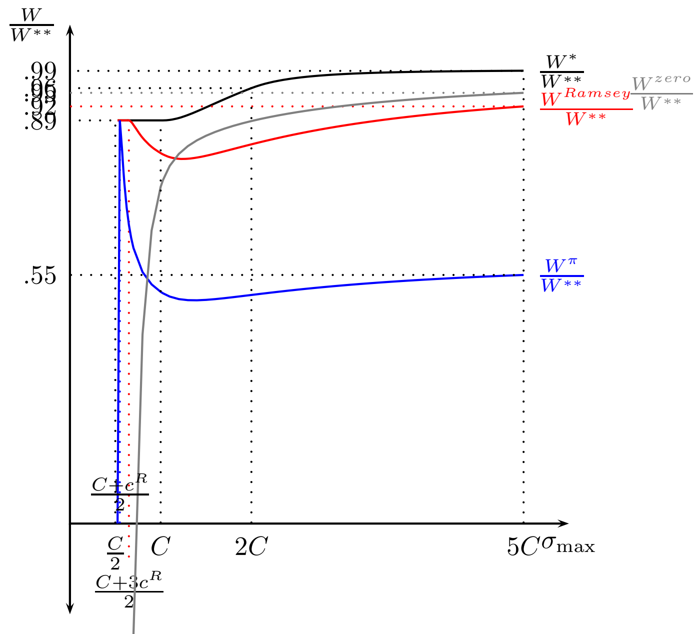

Figure 12: $\frac{W}{W^{**}}$ as a function of $\sigma_{\max} = \rho_{\max}$ for optimal uniform, Ramsey, monopoly and zero prices. $c^U = 0.35,\ c^S = .55,\ c^R = .1$ $(C = 1)$ and $\sigma_{\min} = \rho_{\min} = -2$

6.2 Asymmetric message distributions

Now we look at distributions where senders get more utility for messages than receivers do. Specifically, assume that $\sigma_{\max} = 2\rho_{\max}$ and $\sigma_{\min} = 2\rho_{\min}$. Given these assumptions and our uniform distribution, we can express the second best welfare in (11) and (12), achievable

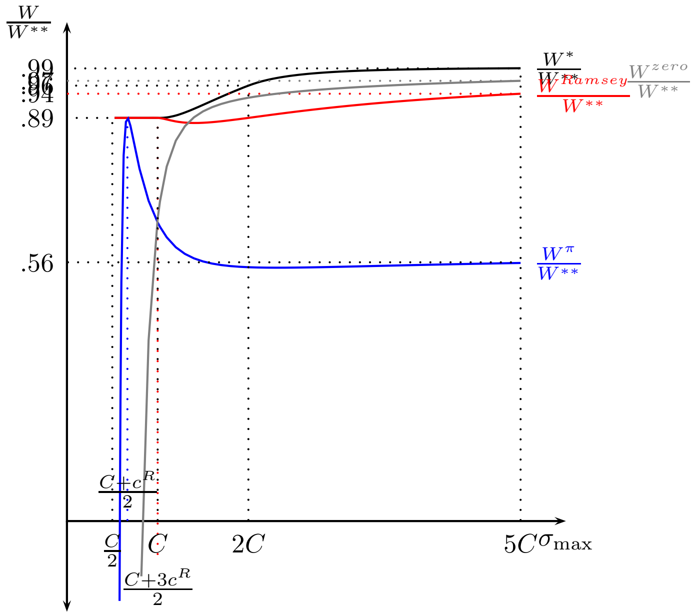

Figure 13: $\frac{W}{W^{**}}$ as a function of $\sigma_{\max} = \rho_{\max}$ for optimal uniform, Ramsey, monopoly and zero prices. $c^U = 0.35$, $c^S = .3$, $c^R = .35$ ($C = 1$) and $\sigma_{\min} = \rho_{\min} = -2$

through perfectly discriminatory prices which are limited to be $p^S \geq -c^S$ and $p^R \geq 0$, by

$$W_1^{**} = \frac{4C^3 - 12C\sigma_{\max}^2 + 9\sigma_{\max}^3}{12(\sigma_{\max} - \sigma_{\min})^2} \tag{37}$$

if $\sigma_{\max} \geq 2C$, by

$$W_2^{**} = \frac{\sigma_{\max}(12C^2 - 30C\sigma_{\max} + 19\sigma_{\max}^2)}{24(\sigma_{\max} - \sigma_{\min})^2} \tag{38}$$

if $C < \sigma_{\max} < 2C$, and by

$$W_3^{**} = \frac{(3\sigma_{\max} - 2C)^3}{24(\sigma_{\max} - \sigma_{\min})^2} \tag{39}$$

if $\frac{2C}{3} \leq \sigma_{\max} \leq C$.

The optimal uniform prices are now $p_a^{S*} = \frac{2C}{3} - c^S$ and $p_a^{R*} = \frac{2C}{3} - \frac{\sigma_{\max}}{2}$ if $\frac{2C}{3} \leq \sigma_{\max} \leq \frac{4C}{3}$, $p_c^{S*} = C - \frac{\sigma_{\max}}{4} - c^S$ and $p_c^{R*} = 0$ if $\frac{4C}{3} < \sigma_{\max} < 4C$, and $p_d^{S*} = -c^S$ and $p_d^{R*} = 0$ if $\sigma_{\max} \geq 4C$. The corresponding welfares are $W_a^* = \frac{(3\sigma_{\max}-2C)^3}{27(\sigma_{\max}-\sigma_{\min})^2}$, $W_c^* = \frac{\sigma_{\max}(5\sigma_{\max}-4C)^2}{32(\sigma_{\max}-\sigma_{\min})^2}$ and $W_d^* = \frac{\sigma_{\max}^2(3\sigma_{\max}-4C)^2}{4(\sigma_{\max}-\sigma_{\min})^2}$ (from rows a, c and d in Table 5.1, respectively).

Ramsey pricing involves the unconstrained optimal uniform prices $p_a^{S*} = \frac{2C}{3} - c^S$ and $p_a^{R*} = \frac{2C}{3} - \frac{\sigma_{\max}}{2}$ if $\frac{2C}{3} \leq \sigma_{\max} \leq \frac{2C}{3} + 2c^R$, the interior solution Ramsey prices $p_f^{S,Ramsey} = \frac{\sigma_{\max}+2c^U-2c^S}{4}$ and $p_f^{R,Ramsey} = \frac{2c^U+2c^S-\sigma_{\max}}{4}$ if $\frac{2C}{3} + 2c^R < \sigma_{\max} < 2C - 2c^R$, and corner solution Ramsey prices $p_h^{S,Ramsey} = c^U$ and $p_h^{R,Ramsey} = 0$ if $\sigma_{max} > 2C - 2c^R$. The corresponding welfares are $W_a^* = \frac{(3\sigma_{\max}-2C)^3}{27(\sigma_{\max}-\sigma_{\min})^2}$, $W_f^{Ramsey} = \frac{(3\sigma_{\max}-2C+2c^R)^2(3\sigma_{\max}-2C-2c^R)^2}{16(\sigma_{\max}-\sigma_{\min})^2}$ and $W_h^{Ramsey} = \frac{\sigma_{\max}(\sigma_{\max}-c^U-c^S)(3\sigma_{\max}-4C+2c^S+2c^U)}{4(\sigma_{\max}-\sigma_{\min})^2}$. (from rows a, f and h in Table 5.2, respectively).

The uniform monopoly prices are $p_i^{S,\pi} = \frac{3\sigma_{\max}+2c^U-4c^S}{6}$ and $p_i^{R,\pi} = \frac{c^U+c^S}{3}$ generating welfare $W_i^\pi = \frac{(3\sigma_{\max}-2C+2c^R)^2(3\sigma_{\max}-2C-c^R)}{54(\sigma_{\max}-\sigma_{\min})^2}$ (from row i in Table 5.3.

Last, the welfare with zero prices is

$$W^{zero} = \frac{\sigma_{\max}(\sigma_{\max} - c^S)(3\sigma_{\max} - 4C + 2c^S)}{4(\sigma_{\max} - \sigma_{\min})^2}. \tag{40}$$

Figure 14 plots the three types of prices and Figure 15 plots the welfare ratios as functions of $\sigma_{\max} = 2\rho_{\max}$ for a given set of cost parameters. It is evident that total welfare maximizing uniform prices do fairly well in mimicking perfectly discriminatory pricing. For the first set of parameters, these prices are achieve at least 89 % of the achievable welfare, and this ratio approaches 100 % as $\sigma_{\max}$ increases. Ramsey prices are the total welfare maximizing prices for $\sigma_{\max} \leq \frac{C+3c^R}{2}$, but beyond that point do somewhat worse due to the break-even constraint becoming binding. However, as the total welfare maximizing prices sum up to $-c^S$ when $\sigma_{\max} \geq 2C$, and because Ramsey prices always sum up to c^U, both of which are likely to be small, there is not a great amount of difference between the unconstrained and constrained total welfare-maximizing prices.

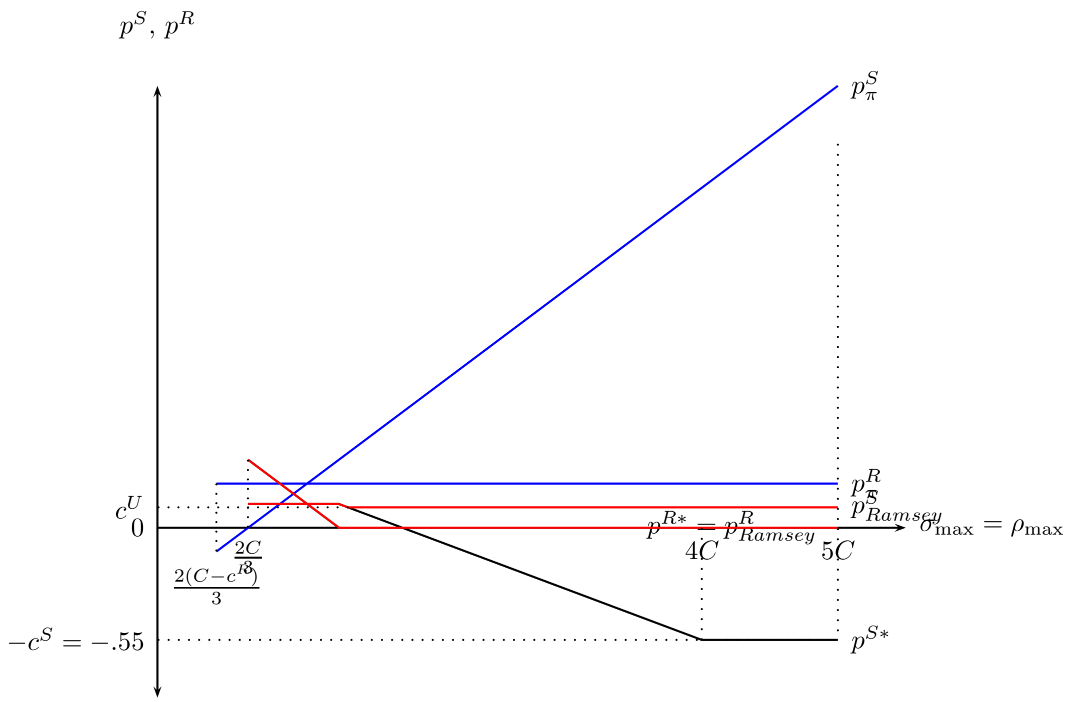

Figure 14: Optimal uniform, Ramsey and monopoly prices as functions of $\sigma_{\max} = 2\rho_{\max}$. $c^U = 0.1,\ c^S = .55,\ c^R = .35\ (C = 1)$ and $\sigma_{\min} = \rho_{\min} = -2$

The comparison of the welfare generated by uniform monopoly pricing to second best total welfare is more interesting. First, when $\sigma_{\max}$ is small, the ratio is negative reflecting the fact that the ISP chooses to operate when total welfare is negative. The prices the ISP charges are smaller than the total welfare maximizing prices for $\sigma_{\max} \leq \frac{C+c^R}{2}$. In that region, as $\sigma_{\max}$ increases, the total welfare maximizing prices fall and the monopoly prices rise leading to monopoly welfare doing relatively well compared to uniform total welfare-maximization at first. The monopoly prices equal the welfare-maximizing prices at $\sigma_{\max} = \frac{C+c^R}{2}$, and beyond that the monopoly prices surpass the total welfare-maximizing prices leading to the welfare ratio to plunge until it settles at around %56 when $\sigma_{\max} = 5C$. Thus, a monopoly ISP does rather well when the maxima in the preference distributions are small, but quite poorly when the maxima are large.

The other interesting comparison is zero pricing (status quo) to second best. When $\sigma_{\max}$ is small, zero prices result in many total welfare reducing messages being exchanged leading to negative total welfare. As $\sigma_{\max}$ increases, however, the total welfare maximizing uniform prices fall and get closer to the status quo, zero prices. Thus, zero prices do reasonably well when $\sigma_{\max}$ is large but not so well when it is small.

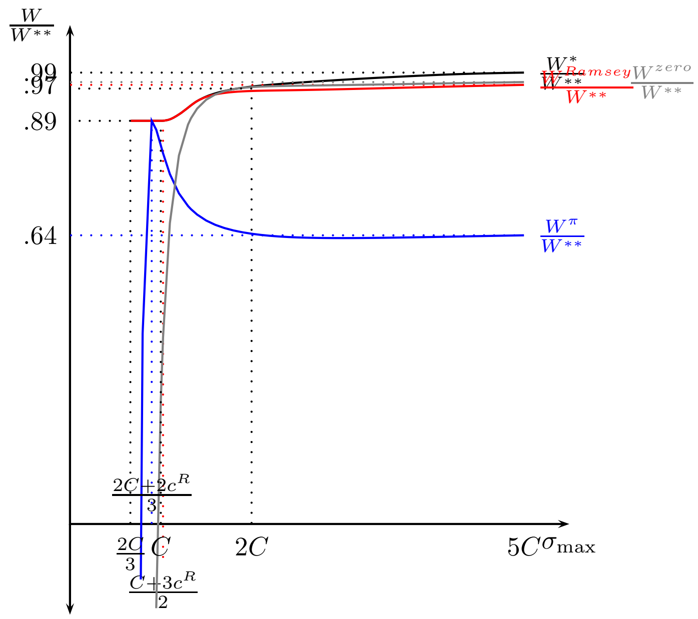

Figure 15: $\frac{W}{W^{**}}$ as a function of $\sigma_{\max} = 2\rho_{\max}$ for uniform, Ramsey, monopoly and zero prices. $c^U = 0.1$, $c^S = .55$, $c^R = .35$ $(C = 1)$ and $\sigma_{\min} = 2\rho_{\min} = -2$

7 Conclusions

In this paper we have shown how the nature of email technology affects the level of efficient and monopoly prices as well as the optimal mix between sender and receiver pricing. The first-best outcome may not be achievable even with perfectly discriminatory prices because consumers cannot be induced to read efficient messages with negative receiver prices and there is a limit in the size of the subsidy that can induce senders to send efficient messages.

Efficient uniform sender prices are decreasing in the magnitude of maximum sender value of the message distribution and at a minimum are equal to the negative of the sender's processing cost. Efficient uniform receiver prices are similarly decreasing in the magnitude of the maximum receiver value and at a minimum are equal to zero. Even with perfectly symmetric message preference distributions the optimal uniform prices are asymmetric in that the receiver pays more than the sender. However, the sender price is relatively large compared to the receiver price when the message preference distribution has a lot of mass for messages with high sender value and low receiver value.

The sum of the efficient uniform receiver and sender prices fails to cover all the message costs but does cover the ISP's costs when one or both the maxima in the message preference distributions are sufficiently small. With symmetric message preference distributions, the efficient prices given an ISP break-even constraint are constant in the maximum of the distributions. These prices are also asymmetric so that receivers pay more than senders do.

Monopoly prices are also asymmetric in that receivers pay more than the senders when the message preference distributions are symmetric. Given such a distribution, the monopoly prices increase without bound as the maximum of the message preference distributions is increased.

Finally we show that uniform and Ramsey prices generally get closest to the second-best welfare (welfare generated by perfectly discriminatory prices subject to non-negativity constraints). Zero prices have the smallest welfare loss when the message preference maxima are large relative to the message costs, in which case zero prices are close to the optimal uniform prices. Monopoly prices, however, do well only for a small range of parameter values when the monopoly prices equal or are close to the efficient uniform prices. When the maxima in the message preference distributions get larger than these values, the ratio of monopoly welfare to the second best welfare first declines fast but then becomes fairly steady at 55-65 % in our examples.

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