

Matching Users' Rights to Available Ground Water

John F. Raffensperger

john.raffensperger@canterbury.ac.nz, Dept. of Management, Private Bag 3800,
University of Canterbury, New Zealand

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Abstract

The available water in a catchment depends on where and when the water is taken. This raises considerable man-made uncertainty in water availability. Partly due to this uncertainty, catchments are over-allocated almost everywhere. The over-allocation problem could be managed more effectively if catchment managers could match users' rights to the available water. In this paper, I show how this dilemma can be solved, using the framework of a smart market. A market manager serves as a broker managing a common pool, to buy and sell water. The market manager's revenue reflects the available water relative to users' rights. When the market manager is revenue neutral, the catchment may be viewed as perfectly allocated. I suggest ways in which a catchment manager can find this revenue-neutral allocation, assuming the manager has authority to adjust initial rights.

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Key words

Auctions, smart markets, groundwater, water allocation, initial rights

1 Introduction to the smart market for ground water.

Water availability is a contentious issue. How much is available? Who has the rights to it? Despite a great deal of law on the matter, government still has no sure way to allocate water rights. Indeed, many catchments are tragically over-allocated, where users have more water rights than there is water. The problem is less difficult for reservoir water, which can be measured easily, and much more difficult for ground water, which is difficult to measure.

A closely related problem to water allocation is protection of the environment. Who should pay for environmental services? Should users pay government for the right to damage the environment, or should government pay users to protect the environment? In many parts of the world, water use is subsidized, but government also pays users to protect the environment. These perverse behaviors need overhaul in order to manage the environment sustainably. Unfortunately, government management is restricted by the available tools. Too often, the tools are simple policies that can be put into a few words, such as "use it or lost it," "first come first served," "the market will sort it out," and, of course, "anything goes." Increasingly, however, water management authorities are using sophisticated hydrological modeling. In some places, this modeling is used to assist pair-wise water trades. But even with this modeling, catchments are often over-allocated, and as a result, the environment is damaged and water is mis-allocated.

In this article, I will assume that the government is operating a smart market for water. (A smart market is a periodic auction cleared by an optimization model, such as a linear program.) I will use this framework to show how government can adjust initial rights so that a given catchment can be perfectly allocated. I will first discuss some of the relevant literature, give an overview of the smart market for ground water, and explain the problem of revenue sufficiency. In Section 2, the main body of the article, I present a range of methods to adjust initial rights to the available water. These methods

44 include proportional scaling, quantity adjustments, a method for appropriative rights, financial
45 adjustments, and constraint-based adjustments. We shall see that some existing methods for scaling
46 rights result in surprising and counter-intuitive outcomes. We shall see that users might find
47 proportional scaling to be unfair. Some methods result in unclaimed water, or a junior user obtaining
48 more water than a senior user. The contribution here should generalize to management of the
49 commons generally.

50 **Literature review**

51 The work at hand necessarily draws together a wide range of literature, including management of the
52 commons, mechanism design, and hydrogeology.

53 That the commons requires government management seems no longer in dispute (see for example
54 McAfee (1999), and Baliga and Maskin (2003)). The large literature on mechanism design is heavily
55 focused on auctions for single items, with analyses especially of market power. Most of this work seems
56 irrelevant to the problem of allocating initial rights for water. Water is divisible, obviously, so it makes
57 little sense to discuss Vickrey or combinatorial auctions. As for market power, ground water users are
58 likely to have little of it, because a given catchment is likely to have many participants, and research on
59 protecting the commons has shown that market power declines quickly with the number of market
60 participants (McAfee 1998, Montero 2008). The small catchment of Marlborough, New Zealand, for
61 example, has about eight hundred well owners (mostly vineyards growing fantastic sauvignon blanc).

62 Disegni-Eshel (2005) showed how to allocate initial rights to a group of competitive and non-
63 competitive firms, for free, to balance efficiency with market power. Again, more participants resulted
64 in less market power. Furthermore, groundwater is spatially dispersed; a well owner has the most
65 control of water nearest the well, and less control of water further away. While market power still needs
66 study in the proposed smart market for ground water, I will ignore issues of market power in this
67 article.

68 Initial rights can affect efficiency in the presence of large transaction costs (Stavins 1995). However, as
69 I shall explain shortly, I will assume zero transaction costs. Nevertheless, despite Coase (1960), and
70 even assuming only price-taking behavior, the process for setting initial rights does matter (Neuhoff,
71 Martinez & Sato 2006), even when the overall cap is known. For water, and this is the issue at hand, the
72 overall cap is not known in advance, because the catchment manager does not know how much rain
73 the catchment will get. So, even with zero transaction costs, rights should be adjusted in some manner
74 to the available water.

75 Some researchers have used optimization to adjust initial rights, including for water rights. Wang et al
76 (2007) used network linear programming to allocate initial rights for primarily surface water, accounting
77 for return flows, in-stream uses, reservoir storage rights, and pollution. They calculated “fairness” as
78 weighted water shortage, recognizing different seniorities of rights. Environmental flows were viewed
79 as normal demands. They did not envision their system used for a water market, and rights were not
80 adjusted to the available water; rather, users were allocated quantities short of their rights. Lozano,
81 Villa, & Brännlund (2009) proposed to reallocate pollution emission permits with data envelope
82 analysis, trying to avoid decreases in production and increases in pollutants; a similar procedure could
83 be done with water quantity, but because their method requires private data, I do not think it is
84 practical for market operation.

85 For a market orientation on water, we should look to the literature on smart markets, such as McCabe
86 et al (1991) and especially Murphy et al (2000). The latter work describes an economics experiment with
87 a small stylized surface water problem. Raffensperger & Milke (2005) and Raffensperger, Milke & Read
88 (2009) described a smart market in which users buy and sell rights to take ground water via a shared
89 pool. That work was inspired by the well-established field of hydrological optimization, which applies
90 math programming to solve hydrogeological problems, such as contaminant remediation (e.g., Ahlfeld
91 and Mulligan 2000). We found that a smart market design based on hydrological optimization elegantly
92 handled the physics, sustainability, and economics. The shared “pool” of water would be managed by a
93 market manager. I will depend on the model proposed in the latter article in this one.

94 The large literature on electricity markets may appear relevant, due to an almost identical mechanism: a
95 periodic auction cleared by a linear program (see, for example, Read, Drayton-Bright & Ring 1998). But
96 despite the similarity as smart markets, electricity and water markets will be quite different. The number
97 of market participants is different by orders of magnitude, and ground water has only a natural supply.
98 The informed reader may immediately think of the financial trades in the electricity markets, and try to
99 make the analogy to water. Joskow and Tirole (2000) and Gilbert, Neuhoff, and Newberry (2004)
100 discussed financial transmission rights and market power, and these issues are critical in a market with
101 few players. But unlike ground water, initial rights and the available quantities are clear in electricity
102 markets, as generators have initial rights for generation, and the network operator usually owns the
103 initial transmission rights. In any case, the water rights discussed in this article are not financial
104 ownership of the rent on a constraint, nor contracts about price changes, but rights (options) to take
105 physical water. Following establishment of the spot market for ground water, participants will want to
106 develop financial trading instruments, such as to hedge against price fluctuations, but the discussion
107 here is only about the physical rights.

108 Most relevant to this article, and water allocation, is that a smart market can slash transaction costs,
109 while managing rights and a range of complicated constraints. I will mention additional articles in the
110 remainder.

111 **Review of the spot market design for ground water**

112 To motivate the main part of the article, this section briefly reviews the smart market for ground water
113 in Raffensperger & Milke (2005) and Raffensperger, Milke & Read (2009).

114 *Assumptions and terminology*

115 A *user* is someone who takes water for a known purpose, e.g., irrigating a vineyard. All users are within
116 one *catchment*, which is a region of land that is relatively isolated hydrologically from other regions. I
117 assume that government has authority to control user's rights to and use of water, and that government
118 has the means and will to enforce market rules. I assume that water is metered and generally short.

119 I assume that water management authorities can allocate two types of rights, which differ mainly in
120 their term.

- 121 • *Quota* is a legal option to take water for a short period, e.g., a week or a month. This quota is
122 administrative permission to a particular user to use a certain amount of water for a particular
123 application for a particular period of time. Water itself is not traded, but rather permission to take
124 water is traded.
- 125 • *Consent* is a legal option to take water at a given location, perhaps with a range of restrictions, for a
126 long period, e.g., 10 to 30 years, or permanently. Consent confers quota automatically.

127 I assume that a *market manager* operates a spot market for ground water quota within the catchment.
128 The market manager clears the market with the help of a linear program (GWMarket below) which
129 includes explicit constraints for environmental flows. The market manager requires knowledge of the
130 user's return flows to calculate the effects of the user's abstraction. Thus, I assume that consent and
131 quota are for a known purpose. I assume that every user's demand function is independent of the user's
132 quota, and that a deterministic market clearing model is acceptable.

133 The market does not manage bilateral transactions between participants. Rather, trades are to and from
134 a common pool through the market manager. During bidding for a given auction, users give bids via
135 the auction web site, for each period of the planning horizon, e.g., each week through the end of the
136 season. Then the market manager solves GWMarket and informs users of prices and allocations. Users
137 have an initial quota, so purchases and sales are the difference of the initial quota to the market
138 allocation.

139 Through the spot market, a consent holder may lease the associated quota to another user, and a user
140 without consent may rent quota from another user. I will use the terms "buy" and "sell," but it should

141 be understood that these transactions are short-term rentals in a spot market. After market clearing,
142 sales or purchases are calculated as the difference of the market allocation to the initial quota.

143 A person who does not hold consent (or even land) could get information to choose a location for
144 operations. To enter the market, a new user could review a map, made available by the market manager,
145 showing prices by location (adjusted for type of use) within the catchment, seeking a well with a
146 relatively low price for water. The user then applies to the market manager to be accepted into the
147 market, satisfying such criteria as reasonable use for the water, sufficient financial stability for the
148 transactions, notification of the location of the user's well, guarantees about metering, etc. The new
149 user does not need to apply for a quantity, but instead can simply rent quota from the market.

150 *The linear program*

151 I next introduce the gross pool market-clearing model for groundwater.

152 **Indices**

153 $b=1, \dots, B$, demand tranche.

154 $i, j=1, \dots, I$, wells, assumed to be in one-to-one correspondence with users.

155 $u, t=1, \dots, T$, time periods. Period 1 is the present period (e.g., week). Each auction occurs at the
156 beginning of period 1, and the subscript is updated by an external program. Period T is the last period
157 of the season or hydrological year. Thus, the first auction of the hydrological year could have 52 weeks;
158 the second auction could have 51 weeks, and so on, until the final auction is for week $T=52$ only. (A
159 rolling horizon may be more appropriate. See Raffensperger, Milke & Read (2009) for a related
160 discussion.)

161 **Parameters**

162 C_i^t = initial quota for user i in time period t . This is not used in Model GWMarket below, but is
163 required to calculate net sales following market clearing. Managing this initial quota is the key issue
164 addressed in this article.

165 F_{ik}^t = drawdown rate at control point k , $t-1$ periods after abstraction at well i , in [meters of head] per
166 [cubic meter of water per second]. Note that F_{ik}^1 is the drawdown in the period of abstraction, not
167 necessarily in period 1. Almost always $F_{ik}^t \geq 0$. Hydrologists can obtain F_{ik}^t with standard hydrology
168 software (Ahlfeld, Barlow, & Mulligan, 2006).

169 P_{ib}^t = the bid price for demand quantity Q_{ib}^t at user i , period t .

170 Q_{ib}^t = the bound on demand tranche b for abstraction by user i , period t .

171 U_k^t = the upper bound on drawdown at control point k , period t , in meters.

172 V = the minimum net revenue that the market manager requires from users for a given auction. This is
173 not used in Model GWMarket below, but will be used in the market clearing process. V need not be
174 positive if the market manager is willing to make a net payment to users. V is positive, if, for example,
175 the market manager wishes to have net revenue to cover the costs of running the auction. Users will be
176 inclined to ensure that the market manager recovers only what is needed for market operation.
177 However, payment for market operations should be seen as separate from adjusting initial rights to
178 match the available resource.

179 **Variables**

180 $p_i^t \leq 0$, market price per unit of water faced by well i , period t . This is the dual price on constraint 3
181 below.

182 $\lambda_k^t \geq 0$, price per unit of head at control point k , period t . This is the dual price on constraint 4 below.

183 q_{ib}^t = abstraction at bid price P_{ib}^t for the well i , period t .

184 q_i^t = total abstraction by well i during period t .

185 **Model GWMarket**

186 Maximize $\sum_{t=1}^T \sum_{i=1}^I \sum_{b=1}^B P_{i,b}^t q_{i,b}^t$, subject to (1)

187 $0 \leq q_{ib}^t \leq Q_{ib}^t$ for tranches $b=1, \dots, B$, users $i=1, \dots, I$, and periods $t=1, \dots, T$. (2)

188 $q_i^t = \sum_{b=1}^B q_{ib}^t$ for users $i=1, \dots, I$, and periods $t=1, \dots, T$. (Dual variable p_i^t) (3)

189 $\sum_{i=1}^I \sum_{u=1}^t F_{i,k}^{t-u+1} q_i^u \leq U_k^t$, for control points $k=1, \dots, K$, and periods $t=1, \dots, T$. (Dual variable $\lambda_{k,t}^t$) (4)

190 **Explanation**

191 1. The objective function maximizes total profit from water abstraction over the planning horizon,
192 assuming users bid truthfully.

193 2. Abstraction in each tranche is bounded by bid quantities.

194 3. The abstraction equals the total cleared. The dual variable p_i^t of this constraint is the marginal profit
195 to the market for another unit of water taken by well i in period t .

196 4. The drawdown at each control point is limited, e.g., to prevent coastal salt water intrusion, to
197 maintain minimum stream flows, or to limit aquifer drawdown. This constraint set guarantees that
198 every solution is environmentally sustainable, as defined by the constraints.

199 I assume $U_k^t \geq 0$, so Model GWMarket is always feasible mathematically. That is, I assume that all
200 environmental constraints can be satisfied when no one abstracts water. The market model manages
201 human-induced impact on drawdown, not natural events.

202 Because the hydrology data comes from standard hydrology models and software, this system can be
203 set up in any catchment for which a standard hydrology model (e.g. MODFLOW) is available. Many
204 U.S. locations already have such models, and they are often used to help settle legal disputes.

205 *Auction operation*

206 During bidding for a given auction, users specify bids for every remaining period of the planning
207 horizon, e.g., each week through the end of the season. After bids are entered, the market manager
208 solves the linear program, then informs users of the final prices and allocations. Under marginal cost
209 pricing, the user at well i pays $\sum_{t=1}^T p_i^t (C_i^t - q_i^t)$, assuming initial rights C_i^t are not scaled or adjusted.
210 Note that the market manager's *required* net revenue V is not automatically the same as the actual net
211 revenue $\sum_{i=1}^I \sum_{t=1}^T p_i^t (C_i^t - q_i^t)$.

212 The user would then have firm rights to take up to q_i^1 in the current period, i.e., period 1, and would
213 have quota for q_i^t for periods $t = 2, \dots, T$. Because all remaining periods are open for bid in every period,
214 users will tend to trade only differences between their adjusted right and what they actually wish to
215 have. More importantly, inflows are uncertain, so future quota are subject to adjustment by the market
216 manager. This problem of adjustment is the main issue addressed in this article.

217 **Revenue sufficiency**

218 *Under-allocated catchment*

219 If water is in excess supply, the constraints in set 4 are slack, so $p_i^t = 0$ and $\sum_{i=1}^I \sum_{t=1}^T p_i^t (C_i^t - q_i^t) = 0$. In
220 this case, neither users nor policy makers have much interest in market operation. For this article, I
221 assume that water is generally short.

222 *Over-allocated catchment*

223 Worldwide, governments have given more quota than is sustainable, whether because environmental
224 standards have tightened, because catchments face increased drought, or because governments have

225 been careless. The case of an over-allocated catchment is important, as it is the root of the world water
226 crisis.

227 If $\sum_{i=1}^I \sum_{t=1}^T C_i^t > 0$, feasibility ($q_i^t = 0$ in the most extreme situation) may require that some users are
228 willing to sell. In this case, our design requires the market manager to buy quota from users – paying
229 users to forego water – to protect the environment. Over-allocation may not be a problem in the short
230 run if users are willing to sell for a “reasonable” price. If $V < 0$, the manager pays out large amounts,
231 and the market may be viewed as a procurement (or reverse auction) where users are paid not to
232 damage the environment.

233 Governments sometimes buy rights from users to protect the environment (McGauran 2006, NSW
234 DNR 2006). Cummings, Holt & Laury (2004) developed a web-based auction system for government
235 to pay farmers to reduce irrigation during drought, in the Flint River Basin, Georgia. The authors’
236 system was not a water trading system, but an irregular drought management system without hydrology
237 modeling. The budget was set in advance, and government accepted the lowest bids until the budget
238 was gone. However, the environment absorbed the difference between the government’s decision and
239 any over-allocation. Similarly, Stoneham et al (2003) described the Australian government’s
240 environmental services procurement program. Government has given too many rights to commerce,
241 and now must buy back those rights to protect the environment.

242 If the catchment is over-allocated, users may realize that they can demand a high price. This is likely to
243 happen as a user observes that her offer to sell was accepted, so she raises the offer price and observes
244 that she improves her revenue. This effect would be enhanced if users collude, but users may observe
245 the effect independently over many auctions, especially if the catchment is over-allocated regularly. The
246 users are likely to catch on even faster if the market manager operates tentative auctions to give users
247 opportunity for price discovery. An over-allocated catchment can result in high payments from the
248 market manager to users, and it is easy to show that under modest assumptions, for a large catchment
249 with many users, these payments can be arbitrarily high.

250 If users set P_{ib}^t to a high enough price that $\sum_{i=1}^I \sum_{t=1}^T p_i^t(C_i^t - q_i^t) < V$, the market manager cannot clear
251 the market within budget. In this case, I will call the catchment is *over-allocated* and will say that the
252 associated auction model is infeasible.

253 It is my opinion that giving people rights to assets which do not exist is bad policy. Paying people to
254 forego environmental damage is politically expedient in the short run, but such payments cannot be
255 viable over a long term through a range of economic conditions. Eventually, perhaps due to public
256 outcry, the government will not continue paying business to prevent damage to the commons.

257 If local law allows the market manager to adjust C_i^t , policy makers can choose V as part of auction
258 design and operation. As we shall see, to find a feasible solution at reasonable cost to the market
259 manager, the manager requires authority to adjust quota. I therefore assume that the market manager
260 has such authority, and I define a parameter corresponding to this adjustment.

261 $\alpha_i^t =$ % of initial quota C_i^t available to the user at well i in period t .

262 In the following, I will occasionally change subscripts on α_i^t . For example, the scalar α implies the same
263 scaling for all users in all time periods.

264 If the market manager adjusts quota, then net revenue can be chosen by setting α_i^t in such a way that
265 revenue exactly equals the desired target V , after auction clearing: $\sum_{i=1}^I \sum_{t=1}^T p_i^t(\alpha_i^t C_i^t - q_i^t) = V$. Clearly,
266 α_i^t allows flexibility as to which users will get the most adjustment. How should α_i^t be set? Which users
267 should be allocated the most rights? I shall address this shortly.

268 **Optimal allocations and prices are independent of initial allocations**

269 Model GWMarket results in the same flows q_i^t and the same prices p_i^t , for any feasible quota allocation
270 C_i^t , except for alternative optima, and assuming all users participate. Mathematically, this follows
271 trivially because buy and sell quantities are calculated after the model is solved. We also need the
272 assumptions that every user’s demand function is independent of the user’s quota, and is willing to bid

273 so that $\sum_{i=1}^I \sum_{t=1}^T p_i^t (\alpha_i^t C_i^t - q_i^t) \geq V$, as stated earlier. Economists will note that Coase (1960) proved the
274 same theorem more generally, assuming sufficiently small transaction costs. We should not pass over
275 the assumption too quickly.

276 Existing water markets worldwide have high transactions costs due to the need to find a trading
277 partner, negotiate and enforce a contract, and manage externalities through protracted government
278 approval processes. The smart market eliminates these transactions costs, thus satisfying the
279 assumptions for Coase's theorem. Hence, if all users participate, the market manager need not re-solve
280 Model GWMarket to find the preferred α_i^t .

281 To relax the assumption that all participate, we can require that non-participants and participants are
282 scaled similarly. Otherwise, a non-participant could get 100% allocation for free, while the participant
283 gets scaled to 80%, then buys back to 100%, in which case the participant would have been better off
284 without the market. The effect of non-participants is to fix $q_i^t = \alpha_i^t C_i^t$ in Model GWMarket. These users
285 are actually participating, in that they will be taking water; they simply have bids with a sell price of
286 infinity and a buy price of zero. Yet scaling is done *after* the market model is solved, to achieve the
287 revenue goal V . So there is a "chicken-and-egg" problem of requiring α_i^t before the model is solved,
288 but calculating it afterwards. This issue may be resolved through a variety of mechanisms, including
289 requiring all users to participate (perhaps for sufficiently large C_i^t and ignoring those below that), using
290 approximate scaling factors for non-participants (perhaps a bit on the low side to incentivise
291 participation), simply charging each non-participant i the amount $p_i^1(1 - \alpha_i^1)C_i^1$, or solving Model
292 GWMarket iteratively to find the right α_i^t .

293 2 Methods of adjusting quota

294 This section describes methods to choose α_i^t so that the auction model is feasible, i.e., $\sum_{i=1}^I \sum_{t=1}^T p_i^t (\alpha_i^t C_i^t$
295 $- q_i^t) = V$.

296 **User-pays method, revenue positive**

297 Perhaps the most obvious value for α_i^t is $\alpha_i^t = 0$. This corresponds to a user pays market, where $\alpha_i^t C_i^t =$
298 0 for all i and t . In this method, $\sum_{i=1}^I \sum_{t=1}^T p_i^t (\alpha_i^t C_i^t - q_i^t) = -\sum_{i=1}^I \sum_{t=1}^T p_i^t q_i^t = V \gg 0$, and the manager
299 receives a large revenue. The market may be viewed as an auction in which users buy all quota from the
300 market manager. (For simplicity of exposition, I will continue to use the word auction to mean one
301 event of this market's operation, regardless of the manager's revenue.) Auctions for public resources
302 are, of course, widely implemented. The revenue raised can be used to offset other distorting taxes, as
303 could be done with carbon emissions (Cramton & Kerr 2002).

304 It is important to remember, however, that water has already been allocated almost everywhere. If V
305 $\gg 0$, existing users may not want to implement the proposed market, because they face a significant
306 outlay. If government no longer issued consents for water, prospective users would wish to have the
307 market, as it would give them an opportunity to get quota for water. With centuries of hard-won rights
308 already in place, I think a user-pays system will be unacceptable to users.

309 Even if we assume a user-pays system, the need to scale rights to the available water remains. Reservoir
310 and ground water have storage components, so the effects can lag over many periods. Furthermore,
311 users will want some certainty over the full growing season at least, and would not want to commit to a
312 crop without a belief that water will be available for those crops. The allocation problem is therefore
313 multi-period. After period 1, the market manager has granted quota to every user for future periods
314 $2, \dots, T$. Thus, when period 2 arrives, users will already hold quota for which they paid in period 1. But
315 period 2 flows are unlikely to match their forecast, so the market manager will have to adjust rights
316 again in some fashion, and the scaling problem reappears.

317 **Proportional adjustment**

318 People sometimes like to allocate goods proportionally. We can adjust quotas proportionally in a variety
319 of ways.

320 *Myopic maximum proportion*

321 One intuitive method of scaling initial rights is to find the maximum proportion α that all users could
 322 take sustainably, found with Model MaxProportion below. I drop the subscripts on α_i^t , because all users
 323 have the same proportion. As we shall see, this solution is myopic, because it ignores the economics.
 324 Denote the solution to MaxProportion as $\underline{\alpha}$.

325 Model MaxProportion: maximize $\underline{\alpha}$, subject to (5)

326 $q_i^t = \underline{\alpha}C_i^t$, for all wells $i=1, \dots, N$, and periods $t=1, \dots, T$, (6)

327 and constraints 4.

328 Because I assumed that a feasible solution exists with $q_i^t = 0$, then MaxProportion has a feasible
 329 solution. Furthermore, if the optimal $\underline{\alpha}$ is positive and if all $C_i^t > 0$, then every user is guaranteed some
 330 initial quota.

331 Now the thoughtful reader may think that the lesson from this exercise is that quota should not be a
 332 quantity, but rather a fraction of the available water. Another guess is that net revenue to the market
 333 manager will be zero, so $\sum_{i=1}^I \sum_{t=1}^T p_i^t (\underline{\alpha}C_i^t - q_i^t) = 0$. Neither guess is true.

334 Consider the following example (which will appear throughout the article).

335 Control point 1: $q_1 + q_2 \leq 10$ (7)

336 Control point 2: $q_2 + q_3 \leq 12$ (8)

337 The solution (10, 0, 12) maximizes q_1 ; it also maximizes q_3 . The solution (0, 10, 2) maximizes q_2 . These
 338 solutions allocate different quantities of water, 22 units in the first solution, and only 12 units in the
 339 second. Even for a trivial problem, the sustainable available water depends on *where* it is taken. For real
 340 problems, available water also depends on *when* it is taken; a price may be high now to prevent future
 341 damage. Furthermore, available water changes quickly and variably over space and time. Thus, speaking
 342 of the fraction of available water makes little sense. Consent and quota should not be registered or
 343 recorded as fractions of some notional “total water in the catchment”.

344 Regarding net revenue, suppose $C_i = 4, 8$, and 4 for users 1, 2, and 3 respectively. The catchment is
 345 over-allocated at control point 1, because $C_1 + C_2 = 4 + 8 > 10$, but is perfectly balanced at control
 346 point 2, because $C_2 + C_3 = 8 + 4 = 12$. The solution to Model MaxProportion is $\underline{\alpha} = 10/12$, based on
 347 control point 1. User 3 will complain, because she has responsibility only to control point 2, which was
 348 perfectly allocated. In the market, she would have to buy back her original quantity, if she wanted it.

349 If each user bids \$1/unit quota, the solution is (10, 0, 12). User 3 *does* buy back her original allocation,
 350 plus more. The dual prices are $p_1 = -\$1, p_2 = -\$2, p_3 = -\$1$. Net revenue to the market manager is –
 351 $\$1(4*10/12 - 10) + -\$2(8*10/12 - 0) + -\$1(4*10/12 - 12) = \2 . Thus, the auction is revenue positive.

352 In general, Model MaxProportion will choose $\underline{\alpha}$ based on the “driest” well. This is most likely the user
 353 closest to a particular most-stressed environmental control point. The optimal $\underline{\alpha}$ and the suspect user
 354 may be identified as follows.

355 Substitute $q_i^t = \underline{\alpha}C_i^t$ into constraint set 4: $\sum_{i=1}^I \sum_{u=1}^t F_{i,k}^{t-u+1} \underline{\alpha}C_i^t = \underline{\alpha} \sum_{i=1}^I \sum_{u=1}^t F_{i,k}^{t-u+1} C_i^t \leq U_k^t$, and so:

$$356 \quad \underline{\alpha} = \min_{k,t} \left\{ U_k^t / \left(\sum_{i=1}^I \sum_{u=1}^t F_{i,k}^{t-u+1} C_i^t \right) \right\}. \quad (9)$$

357 Denote by (\bar{k}, \bar{t}) as the control point and time period which most constrain $\underline{\alpha}$. The user with the

358 “driest” well is user $\bar{i} = \operatorname{argmax}_i \sum_{u=1}^{\bar{t}} F_{i,\bar{k}}^{\bar{t}-u+1} C_i^{\bar{t}}$.

359 Questions of fairness arise. All users’ quota have been reduced to that of the driest user, but if this user
 360 \bar{i} took one unit less, users away from the environmentally sensitive areas could have more than a unit

361 of water. As a result, $\underline{\alpha}$ almost always results in an auction which is revenue positive for the market
 362 manager, because most users will buy back some quota.

363 *User trades, well quota proportion, revenue neutrality*

364 A particularly interesting case is where $V \approx 0$. In this case, the market manager is serving as a broker to
 365 clear the market, maintaining no financial position; the catchment may be viewed as perfectly allocated.
 366 Users are likely to participate voluntarily, unilaterally choosing to buy or sell without government
 367 coercion. Non-participating users are likely to have a marginal value for water that is different to the
 368 price, and thus they will have incentive to trade. Thus, the user trades method gives the policymaker a
 369 simple way to start and expand the market.

370 The α^0 we seek, indicating revenue neutrality with the superscript 0, solves the following equation.

$$371 \sum_{i=1}^I \sum_{t=1}^T p_i^t (\alpha^0 C_i^t - q_i^t) = V = 0. \quad (10)$$

372 This is easily solved: $\sum_{i=1}^I \sum_{t=1}^T p_i^t (\alpha^0 C_i^t - q_i^t) = \sum_{i=1}^I \sum_{t=1}^T (\alpha^0 p_i^t C_i^t - p_i^t q_i^t)$

373 $= \alpha^0 \sum_{i=1}^I \sum_{t=1}^T p_i^t C_i^t - \sum_{i=1}^I \sum_{t=1}^T p_i^t q_i^t = 0$, which implies

$$374 \alpha^0 = (\sum_{i=1}^I \sum_{t=1}^T p_i^t q_i^t) / (\sum_{i=1}^I \sum_{t=1}^T p_i^t C_i^t). \quad (11)$$

375 Using our earlier example, the solution (10, 0, 12) maximized $q_1 + q_2 + q_3$, supposing identical bids. Now
 376 solve for α^0 :

$$377 -\$1(4*\alpha^0 - 10) + -\$2(8*\alpha^0 - 0) + -\$1(4*\alpha^0 - 12) = 0, \text{ so } \alpha^0 = 11/12.$$

378 Users 1, 2, and 3 buy 6.333, -7.333, and 8.667, respectively. Users 1 and 3 buy back only 1/12th of
 379 their original quota. User 2 sells everything, but is paid for only 11/12ths. Still, we can imagine user 3
 380 still complaining (as control point 2 was never over-allocated), just not as loud as with the solution to
 381 Model MaxProportion. The same problem of unfairness occurs, but to a lesser degree only because less
 382 money goes to the market manager.

383 Every user will prefer the user trades proportion to the myopic proportion. This is easily proved, and it
 384 suffices to show that $\underline{\alpha} \leq \alpha^0$. Simply observe that by linear programming duality, Model GWMarket
 385 simultaneously maximizes $\sum_{i=1}^I \sum_{t=1}^T p_i^t q_i^t$ and minimizes $\sum_{i=1}^I \sum_{t=1}^T p_i^t C_i^t$. Therefore, the ratio of these two
 386 terms is maximized by the optimal GWMarket solution q_i^* , which means α^0 is the largest possible ratio,
 387 so $\sum_{i=1}^I \sum_{t=1}^T p_i^t \underline{\alpha} C_i^t \leq \sum_{i=1}^I \sum_{t=1}^T p_i^t q_i^*$, and the result follows. With a larger proportion of initial consent,
 388 sellers receive more money, buyers pay less money, and the market manager nets less money.

389 Intuitively, because $\underline{\alpha}$ is based on the tightest constraint in set 4, $\underline{\alpha}$ is likely to be quite small, say, 10%. If
 390 the initial quota are remotely close the right value, then α^0 will be close to 100%. Hence, users will
 391 prefer the user trades method to the myopic method.

392 The user trades method may be viewed as distributing the scarcity rents among the users. In a market
 393 where binding constraints correspond to private assets, such as power line capacity in an electricity
 394 market, this point of view makes sense. The owner of the capacity has incentive and compensation to
 395 increase capacity. (Interestingly, in New Zealand, the electricity network operator Transpower returns a
 396 sizeable fraction of line capacity rents to their customers (ECNZ 2008).) However, environmental
 397 constraints may be viewed as public goods; the ‘‘capacity’’ is the allowable draw down of the
 398 environmental resource. Government could allocate all sustainable water to users, but (barring
 399 technological changes that expand the sustainable capacity of the environment) should not allocate
 400 more than that.

401 *User trades, proportion by time period*

402 The market should signal to users and policymakers information about the future levels of quota. To
 403 provide this signal, the market manager can choose a method of scaling to have revenue neutrality in
 404 every period of the current auction model. This is done easily by adding a subscript for time, α_t^0 , then
 405 solving for α_t^0 .

$$406 \alpha_t^0 = \sum_{i=1}^I p_i^t q_i^t / \sum_{i=1}^I p_i^t C_i^t, \text{ for } t=1, \dots, T. \quad (12)$$

407 In this way, a given user's quota may be scaled differently for each future period. Thus, in period 1, if a
 408 user's quota for period 2 were scaled by $\alpha_2^0 = 15\%$, the user is signaled that the future period is
 409 expected to be relatively dry.

410 The market manager could choose to achieve revenue neutrality over a sequence of auctions, say, a year
 411 of weekly auctions. Subscripting by time, the manager would wish $\sum_t V^t \approx 0$. But once the precedent
 412 was made for scaling quota, there seems no reason for the market manager to take on the risk.

413 **Quantity adjustments**

414 Rather than adjust users' quota proportionally, quota could be adjusted by quantity. Again, this may be
 415 done in several ways.

416 *Maximize total abstraction*

417 Another intuitive method is to begin with the solution that maximizes total water abstraction from the
 418 catchment. To maximize total abstraction, we can solve Model MaxWater below, which will give an
 419 upper bound on the amount of water available from the catchment. Model MaxWater will be familiar
 420 to many hydrologists, as maximizing total flow subject to constraints is a typical hydrological
 421 optimization problem.

422 Model MaxWater: maximize $\sum_{j=1}^n \sum_{t=1}^T q_i^t$, (13)

423 subject to constraints 4.

424 The market manager then sets $\alpha_i^t = q_i^t / C_i^t$.

425 Following our three-user example, this solution would change user 1's initial allocation from 4 to 10,
 426 user 2's allocation from 8 to zero, and user 3's allocation from 4 to 12. Users 1 and 3 would be thrilled,
 427 and user 2 is likely to begin court proceedings. This simplistic method ignores any initial right and
 428 willingness to pay. Users closest to environmental control points will be most restricted, losing their
 429 quota without regard to previously hard-won battles. Given the decreasing marginal utility for water,
 430 this method will tend to hurt some users financially more than others.

431 *Minimize quantity reduction*

432 We could take an equal quantity from every user. I introduce a scalar variable $r \geq 0$, which is the
 433 reduction in every user's quota. (If we wish, we could subscript r by time.)

434 Model MinReduction1: minimize r , subject to (14)

435 $q_i^t + r = C_i^t$, for all wells $i=1, \dots, N$, and periods $t=1, \dots, T$, (15)

436 $r \geq 0$, (16)

437 and constraints 4.

438 In our example, each user would give up one unit of water. The market manager would therefore
 439 receive $-\$1(3 - 10) + -\$2(7 - 0) + -\$1(3 - 12) = \2 . This is not revenue neutral, just as the myopic
 440 proportional reduction is not revenue neutral. Smaller users will be hurt most by this.

441 *Revenue neutral quantity reduction*

442 We can easily find a revenue neutral solution, as we did with proportional adjustment, by solving the
 443 following equation for r .

444 $\sum_{i=1}^I \sum_{t=1}^T p_i^t (C_i^t - r - q_i^t) = V = 0$. (17)

445 Our example solution was (10, 0, 12). The quantity reduction r is found easily by inspection:

446 $-\$1(4 - r - 10) + -\$2(8 - r - 0) + -\$1(4 - r - 12) = 0$, so $r = 0.5$.

447 Neither MinReduction1 nor its revenue neutral version may be feasible, because $0 \leq r \leq \min_{i,t} C_i^t$,
 448 meaning that the quantity reduction is bounded by the smallest quota, which may not be sufficient for a

449 feasible solution to constraint set 4. We can get around this if we allow r to fall further (with the
 450 understanding that no user would be required to inject water into a well), choosing r to satisfy:

$$451 \sum_{i=1}^I \sum_{t=1}^T p_i^t (\max(0, C_i^t - r) - q_i^t) = V. \quad (18)$$

452 This system will take all quota away from each user with quota of r or less, and r units from each of the
 453 other users.

454 To motivate this method, consider a modified version of our previous three-user example, where the
 455 initial rights are 4, 16, and 4, respectively. The optimal solution was 10, 0, 12, with prices $-\$1$, $-\$2$, and
 456 $-\$1$. Solving for r , $-\$1(\max(0, 4 - r) - 10) + -\$2(\max(0, 16 - r) - 0) + -\$1(\max(0, 4 - r) - 12) = 0$, so r
 457 $= 5$. Now users 1 and 3 lose all their initial rights, and user 2 is reduced from 16 to 11. Users 1 and 2
 458 must purchase their full quantities, while user 2 sells her remaining quantity. This method will hurt
 459 users with small quota, and leave a relatively few large users with water.

460 *Cut the user with heaviest impact.*

461 In New Zealand, water management authorities occasionally require some users to stop pumping on
 462 short notice, especially to prevent coastal salt water intrusion. The selected users are those closest to the
 463 coast, with the immediate impact. The authority has no means to restrict in retrospect users with
 464 delayed impacts; lacking is coordinated planning for the future.

465 Rather than restricting all users equally, we can restrict the user with the highest impact. We can use the
 466 earlier notion of the “driest well,” owned by user $i^* = \operatorname{argmax}_i \sum_{t=1}^{t^*} F_{i,k^*}^{t^*-t+1} C_i^t$, associated with control
 467 point k^* and time period t^* . The argument is that this user has the most impact, so this user should be
 468 first to have C_i^t reduced, perhaps until $\sum_{t=1}^{t^*} F_{i,k^*}^{t^*-t+1} C_i^t$ equals that of the second worst offender. More
 469 generally, the market manager reduces C_i^t for all users with the largest impacts until the manager is
 470 satisfied with the auction revenue.

471 This method has some sense of justice to it, as it includes both marginal impact and quantity. The big
 472 users who also have the biggest impact will be restricted first. However, one could imagine 99 users
 473 who each take 1 unit of water for washing their cars on Sundays, and a 100th user who takes 10 units of
 474 water for growing crops. The farmer could be cut back so severely as to become bankrupt.

475 Other quantity methods include taking water only from large users regardless of impact, taking water
 476 from users with the largest marginal impact F_{i,k^*}^t , taking different quantities from different sized users
 477 (perhaps step-sized packets), etc.

478 *Appropriative rights*

479 U.S. laws in some states specify that rights are acquired by use, and the earliest user has senior rights to
 480 later users. In dry years, senior users get water first; junior users cannot “hurt” senior users. Users are
 481 required to use water “reasonably”, and users argue about what “reasonably” means, especially in dry
 482 years, with resolution through the courts. Appropriative rights are a special kind of quantity adjustment.

483 To create a market for users with appropriative rights, we can adjust Model MaxProportion. (No one is
 484 known to use this anywhere in the world; the smart market model which allows the calculation.)

485 Assume that each user i has some C_i^t at which user i is considered to have full quota. Create an α_i for
 486 each user, for simplicity dropping the subscript for time. Number the users by seniority, so the senior
 487 user is user 1. Beginning with the senior user, for each user j , maximize α_j sequentially:

488 Model Appropriative: for $j = 1, \dots, I$, maximize α_j , subject to (19)

$$489 q_i^t = \alpha_i^* C_i^t, \text{ for all wells } i=1, \dots, j-1, \text{ and periods } t=1, \dots, T. \quad (20)$$

$$490 q_j^t = \alpha_j C_j^t, \text{ for all periods } t=1, \dots, T. \quad (21)$$

$$491 q_i^t \leq 0, \text{ for all wells } i=j+1, \dots, N, \text{ and periods } t=1, \dots, T. \quad (22)$$

$$492 0 \leq \alpha_j \leq 1, \quad (23)$$

493 and constraints 4.

494 Fix α_j to its optimal value α_j^* and go to the next j .

495 Following this initial allocation, the market manager would run the auction as usual with Model
496 GWMarket. Following that run, the market manager may adjust quota again, this time proportionally to
497 achieve revenue neutrality.

498 This method ensures that the senior user has absolute first use. Another interpretation of appropriative
499 rights instead requires that the most senior user obtains at least as much water as the next junior. This
500 can be done by adding constraints $\alpha_i \geq \alpha_{i+1}$, rather than solving for α_i for each trader. Which method is
501 correct? Similarly, the issue of timing of rights must be addressed: is user 1's right to period 2 water
502 superior to user 2's right to period 1 water? This may depend on the length of the period; probably for
503 a day, but perhaps not for a month. Such questions will be answered in the smoke-filled conferences
504 and stolid court rooms. In any case, the appropriative doctrine will provide a windfall for senior users
505 in the spot market. Over time, however, after consent trades hands, the notion of appropriative rights
506 will become meaningless. States have appropriative water rights because government has not worked
507 out how to operate water markets.

508 Following our example, suppose users are numbered in increasing seniority, so user 3 is most senior.
509 Further, assume that each user is deemed to have full quota with 7 units. Then user 3 would obtain 7
510 units, user 2 would receive 5 units, and user 1 would receive 5 units. This follows our intuition that the
511 senior user will get the most water.

512 However, suppose users are numbered in decreasing seniority, so user 1 is most senior. Now user 1
513 would get 7 units, user 2 would get 3, and user 3 would get 7, leaving 2 units unallocated! Seniority does
514 not always confer more water, because the market manager must consider water availability. The
515 unallocated water is a bigger surprise. On reflection, allocation by seniority is unlikely to result in an
516 optimal solution to Model MaxWater.

517 Financial adjustments

518 We can define adjustment methods based on final net payments. The market manager could choose to
519 achieve revenue neutrality by charging users directly rather than changing their initial quota. This may
520 be viewed as a sales tax τ_i^t .

$$521 \tau_i^t = p_i^t |C_i^t - q_i^t| \cdot \sum_{j=1}^I p_j^t (C_j^t - q_j^t) / \sum_{j=1}^I p_j^t |C_j^t - q_j^t|, \text{ for all } i \text{ and } t. \quad (24)$$

522 The sales tax rate is $\sum_{j=1}^I p_j^t (C_j^t - q_j^t) / \sum_{j=1}^I p_j^t |C_j^t - q_j^t|$.

523 Following our example, the optimal solution was:

524	User	C_i	p_i	q_i	User gain
525	1	4	-\$1	10	-\$6
526	2	8	-\$2	0	\$16
527	3	4	-\$1	12	-\$8.

528 Here, $\sum_{j=1}^I p_j^t (C_j^t - q_j^t) = \2 , and $\sum_{j=1}^I p_j^t |C_j^t - q_j^t| = \30 . The market manager can therefore require
529 additional payments of $\$12/30$, $\$32/30$, and $\$16/30$, from users 1, 2, and 3, respectively, thus achieving
530 revenue neutrality. This method charges all participants, including those with no initial quota. It does
531 not specify a way to scale non-participants.

532 The extension to fixed charge payments corresponds to a fixed transaction cost on trades: charge each
533 user $\sum_{j=1}^I \sum_{t=1}^T p_j^t (C_j^t - q_j^t) / I$. Other extensions include the option to charge only buyers, or only sellers,
534 or to adjust each user's initial quota C_i^t to imply the tax τ_i^t , e.g., $4 - 12/30$, $8 - 16/30$, $4 - 16/30$ for
535 each respective user in our example. Indeed, the market manager could use any of many possible
536 methods to achieve the revenue target, but not every method would relate users' rights to the available
537 resource. Unless the initial quota is adjusted, users do not receive direct signals about how their quota
538 will be adjusted in the future. So this method seems appropriate to raise revenue to cover the auction

539 costs, not a way to match rights to the available water. We shall see next a method which relates the
 540 users' rights much more closely to the resource than any thus discussed.

541 **Constraint quota**

542 To now, I have described rights in terms of the user and his or her location: the quantity of water that
 543 can be taken from the user's own well. Rights can instead be defined in terms of the user's impact on
 544 the control point.

545 Montgomery (1972), in the context of air pollution, defined "ambient" permits, perhaps an unfortunate
 546 name, where the regulator specified separate rights for each control point. A user wishing to discharge
 547 at his or her own location had to obtain sufficient separate rights for each control point that his or her
 548 discharge would impact. Montgomery's method allowed for multiple control points, in theory.

549 Systems based on "trading ratios" (Horan, Shortle, and Abler 2002; Kerr, Rutherford, and Lock 2007;
 550 many others) have few control points, ideally one, and sometimes using zonal trading ratios, where all
 551 users in a give zone have the same ratio. These ratios correspond to oversimplified F_{ik}^t coefficients in
 552 Model GWMarket. In a way, the rights adjustment methods that I proposed to now are similar to these
 553 trading ratio systems, in that the right is usually defined in terms of the user's behavior, not the user's
 554 impact. Montgomery's ambient permit system defined rights in terms of the user's impact, not their
 555 behavior.

556 Of course, even with only one control point, these systems suffer from high transaction costs. Market
 557 designers oversimplified the physics in an attempt to reduce the transaction costs, understanding that
 558 by doing so, they were creating fewer commodities, which should allow easier trade. But the transaction
 559 cost was not nearly reduced enough (Stavins 1995).

560 The smart market suffers from none of these limitations. Linear programming can easily manage a rich
 561 set of control points and impact factors, which affect a given control point dynamically over time. The
 562 auction format provides a venue for the market. The market clearing process assures the regulator that
 563 the commons is respected. And the model formulation lets us choose whether to specify rights based
 564 on the user's behavior or the user's impact. So let us now explore the latter.

565 *Quantity constraint quota*

566 Denote the dual price of constraint (k, t) of set 4 as λ_k^t . From linear programming duality,

567
$$\sum_{t=1}^T \sum_{i=1}^I \sum_{b=1}^B P_{i,b}^t q_{i,b}^t = \sum_{t=1}^T \sum_{k=1}^K \lambda_k^t U_k^t.$$
 Thus, the value of the right to take water corresponds to the value of

568 the resource. This insight changes the notion of a right to take water into a right to reduce head at a set
 569 of control points. This latter interpretation corresponds to a claim of $F_{ik}^t C_i^t$ against the right hand side
 570 U_k^t .

571 I will use the term *constraint consent* to mean a long-term right to reduce head at a control point, and
 572 *constraint quota* C_{ik}^t to mean the right of user i to reduce head by C_{ik}^t at control point k in period t . A
 573 constraint quota is for a single period and a single control point. Abstraction of water from a given well
 574 will likely affect many control points over many time periods. To take a unit of water, the user at well i
 575 must have quota to every control point where $F_{i,k}^t > 0$. Rather than holding T quota, one for each
 576 period, the user would be required to hold KT separate constraint quota, a right for every control point
 577 in every period. The market manager now has:

578
$$\text{net revenue} = - \sum_{k=1}^K \sum_{t=1}^T \lambda_k^t \left(\sum_{i=1}^I C_{i,k}^t - \sum_{i=1}^I \sum_{u=1}^t F_{i,k}^{t-u+1} q_i^u \right). \quad (25)$$

579 Scaling quota, then, is necessarily different. To achieve revenue neutrality, the market manager may
 580 wish to scale users' constraint quota C_{ik}^t by α_{ik}^t , in its most general form. It seems appropriate to scale
 581 the quota equally for all users at a given control point and time period, so I drop the subscript i and use
 582 α_k^t . For revenue neutrality,

583 $\lambda_k^t \left(\sum_{i=1}^I \alpha_k^t C_{i,k}^t - \sum_{i=1}^I \sum_{n=1}^t F_{i,k}^{t-n+1} q_i^n \right) = 0$, for each control point k and time period t . (26)

584 The scale factor α_k^t that we seek is easily calculated as

585 $\alpha_k^t = \frac{\sum_{i=1}^I \sum_{n=1}^t F_{i,k}^{t-n+1} q_i^n}{\sum_{i=1}^I C_{i,k}^t}$, for each k and t . (27)

586 Interestingly, α_k^t does not depend directly on the price λ_k^t . It depends on λ_k^t indirectly through the price
 587 impact on the solution q_i^t . This is different to the well quota method, which depended directly on the
 588 price p_i^t .

589 Following market clearing, each user i would be given firm rights to induce drawdown of $F_{ik}^1 q_i^1$ at every
 590 control point k in the current period, and constraint quota of $C_{i,k}^t = \sum_{n=1}^t F_{i,k}^{t-n+1} q_i^n$ for each control
 591 point k and each period $t=2, \dots, T$. A non-participating user i would be restricted to the tightest
 592 constraint: $q_i^1 = \min_{k,t} \{ \alpha_k^t C_{i,k}^t / F_{ik}^1 \}$ for each time period t , which would incentivise participation.

593 Let's look at our continuing example. Recall equations 7 and 8 as

594 Control point 1: $q_1 + q_2 \leq 10$
 595 Control point 2: $q_2 + q_3 \leq 12$

596 Initial quota were 4, 8, and 4, respectively. Suppose instead that the rights were recorded by control
 597 point. Thus, user 1 has rights of 4 to control point 1, and no rights to control point 2. User 2 has rights
 598 of 8 to control point 1 and 8 to control point 2. User 3 has rights of 4 to control point 2 only.

599 Maximizing $q_1 + q_2 + q_3$ as before, the primal solution is (10, 0, 12) as before, and $\lambda_1 = \lambda_2 = \1 , but
 600 those prices will not be required until we calculate the final user payments. We seek a scale factor for
 601 each constraint:

602 For constraint 1, $\alpha_1 = (1*10 + 1*0 + 0*12)/(1*4 + 1*8 + 0*12) = 5/6$.

603 For constraint 2, $\alpha_2 = (0*10 + 1*0 + 1*12)/(0*4 + 1*8 + 1*4) = 1$.

604 Because only control point 1 was over-allocated, we need not adjust the rights for user 3. We set $\alpha_1 =$
 605 $5/6$ and $\alpha_2 = 1$. Table 1 compares the earlier user trades method to this constraint quota method.

606 **Table 1. Comparison of well quota to constraint quota.**

User	C_i	q_i	Well quota method			Constraint quota method		
			p_i	αC_i	User gain	λ_1, λ_2	$\alpha_1 C_{i,1}, \alpha_2 C_{i,2}$	User gain
1	4	10	-\$1	3.67	-\$6.33	\$1, \$1	3.33, 0	-\$6.67
2	8	0	-\$2	7.33	\$14.67	\$1, \$1	6.67, 8	\$14.67
3	4	12	-\$1	3.67	-\$8.33	\$1, \$1	0, 4	-\$8.00

607 Compared to the well quota method, user 1 is slightly worse off and user 3 is slightly better off. In the
 608 well quota method, user 3 was restricted due to the over-allocated control point 1, then bought back
 609 the quota; that money offset user 1's restriction. So user 1 obtained a benefit (better than a free ride) at
 610 user 3's expense. The constraint quota method is likely to be viewed as fairer than the well quota
 611 method, as it assigns the full cost of the environmental impact to the correct users. If environmental
 612 standards were changed, then U_k^t changes, and the constraint quota method would specify which users
 613 would be affected.

614 The constraint quota method could be used with a quantity reduction, reducing first those users with
 615 the largest impact, $\max_i C_{i,k}^t$ for each k and t . In our example, user 2's constraint quota for control point
 616 1 would be reduced from 8 to 6. The solution (10, 0, 12) is unchanged, but user 1 pays \$6, user 2 gains
 617 \$14, and user \$3 pays \$8. Thus, in comparison to the proportional method, user 1 gains at the expense
 618 of user 2.

619 Each user requires constraint quota, and the solution q_i^t applies, for every relevant control point and
620 time period. We therefore might expect that the constraint quota system would converge over some
621 long run to be identical to the well quota system. This would be true if U_k^t (which reflect natural flows)
622 move together, and in the same fashion that rights are adjusted, which is unlikely. Further, if
623 environmental standards change, the market manager will wish to know exactly which users are
624 affected, and by how much, in order to determine any compensation or rights reductions.

625 To clarify how constraint quota C_{ik}^t works over time, consider the following expanded example, in
626 which each user's abstraction in period 1 also has some impact in period 2.

$$627 \text{ Control point 1, period 1: } q_1^1 + q_2^1 \leq 10. \quad (28)$$

$$628 \text{ Control point 1, period 2: } 0.5 q_1^1 + 0.5 q_2^1 + q_1^2 + q_2^2 \leq 10. \quad (29)$$

$$629 \text{ Control point 2, period 1: } q_2^1 + q_3^1 \leq 12. \quad (30)$$

$$630 \text{ Control point 2, period 2: } 0.5 q_2^1 + 0.5 q_3^1 + q_2^2 + q_3^2 \leq 12. \quad (31)$$

631 Translating the users' original well quota of 4, 8, and 4, respectively, in both periods, one might think
632 that the constraint quota is ambiguous in constraint 29 for user 1 in period 2. Is the user's right for
633 $0.5 \cdot 4$ corresponding to q_1^1 in constraint 29, or for $1 \cdot 4$ corresponding to q_1^2 in constraint 29? The
634 answer is both. User 1 has quota of $0.5 \cdot 4 + 1 \cdot 4 = 6$ at control point 1 for period 2. Whether to use
635 that quota in period 1 or period 2 is user 1's choice. Table 2 shows the initial constraint quota.

636 **Table 2. Initial constraint quota for the extended example.**

637 User	1	2	3	1	2	3	1	2	3	1	2	3
638 Control point	1	1	1	1	1	1	2	2	2	2	2	2
639 Period	1	1	1	2	2	2	1	1	1	2	2	2
640 C_{ik}^t	4	8	0	6	12	0	0	8	4	0	12	6

641 Constraint 29 is over-allocated by $6 + 12 - 10 = 8$ units (bold in Table 2). Similarly, constraint 31 is
642 over-allocated by $12 + 6 - 12 = 6$. Calculating α_k^t is straightforward.

643 Some users may find that managing KT different rights is too complicated. Those users could be told
644 their rights in the simpler well quota terms, with the understanding that they would be charged based
645 on the constraint quota method. The constraint quota would be some of the fine print in the contract
646 with the market manager, though savvier users would want to contest their impact coefficients.

647 *Per cent constraint quota*

648 Earlier, it was argued that recording quota as a per cent of available water made little sense, because
649 available water depends on where and when it is taken. But if we specify quota in a way that *does*
650 depend on where and when it is taken, then a percentage quota can work. In this method, we view the
651 constraint quota C_{ik}^t as a per cent of the available drawdown U_k^t , easily calculated from the initial well
652 quota C_i^t .

$$653 C_{i,k}^t = \sum_{u=1}^t F_{i,k}^{t-u+1} C_i^t / \sum_{j=1}^t \sum_{u=1}^t F_{j,k}^{t-u+1} C_j^t. \quad (32)$$

654 Note that $\sum_i C_{ik}^t = 1$. We could have define C_{ik}^t as $C_{i,k}^t = \sum_{u=1}^t F_{i,k}^{t-u+1} C_i^t / U_k^t$, but then this would change
655 when U_k^t changed, and we would also have to rescale C_{ik}^t so that $\sum_i C_{ik}^t = 1$.

656 Following market clearing, each user is given firm rights for the current period to draw down each
657 control point k by the fraction $C_{i,k}^1 = F_{i,k}^1 q_i^1 / \sum_{j=1}^I F_{j,k}^1 q_j^1$, and each user is given quota for periods

658 $2, \dots, T$ to draw down control point k by the fraction $C_{i,k}^t = \sum_{u=1}^t F_{i,k}^{t-u+1} q_i^t / \sum_{j=1}^I \sum_{u=1}^t F_{j,k}^{t-u+1} q_j^t$.

659 Do we need to rewrite constraint 4? No, because all the rights calculations are done after the model is
660 solved. The market manager has

661 net revenue =
$$-\sum_{k=1}^K \sum_{t=1}^T \lambda_k^t U_k^t \left(\sum_{i=1}^I C_{i,k}^t - \sum_{i=1}^I \sum_{u=1}^t F_{i,k}^{t-u+1} q_i^u \right) / \left(\sum_{i=1}^I \sum_{u=1}^t F_{i,k}^{t-u+1} q_i^u \right). \quad (33)$$

662 The terms in parentheses cancel as $(1 - 1)$, so the net revenue is guaranteed to be zero. Thus, this
 663 method needs no scaling to achieve revenue neutrality. If the market manager wants $V \neq 0$, a range of
 664 methods are readily available. This percentage approach simplifies the adjustment process considerably,
 665 and would be handy if U_k^t often changes contrary to forecast.

666 How might users be compensated for tighter environmental standards? In a user pays system, the
 667 policy maker could wait for the beginning of the hydrological year to adjust U_k^t . For systems with long-
 668 term consents, the policy maker could wait for the consent renewal to adjust U_k^t fractionally, as users'
 669 renewals are probably not simultaneous. For systems with permanent consents, the market manager
 670 may be tempted to tell users that their rights were defined as a percentage, and they still have the same
 671 fraction, just of a smaller pie, and therefore no compensation will be forthcoming. But once society has
 672 made a precise agreement with commerce regarding the rights to the commons, changes to the
 673 agreement must follow due process. The policy maker and users, and presumably the courts, would
 674 need to give careful attention to the wording of the rights documents.

675 3 Conclusion

676 Water rights should be adjusted to match the sustainability of the resource, for the sake of future
 677 generations. Within the limits of sustainability, the available water can be allocated many ways, and this
 678 article has demonstrated several of them. The problem of revenue neutrality is identical to that of
 679 matching quota to the available resource.

680 A revenue-positive market will be disliked by users. With a revenue-neutral market, users would be
 681 incentivised to participate voluntarily. A strongly revenue-negative market is almost certainly
 682 unsustainable from the market manager's business point of view. With appropriative rights, a given user
 683 may get less water than a more junior user, due to local conditions and availability, and sometimes
 684 water may be left unallocated. Recording well quota as a per cent of "total water in the catchment" is
 685 not a good idea, as the water available depends on where and when it is taken. Users will probably view
 686 constraint quota to be the fairest method, as free riding is eliminated. In addition, constraint quota
 687 could be recorded as a percentage.

688 Because the methods proposed in this article depend mainly on linear programming duality, results are
 689 not restricted to ground water, and should apply to many smart markets which are cleared by linear
 690 programs, and theoretical market analyses that rely on optimization. Extension, for example, to a smart
 691 market for the combination of surface and water rights, should be entirely straight forward, based on
 692 the appropriate hydrology model, linear programming duality, and definition of rights related to surface
 693 water conveyances.

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