# Matching Users' Rights to Available Ground Water

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24 July 2009

## Abstract

The available water in a catchment depends on where and when the water is taken. This raises considerable man-made uncertainty in water availability. Partly due to this uncertainty, catchments are over-allocated almost everywhere. The over-allocation problem could be managed more effectively if catchment managers could match users' rights to the available water. In this paper, I show how this dilemma can be solved, using the framework of a smart market. A market manager serves as a broker managing a common pool, to buy and sell water. The market manager's revenue reflects the available water relative to users' rights. When the market manager is revenue neutral, the catchment may be viewed as perfectly allocated. I suggest ways in which a catchment manager can find this revenue-neutral allocation, assuming the manager has authority to adjust initial rights.

## Acknowledgements

Thanks to E. Grant Read for many thought-provoking discussions, and for suggesting the notion of constraint rights.

## Key words

Auctions, smart markets, groundwater, water allocation, initial rights

## 1 Introduction to the smart market for ground water.

Water availability is a contentious issue. How much is available? Who has the rights to it? Despite a great deal of law on the matter, government still has no sure way to allocate water rights. Indeed, many catchments are tragically over-allocated, where users have more water rights than there is water. The problem is less difficult for reservoir water, which can be measured easily, and much more difficult for ground water, which is difficult to measure.

A closely related problem to water allocation is protection of the environment. Who should pay for environmental services? Should users pay government for the right to damage the environment, or should government pay users to protect the environment? In many parts of the world, water use is subsidized, but government also pays users to protect the environment. These perverse behaviors need overhaul in order to manage the environment sustainably. Unfortunately, government management is restricted by the available tools. Too often, the tools are simple policies that can be put into a few words, such as "use it or lost it," "first come first served," "the market will sort it out," and, of course, "anything goes." Increasingly, however, water management authorities are using sophisticated hydrological modeling. In some places, this modeling is used to assist pair-wise water trades. But even with this modeling, catchments are often over-allocated, and as a result, the environment is damaged and water is mis-allocated.

In this article, I will assume that the government is operating a smart market for water. (A smart market is a periodic auction cleared by an optimization model, such as a linear program.) I will use this framework to show how government can adjust initial rights so that a given catchment can be perfectly allocated. I will first discuss some of the relevant literature, give an overview of the smart market for ground water, and explain the problem of revenue sufficiency. In Section 2, the main body of the article, I present a range of methods to adjust initial rights to the available water. These methods include proportional scaling, quantity adjustments, a method for appropriative rights, financial adjustments, and constraint-based adjustments. We shall see that some existing methods for scaling rights result in surprising and counter-intuitive outcomes. We shall see that users might find proportional scaling to be unfair. Some methods result in unclaimed water, or a junior user obtaining more water than a senior user. The contribution here should generalize to management of the commons generally.

## Literature review

The work at hand necessarily draws together a wide range of literature, including management of the commons, mechanism design, and hydrogeology.

That the commons requires government management seems no longer in dispute (see for example McAfee (1999), and Baliga and Maskin (2003)). The large literature on mechanism design is heavily focused on auctions for single items, with analyses especially of market power. Most of this work seems irrelevant to the problem of allocating initial rights for water. Water is divisible, obviously, so it makes little sense to discuss Vickrey or combinatorial auctions. As for market power, ground water users are likely to have little of it, because a given catchment is likely to have many participants, and research on protecting the commons has shown that market power declines quickly with the number of market participants (McAfee 1998, Montero 2008). The small catchment of Marlborough, New Zealand, for example, has about eight hundred well owners (mostly vineyards growing fantastic sauvignon blanc).

Disegni-Eshel (2005) showed how to allocate initial rights to a group of competitive and non-competitive firms, for free, to balance efficiency with market power. Again, more participants resulted in less market power. Furthermore, groundwater is spatially dispersed; a well owner has the most control of water nearest the well, and less control of water further away. While market power still needs study in the proposed smart market for ground water, I will ignore issues of market power in this article.

Initial rights can affect efficiency in the presence of large transaction costs (Stavins 1995). However, as I shall explain shortly, I will assume zero transaction costs. Nevertheless, despite Coase (1960), and even assuming only price-taking behavior, the process for setting initial rights does matter (Neuhoff, Martinez & Sato 2006), even when the overall cap is known. For water, and this is the issue at hand, the overall cap is not known in advance, because the catchment manager does not know how much rain the catchment will get. So, even with zero transaction costs, rights should be adjusted in some manner to the available water.

Some researchers have used optimization to adjust initial rights, including for water rights. Wang et al (2007) used network linear programming to allocate initial rights for primarily surface water, accounting for return flows, in-stream uses, reservoir storage rights, and pollution. They calculated “fairness” as weighted water shortage, recognizing different seniorities of rights. Environmental flows were viewed as normal demands. They did not envision their system used for a water market, and rights were not adjusted to the available water; rather, users were allocated quantities short of their rights. Lozano, Villa, & Brännlund (2009) proposed to reallocate pollution emission permits with data envelope analysis, trying to avoid decreases in production and increases in pollutants; a similar procedure could be done with water quantity, but because their method requires private data, I do not think it is practical for market operation.

For a market orientation on water, we should look to the literature on smart markets, such as McCabe et al (1991) and especially Murphy et al (2000). The latter work describes an economics experiment with a small stylized surface water problem. Raffensperger & Milke (2005) and Raffensperger, Milke & Read (2009) described a smart market in which users buy and sell rights to take ground water via a shared pool. That work was inspired by the well-established field of hydrological optimization, which applies math programming to solve hydrogeological problems, such as contaminant remediation (e.g., Ahlfeld and Mulligan 2000). We found that a smart market design based on hydrological optimization elegantly handled the physics, sustainability, and economics. The shared “pool” of water would be managed by a market manager. I will depend on the model proposed in the latter article in this one.

The large literature on electricity markets may appear relevant, due to an almost identical mechanism: a periodic auction cleared by a linear program (see, for example, Read, Drayton-Bright & Ring 1998). But despite the similarity as smart markets, electricity and water markets will be quite different. The number of market participants is different by orders of magnitude, and ground water has only a natural supply. The informed reader may immediately think of the financial trades in the electricity markets, and try to make the analogy to water. Joskow and Tirole (2000) and Gilbert, Neuhoff, and Newberry (2004) discussed financial transmission rights and market power, and these issues are critical in a market with few players. But unlike ground water, initial rights and the available quantities are clear in electricity markets, as generators have initial rights for generation, and the network operator usually owns the initial transmission rights. In any case, the water rights discussed in this article are not financial ownership of the rent on a constraint, nor contracts about price changes, but rights (options) to take physical water. Following establishment of the spot market for ground water, participants will want to develop financial trading instruments, such as to hedge against price fluctuations, but the discussion here is only about the physical rights.

Most relevant to this article, and water allocation, is that a smart market can slash transaction costs, while managing rights and a range of complicated constraints. I will mention additional articles in the remainder.

## Review of the spot market design for ground water

To motivate the main part of the article, this section briefly reviews the smart market for ground water in Raffensperger & Milke (2005) and Raffensperger, Milke & Read (2009).

### *Assumptions and terminology*

A *user* is someone who takes water for a known purpose, e.g., irrigating a vineyard. All users are within one *catchment*, which is a region of land that is relatively isolated hydrologically from other regions. I assume that government has authority to control user's rights to and use of water, and that government has the means and will to enforce market rules. I assume that water is metered and generally short.

I assume that water management authorities can allocate two types of rights, which differ mainly in their term.

- *Quota* is a legal option to take water for a short period, e.g., a week or a month. This quota is administrative permission to a particular user to use a certain amount of water for a particular application for a particular period of time. Water itself is not traded, but rather permission to take water is traded.
- *Consent* is a legal option to take water at a given location, perhaps with a range of restrictions, for a long period, e.g., 10 to 30 years, or permanently. Consent confers quota automatically.

I assume that a *market manager* operates a spot market for ground water quota within the catchment. The market manager clears the market with the help of a linear program (GWMarket below) which includes explicit constraints for environmental flows. The market manager requires knowledge of the user's return flows to calculate the effects of the user's abstraction. Thus, I assume that consent and quota are for a known purpose. I assume that every user's demand function is independent of the user's quota, and that a deterministic market clearing model is acceptable.

The market does not manage bilateral transactions between participants. Rather, trades are to and from a common pool through the market manager. During bidding for a given auction, users give bids via the auction web site, for each period of the planning horizon, e.g., each week through the end of the season. Then the market manager solves GWMarket and informs users of prices and allocations. Users have an initial quota, so purchases and sales are the difference of the initial quota to the market allocation.

Through the spot market, a consent holder may lease the associated quota to another user, and a user without consent may rent quota from another user. I will use the terms "buy" and "sell," but it should be understood that these transactions are short-term rentals in a spot market. After market clearing, sales or purchases are calculated as the difference of the market allocation to the initial quota.

A person who does not hold consent (or even land) could get information to choose a location for operations. To enter the market, a new user could review a map, made available by the market manager, showing prices by location (adjusted for type of use) within the catchment, seeking a well with a relatively low price for water. The user then applies to the market manager to be accepted into the market, satisfying such criteria as reasonable use for the water, sufficient financial stability for the transactions, notification of the location of the user's well, guarantees about metering, etc. The new user does not need to apply for a quantity, but instead can simply rent quota from the market.

*The linear program*

I next introduce the gross pool market-clearing model for groundwater.

**Indices**

$b$=1,…, $B$, demand tranche.

$i, j$ =1,…, $I$, wells, assumed to be in one-to-one correspondence with users.

$u$, $t$=1,…, $T$, time periods. Period 1 is the present period (e.g., week). Each auction occurs at the beginning of period 1, and the subscript is updated by an external program. Period $T$ is the last period of the season or hydrological year. Thus, the first auction of the hydrological year could have 52 weeks; the second auction could have 51 weeks, and so on, until the final auction is for week $T$=52 only. (A rolling horizon may be more appropriate. See Raffensperger, Milke & Read (2009) for a related discussion.)

**Parameters**

$C_i^t$ = initial quota for user $i$ in time period $t$. This is not used in Model GWMarket below, but is required to calculate net sales following market clearing. Managing this initial quota is the key issue addressed in this article.

$F^t_{ik}$ = drawdown rate at control point $k$, $t-1$ periods after abstraction at well $i$, in [meters of head] per [cubic meter of water per second]. Note that $F^1_{ik}$ is the drawdown in the period of abstraction, not necessarily in period 1. Almost always $F^t_{ik} \geq 0$. Hydrologists can obtain $F^t_{ik}$ with standard hydrology software (Ahlfeld, Barlow, & Mulligan, 2006).

$P^t_{ib}$ = the bid price for demand quantity $Q^t_{ib}$ at user $i$, period $t$.

$Q^t_{ib}$ = the bound on demand tranche $b$ for abstraction by user $i$, period $t$.

$U_k^t$ = the upper bound on drawdown at control point $k$, period $t$, in meters.

$V$ = the minimum net revenue that the market manager requires from users for a given auction. This is not used in Model GWMarket below, but will be used in the market clearing process. $V$ need not be positive if the market manager is willing to make a net payment to users. $V$ is positive, if, for example, the market manager wishes to have net revenue to cover the costs of running the auction. Users will be inclined to ensure that the market manager recovers only what is needed for market operation. However, payment for market operations should be seen as separate from adjusting initial rights to match the available resource.

**Variables**

$p_i^t \leq 0$, market price per unit of water faced by well $i$, period $t$. This is the dual price on constraint 3 below.

$\lambda_k^t \geq 0$, price per unit of head at control point $k$, period $t$. This is the dual price on constraint 4 below.

$q^t_{ib}$ = abstraction at bid price $P^t_{ib}$ for the well $i$, period $t$.

$q_i^t$ = total abstraction by well $i$ during period $t$.

**Model GWMarket**

Maximize $\sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{b=1}^{B} P_{i,b}^{t} q_{i,b}^{t}$, subject to (1)

$0 \le q_{ib}^{t} \le Q_{ib}^{t}$, for tranches $b=1,\ldots,B$, users $i=1,\ldots,I$, and periods $t=1,\ldots,T$. (2)

$q_i^t = \sum_{b=1}^{B} q_{ib}^{t}$, for users $i=1,\ldots,I$, and periods $t=1,\ldots,T$. (Dual variable $p_i^t$.) (3)

$\sum_{i=1}^{I}\sum_{u=1}^{t} F_{i,k}^{t-u+1} q_i^u \le U_k^t$, for control points $k=1,\ldots,K$, and periods $t=1,\ldots,T$. (Dual variable $\lambda_k^t$.) (4)

**Explanation**

1. The objective function maximizes total profit from water abstraction over the planning horizon, assuming users bid truthfully.

2. Abstraction in each tranche is bounded by bid quantities.

3. The abstraction equals the total cleared. The dual variable $p_i^t$ of this constraint is the marginal profit to the market for another unit of water taken by well $i$ in period $t$.

4. The drawdown at each control point is limited, e.g., to prevent coastal salt water intrusion, to maintain minimum stream flows, or to limit aquifer drawdown. This constraint set guarantees that every solution is environmentally sustainable, as defined by the constraints.

I assume $U_k^t \ge 0$, so Model GWMarket is always feasible mathematically. That is, I assume that all environmental constraints can be satisfied when no one abstracts water. The market model manages human-induced impact on drawdown, not natural events.

Because the hydrology data comes from standard hydrology models and software, this system can be set up in any catchment for which a standard hydrology model (e.g. MODFLOW) is available. Many U.S. locations already have such models, and they are often used to help settle legal disputes.

*Auction operation*

During bidding for a given auction, users specify bids for every remaining period of the planning horizon, e.g., each week through the end of the season. After bids are entered, the market manager solves the linear program, then informs users of the final prices and allocations. Under marginal cost pricing, the user at well $i$ pays $\sum_{t=1}^{T} p_i^t (C_i^t - q_i^t)$, assuming initial rights $C_i^t$ are not scaled or adjusted. Note that the market manager's *required* net revenue $V$ is not automatically the same as the actual net revenue $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t (C_i^t - q_i^t)$.

The user would then have firm rights to take up to $q_i^1$ in the current period, i.e., period 1, and would have quota for $q_i^t$ for periods $t = 2,\ldots,T$. Because all remaining periods are open for bid in every period, users will tend to trade only differences between their adjusted right and what they actually wish to have. More importantly, inflows are uncertain, so future quota are subject to adjustment by the market manager. This problem of adjustment is the main issue addressed in this article.

**Revenue sufficiency**

*Under-allocated catchment*

If water is in excess supply, the constraints in set 4 are slack, so $p_i^t = 0$ and $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t (C_i^t - q_i^t) = 0$. In this case, neither users nor policy makers have much interest in market operation. For this article, I assume that water is generally short.

*Over-allocated catchment*

Worldwide, governments have given more quota than is sustainable, whether because environmental standards have tightened, because catchments face increased drought, or because governments have been careless. The case of an over-allocated catchment is important, as it is the root of the world water crisis.

If $\sum_{i=1}^{I}\sum_{t=1}^{T} C_i^t > 0$, feasibility ($q_i^t = 0$ in the most extreme situation) may require that some users are willing to sell. In this case, our design requires the market manager to buy quota from users – paying users to forego water – to protect the environment. Over-allocation may not be a problem in the short run if users are willing to sell for a "reasonable" price. If $V << 0$, the manager pays out large amounts, and the market may be viewed as a procurement (or reverse auction) where users are paid not to damage the environment.

Governments sometimes buy rights from users to protect the environment (McGauran 2006, NSW DNR 2006). Cummings, Holt & Laury (2004) developed a web-based auction system for government to pay farmers to reduce irrigation during drought, in the Flint River Basin, Georgia. The authors' system was not a water trading system, but an irregular drought management system without hydrology modeling. The budget was set in advance, and government accepted the lowest bids until the budget was gone. However, the environment absorbed the difference between the government's decision and any over-allocation. Similarly, Stoneham et al (2003) described the Australian government's environmental services procurement program. Government has given too many rights to commerce, and now must buy back those rights to protect the environment.

If the catchment is over-allocated, users may realize that they can demand a high price. This is likely to happen as a user observes that her offer to sell was accepted, so she raises the offer price and observes that she improves her revenue. This effect would be enhanced if users collude, but users may observe the effect independently over many auctions, especially if the catchment is over-allocated regularly. The users are likely to catch on even faster if the market manager operates tentative auctions to give users opportunity for price discovery. An over-allocated catchment can result in high payments from the market manager to users, and it is easy to show that under modest assumptions, for a large catchment with many users, these payments can be arbitrarily high.

If users set $P_{ib}^t$ to a high enough price that $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t(C_i^t - q_i^t) < V$, the market manager cannot clear the market within budget. In this case, I will call the catchment is *over-allocated* and will say that the associated auction model is infeasible.

It is my opinion that giving people rights to assets which do not exist is bad policy. Paying people to forego environmental damage is politically expedient in the short run, but such payments cannot be viable over a long term through a range of economic conditions. Eventually, perhaps due to public outcry, the government will not continue paying business to prevent damage to the commons.

If local law allows the market manager to adjust $C_i^t$, policy makers can choose $V$ as part of auction design and operation. As we shall see, to find a feasible solution at reasonable cost to the market manager, the manager requires authority to adjust quota. I therefore assume that the market manager has such authority, and I define a parameter corresponding to this adjustment.

$\alpha_i^t$ = % of initial quota $C_i^t$ available to the user at well $i$ in period $t$.

In the following, I will occasionally change subscripts on $\alpha_i^t$. For example, the scalar $\alpha$ implies the same scaling for all users in all time periods.

If the market manager adjusts quota, then net revenue can be chosen by setting $\alpha_i^t$ in such a way that revenue exactly equals the desired target $V$, after auction clearing: $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t(\alpha_i^t C_i^t - q_i^t) = V$. Clearly, $\alpha_i^t$ allows flexibility as to which users will get the most adjustment. How should $\alpha_i^t$ be set? Which users should be allocated the most rights? I shall address this shortly.

## Optimal allocations and prices are independent of initial allocations

Model GWMarket results in the same flows $q_i^t$ and the same prices $p_i^t$, for any feasible quota allocation $C_i^t$, except for alternative optima, and assuming all users participate. Mathematically, this follows trivially because buy and sell quantities are calculated after the model is solved. We also need the assumptions that every user's demand function is independent of the user's quota, and is willing to bid so that $\sum^{I}_{i=1}\sum^{T}_{t=1} p_i^t(\alpha_i^t C_i^t - q_i^t) \geq V$, as stated earlier. Economists will note that Coase (1960) proved the same theorem more generally, assuming sufficiently small transaction costs. We should not pass over the assumption too quickly.

Existing water markets worldwide have high transactions costs due to the need to find a trading partner, negotiate and enforce a contract, and manage externalities through protracted government approval processes. The smart market eliminates these transactions costs, thus satisfying the assumptions for Coase's theorem. Hence, if all users participate, the market manager need not re-solve Model GWMarket to find the preferred $\alpha_i^t$.

To relax the assumption that all participate, we can require that non-participants and participants are scaled similarly. Otherwise, a non-participant could get 100% allocation for free, while the participant gets scaled to 80%, then buys back to 100%, in which case the participant would have been better off without the market. The effect of non-participants is to fix $q_i^t = \alpha_i^t C_i^t$ in Model GWMarket. These users are actually participating, in that they will be taking water; they simply have bids with a sell price of infinity and a buy price of zero. Yet scaling is done *after* the market model is solved, to achieve the revenue goal $V$. So there is a "chicken-and-egg" problem of requiring $\alpha_i^t$ before the model is solved, but calculating it afterwards. This issue may be resolved through a variety of mechanisms, including requiring all users to participate (perhaps for sufficiently large $C_i^t$ and ignoring those below that), using approximate scaling factors for non-participants (perhaps a bit on the low side to incentivise participation), simply charging each non-participant $i$ the amount $p_i^1(1 - \alpha_i^1)C_i^1$, or solving Model GWMarket iteratively to find the right $\alpha_i^t$.

## 2 Methods of adjusting quota

This section describes methods to choose $\alpha_i^t$ so that the auction model is feasible, i.e., $\sum^{I}_{i=1}\sum^{T}_{t=1} p_i^t(\alpha_i^t C_i^t - q_i^t) = V$.

**User-pays method, revenue positive**

Perhaps the most obvious value for $\alpha_i^t$ is $\alpha_i^t = 0$. This corresponds to a user pays market, where $\alpha_i^t C_i^t = 0$ for all $i$ and $t$. In this method, $\sum^{I}_{i=1}\sum^{T}_{t=1} p_i^t(\alpha_i^t C_i^t - q_i^t) = -\sum^{I}_{i=1}\sum^{T}_{t=1} p_i^t\, q_i^t = V >> 0$, and the manager receives a large revenue. The market may be viewed as an auction in which users buy all quota from the market manager. (For simplicity of exposition, I will continue to use the word auction to mean one event of this market's operation, regardless of the manager's revenue.) Auctions for public resources are, of course, widely implemented. The revenue raised can be used to offset other distorting taxes, as could be done with carbon emissions (Cramton & Kerr 2002).

It is important to remember, however, that water has already been allocated almost everywhere. If $V >> 0$, existing users may not want to implement the proposed market, because they face a significant outlay. If government no longer issued consents for water, prospective users would wish to have the market, as it would give them an opportunity to get quota for water. With centuries of hard-won rights already in place, I think a user-pays system will be unacceptable to users.

Even if we assume a user-pays system, the need to scale rights to the available water remains. Reservoir and ground water have storage components, so the effects can lag over many periods. Furthermore, users will want some certainty over the full growing season at least, and would not want to commit to a crop without a belief that water will be available for those crops. The allocation problem is therefore multi-period. After period 1, the market manager has granted quota to every user for future periods $2,\ldots,T$. Thus, when period 2 arrives, users will already hold quota for which they paid in period 1. But period 2 flows are unlikely to match their forecast, so the market manager will have to adjust rights again in some fashion, and the scaling problem reappears.

**Proportional adjustment**

People sometimes like to allocate goods proportionally. We can adjust quotas proportionally in a variety of ways.

*Myopic maximum proportion*

One intuitive method of scaling initial rights is to find the maximum proportion $\alpha$ that all users could take sustainably, found with Model MaxProportion below. I drop the subscripts on $\alpha_i^t$, because all users have the same proportion. As we shall see, this solution is myopic, because it ignores the economics. Denote the solution to MaxProportion as $\underline{\alpha}$.

Model MaxProportion: maximize $\underline{\alpha}$, subject to (5)

$q_i^t = \underline{\alpha} C_i^t$, for all wells $i$=1,…, $N$, and periods $t$=1,…, $T$, (6)

and constraints 4.

Because I assumed that a feasible solution exists with $q_i^t = 0$, then MaxProportion has a feasible solution. Furthermore, if the optimal $\underline{\alpha}$ is positive and if all $C_i^t > 0$, then every user is guaranteed some initial quota.

Now the thoughtful reader may think that the lesson from this exercise is that quota should not be a quantity, but rather a fraction of the available water. Another guess is that net revenue to the market manager will be zero, so $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t(\underline{\alpha} C_i^t - q_i^t) = 0$. Neither guess is true.

Consider the following example (which will appear throughout the article).

$$\text{Control point 1:} \qquad q_1 + q_2 \leq 10 \tag{7}$$

$$\text{Control point 2:} \qquad q_2 + q_3 \leq 12 \tag{8}$$

The solution (10, 0, 12) maximizes $q_1$; it also maximizes $q_3$.The solution (0, 10, 2) maximizes $q_2$. These solutions allocate different quantities of water, 22 units in the first solution, and only 12 units in the second. Even for a trivial problem, the sustainable available water depends on *where* it is taken. For real problems, available water also depends on *when* it is taken; a price may be high now to prevent future damage. Furthermore, available water changes quickly and variably over space and time. Thus, speaking of the fraction of available water makes little sense. Consent and quota should not be registered or recorded as fractions of some notional "total water in the catchment".

Regarding net revenue, suppose $C_i$ = 4, 8, and 4 for users 1, 2, and 3 respectively. The catchment is over-allocated at control point 1, because $C_1 + C_2 = 4 + 8 > 10$, but is perfectly balanced at control point 2, because $C_2 + C_3 = 8 + 4 = 12$. The solution to Model MaxProportion is $\underline{\alpha} = 10/12$, based on control point 1. User 3 will complain, because she has responsibility only to control point 2, which was perfectly allocated. In the market, she would have to buy back her original quantity, if she wanted it.

If each user bids \$1/unit quota, the solution is (10, 0, 12). User 3 *does* buy back her original allocation, plus more. The dual prices are $p_1 = -\$1, p_2 = -\$2, p_3 = -\$1$. Net revenue to the market manager is –\$1(4*10/12 – 10) + –\$2(8*10/12 – 0) + –\$1(4*10/12 – 12) = \$2. Thus, the auction is revenue positive.

In general, Model MaxProportion will choose $\underline{\alpha}$ based on the "driest" well. This is most likely the user closest to a particular most-stressed environmental control point. The optimal $\underline{\alpha}$ and the suspect user may be identified as follows.

Substitute $q_i^t = \underline{\alpha} C_i^t$ into constraint set 4: $\sum_{i=1}^{I}\sum_{u=1}^{t} F_{i,k}^{t-u+1}\underline{\alpha} C_i^t = \underline{\alpha}\sum_{i=1}^{I}\sum_{u=1}^{t} F_{i,k}^{t-u+1} C_i^t \leq U_k^t$, and so:

$$\underline{\alpha} = \min_{k,t}\left\{ U_k^t \Big/ \left( \sum_{i=1}^{I}\sum_{u=1}^{t} F_{i,k}^{t-u+1} C_i^t \right) \right\}. \tag{9}$$

Denote by $(k^-, t^-)$ as the control point and time period which most constrain $\underline{\alpha}$. The user with the "driest" well is user $i^- = \text{argmax}_i \sum_{u=1}^{t^-} F_{i,k^-}^{t^- -u+1} C_i^{t^-}$ .

Questions of fairness arise. All users' quota have been reduced to that of the driest user, but if this user $i^-$ took one unit less, users away from the environmentally sensitive areas could have more than a unit of water. As a result, $\underline{\alpha}$ almost always results in an auction which is revenue positive for the market manager, because most users will buy back some quota.

*User trades, well quota proportion, revenue neutrality*

A particularly interesting case is where $V \approx 0$. In this case, the market manager is serving as a broker to clear the market, maintaining no financial position; the catchment may be viewed as perfectly allocated. Users are likely to participate voluntarily, unilaterally choosing to buy or sell without government coercion. Non-participating users are likely to have a marginal value for water that is different to the price, and thus they will have incentive to trade. Thus, the user trades method gives the policymaker a simple way to start and expand the market.

The $\alpha^0$ we seek, indicating revenue neutrality with the superscript 0, solves the following equation.

$$\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t(\alpha^0 C_i^t - q_i^t) = V = 0. \qquad (10)$$

This is easily solved: $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t(\alpha^0 C_i^t - q_i^t) = \sum_{i=1}^{I}\sum_{t=1}^{T} (\alpha^0 p_i^t C_i^t - p_i^t q_i^t)$

$= \alpha^0 \sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t C_i^t - \sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t q_i^t = 0$, which implies

$$\alpha^0 = \left(\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t q_i^t\right) / \left(\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t C_i^t\right). \qquad (11)$$

Using our earlier example, the solution (10, 0, 12) maximized $q_1+q_2+q_3$, supposing identical bids. Now solve for $\alpha^0$:

$-\$1(4*\alpha^0 - 10) + -\$2(8*\alpha^0 - 0) + -\$1(4*\alpha^0 - 12) = 0$, so $\alpha^0 = 11/12$.

Users 1, 2, and 3 buy 6.333, –7.333, and 8.667, respectively. Users 1 and 3 buy back only 1/12th of their original quota. User 2 sells everything, but is paid for only 11/12ths. Still, we can imagine user 3 still complaining (as control point 2 was never over-allocated), just not as loud as with the solution to Model MaxProportion. The same problem of unfairness occurs, but to a lesser degree only because less money goes to the market manager.

Every user will prefer the user trades proportion to the myopic proportion. This is easily proved, and it suffices to show that $\underline{\alpha} \le \alpha^0$. Simply observe that by linear programming duality, Model GWMarket simultaneously maximizes $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t q_i^t$ and minimizes $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t C_i^t$. Therefore, the ratio of these two terms is maximized by the optimal GWMarket solution $q_i^{t*}$, which means $\alpha^0$ is the largest possible ratio, so $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t \underline{\alpha} C_i^t \le \sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t q_i^{t*}$, and the result follows. With a larger proportion of initial consent, sellers receive more money, buyers pay less money, and the market manager nets less money.

Intuitively, because $\underline{\alpha}$ is based on the tightest constraint in set 4, $\underline{\alpha}$ is likely to be quite small, say, 10%. If the initial quota are remotely close the right value, then $\alpha^0$ will be close to 100%. Hence, users will prefer the user trades method to the myopic method.

The user trades method may be viewed as distributing the scarcity rents among the users. In a market where binding constraints correspond to private assets, such as power line capacity in an electricity market, this point of view makes sense. The owner of the capacity has incentive and compensation to increase capacity. (Interestingly, in New Zealand, the electricity network operator Transpower returns a sizeable fraction of line capacity rents to their customers (ECNZ 2008).) However, environmental constraints may be viewed as public goods; the "capacity" is the allowable draw down of the environmental resource. Government could allocate all sustainable water to users, but (barring technological changes that expand the sustainable capacity of the environment) should not allocate more than that.

*User trades, proportion by time period*

The market should signal to users and policymakers information about the future levels of quota. To provide this signal, the market manager can choose a method of scaling to have revenue neutrality in every period of the current auction model. This is done easily by adding a subscript for time, $\alpha_t^0$, then solving for $\alpha_t^0$.

$$\alpha_t^0 = \sum_{i=1}^{I} p_i^t q_i^t / \sum_{i=1}^{I} p_i^t C_i^t, \text{ for } t=1, \ldots, T. \qquad (12)$$

In this way, a given user's quota may be scaled differently for each future period. Thus, in period 1, if a user's quota for period 2 were scaled by $\alpha_2^0 = 15\%$, the user is signaled that the future period is expected to be relatively dry.

The market manager could choose to achieve revenue neutrality over a sequence of auctions, say, a year of weekly auctions. Subscripting by time, the manager would wish $\sum_t V^t \approx 0$. But once the precedent was made for scaling quota, there seems no reason for the market manager to take on the risk.

### Quantity adjustments

Rather than adjust users' quota proportionally, quota could be adjusted by quantity. Again, this may be done in several ways.

*Maximize total abstraction*

Another intuitive method is to begin with the solution that maximizes total water abstraction from the catchment. To maximize total abstraction, we can solve Model MaxWater below, which will give an upper bound on the amount of water available from the catchment. Model MaxWater will be familiar to many hydrologists, as maximizing total flow subject to constraints is a typical hydrological optimization problem.

Model MaxWater: maximize $\sum_{j=1}^{n}\sum_{t=1}^{T} q_i^t$, (13)

subject to constraints 4.

The market manager then sets $\alpha_i^t = q_i^t / C_i^t$.

Following our three-user example, this solution would change user 1's initial allocation from 4 to 10, user 2's allocation from 8 to zero, and user 3's allocation from 4 to 12. Users 1 and 3 would be thrilled, and user 2 is likely to begin court proceedings. This simplistic method ignores any initial right and willingness to pay. Users closest to environmental control points will be most restricted, losing their quota without regard to previously hard-won battles. Given the decreasing marginal utility for water, this method will tend to hurt some users financially more than others.

*Minimize quantity reduction*

We could take an equal quantity from every user. I introduce a scalar variable $r \geq 0$, which is the reduction in every user's quota. (If we wish, we could subscript $r$ by time.)

Model MinReduction1: minimize $r$, subject to (14)

$q_i^t + r = C_i^t$, for all wells $i=1,\ldots, N$, and periods $t=1,\ldots, T$, (15)

$r \geq 0$, (16)

and constraints 4.

In our example, each user would give up one unit of water. The market manager would therefore receive –\$1(3 – 10) + –\$2(7 – 0) + –\$1(3 – 12) = \$2. This is not revenue neutral, just as the myopic proportional reduction is not revenue neutral. Smaller users will be hurt most by this.

*Revenue neutral quantity reduction*

We can easily find a revenue neutral solution, as we did with proportional adjustment, by solving the following equation for $r$.

$\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t (C_i^t - r - q_i^t) = V = 0.$ (17)

Our example solution was (10, 0, 12). The quantity reduction $r$ is found easily by inspection:

$-\$1(4 - r - 10) + -\$2(8 - r - 0) + -\$1(4 - r - 12) = 0$, so $r = 0.5$.

Neither MinReduction1 nor its revenue neutral version may be feasible, because $0 \leq r \leq \min_{i,t} C_i^t$, meaning that the quantity reduction is bounded by the smallest quota, which may not be sufficient for a feasible solution to constraint set 4. We can get around this if we allow $r$ to fall further (with the understanding that no user would be required to inject water into a well), choosing $r$ to satisfy:

$$\sum^{I}_{i=1}\sum^{T}_{t=1} p_i^t(\max(0, C_i^t - r) - q_i^t) = V. \tag{18}$$

This system will take all quota away from each user with quota of $r$ or less, and $r$ units from each of the other users.

To motivate this method, consider a modified version of our previous three-user example, where the initial rights are 4, 16, and 4, respectively. The optimal solution was 10, 0, 12, with prices –\$1, –\$2, and –\$1. Solving for $r$, $-\$1(\max(0, 4 - r) - 10) + -\$2(\max(0, 16 - r) - 0) + -\$1(\max(0, 4 - r) - 12) = 0$, so $r = 5$. Now users 1 and 3 lose all their initial rights, and user 2 is reduced from 16 to 11. Users 1 and 2 must purchase their full quantities, while user 2 sells her remaining quantity. This method will hurt users with small quota, and leave a relatively few large users with water.

*Cut the user with heaviest impact.*

In New Zealand, water management authorities occasionally require some users to stop pumping on short notice, especially to prevent coastal salt water intrusion. The selected users are those closest to the coast, with the immediate impact. The authority has no means to restrict in retrospect users with delayed impacts; lacking is coordinated planning for the future.

Rather than restricting all users equally, we can restrict the user with the highest impact. We can use the earlier notion of the "driest well," owned by user $i^- = \text{argmax}_i \sum_{u=1}^{t^-} F_{i,k^-}^{t^- - u+1} C_i^{t^-}$, associated with control point $k^-$ and time period $t^-$. The argument is that this user has the most impact, so this user should be first to have $C_i^t$ reduced, perhaps until $\sum_{u=1}^{t^-} F_{i,k^-}^{t-u+1} C_i^t$ equals that of the second worst offender. More generally, the market manager reduces $C_i^t$ for all users with the largest impacts until the manager is satisfied with the auction revenue.

This method has some sense of justice to it, as it includes both marginal impact and quantity. The big users who also have the biggest impact will be restricted first. However, one could imagine 99 users who each take 1 unit of water for washing their cars on Sundays, and a 100th user who takes 10 units of water for growing crops. The farmer could be cut back so severely as to become bankrupt.

Other quantity methods include taking water only from large users regardless of impact, taking water from users with the largest marginal impact $F^t_{i,k}$, taking different quantities from different sized users (perhaps step-sized packets), etc.

*Appropriative rights*

U.S. laws in some states specify that rights are acquired by use, and the earliest user has senior rights to later users. In dry years, senior users get water first; junior users cannot "hurt" senior users. Users are required to use water "reasonably", and users argue about what "reasonably" means, especially in dry years, with resolution through the courts. Appropriative rights are a special kind of quantity adjustment.

To create a market for users with appropriative rights, we can adjust Model MaxProportion. (No one is known to use this anywhere in the world; the smart market model which allows the calculation.) Assume that each user $i$ has some $C_i^t$ at which user $i$ is considered to have full quota. Create an $\alpha_i$ for each user, for simplicity dropping the subscript for time. Number the users by seniority, so the senior user is user 1. Beginning with the senior user, for each user $j$, maximize $\alpha_j$ sequentially:

Model Appropriative: for $j = 1, \ldots, I$, maximize $\alpha_j$, subject to (19)

$q_i^t = \alpha_i^* C_i^t$, for all wells $i=1,\ldots,j-1$, and periods $t=1,\ldots, T$. (20)

$q_j^t = \alpha_j C_j^t$, for all periods $t=1,\ldots, T$. (21)

$q_i^t \leq 0$, for all wells $i=j+1,\ldots, N$, and periods $t=1,\ldots, T$. (22)

$0 \leq \alpha_j \leq 1$, (23)

and constraints 4.

Fix $\alpha_j$ to its optimal value $\alpha_j^*$ and go to the next $j$.

Following this initial allocation, the market manager would run the auction as usual with Model GWMarket. Following that run, the market manager may adjust quota again, this time proportionally to achieve revenue neutrality.

This method ensures that the senior user has absolute first use. Another interpretation of appropriative rights instead requires that the most senior user obtains at least as much water as the next junior. This can be done by adding constraints $\alpha_i \geq \alpha_{i+1}$, rather than solving for $\alpha_i$ for each trader. Which method is correct? Similarly, the issue of timing of rights must be addressed: is user 1's right to period 2 water superior to user 2's right to period 1 water? This may depend on the length of the period; probably for a day, but perhaps not for a month. Such questions will be answered in the smoke-filled conferences and stolid court rooms. In any case, the appropriative doctrine will provide a windfall for senior users in the spot market. Over time, however, after consent trades hands, the notion of appropriative rights will become meaningless. States have appropriative water rights because government has not worked out how to operate water markets.

Following our example, suppose users are numbered in increasing seniority, so user 3 is most senior. Further, assume that each user is deemed to have full quota with 7 units. Then user 3 would obtain 7 units, user 2 would receive 5 units, and user 1 would receive 5 units. This follows our intuition that the senior user will get the most water.

However, suppose users are numbered in decreasing seniority, so user 1 is most senior. Now user 1 would get 7 units, user 2 would get 3, and user 3 would get 7, leaving 2 units unallocated! Seniority does not always confer more water, because the market manager must consider water availability. The unallocated water is a bigger surprise. On reflection, allocation by seniority is unlikely to result in an optimal solution to Model MaxWater.

## Financial adjustments

We can define adjustment methods based on final net payments. The market manager could choose to achieve revenue neutrality by charging users directly rather than changing their initial quota. This may be viewed as a sales tax $\tau_i^t$.

$$\tau_i^t = p_i^t \,|\, C_i^t - q_i^t \,|\cdot \textstyle\sum_{j=1}^{I} p_j^t (C_j^t - q_j^t) \,/ \textstyle\sum_{j=1}^{I} p_j^t \,|\, C_j^t - q_j^t \,|, \text{ for all } i \text{ and } t. \quad (24)$$

The sales tax rate is $\sum_{j=1}^{I} p_j^t (C_j^t - q_j^t) \,/ \sum_{j=1}^{I} p_j^t \,|\, C_j^t - q_j^t \,|$.

Following our example, the optimal solution was:

| User | $C_i$ | $p_i$ | $q_i$ | User gain |
|---|---|---|---|---|
| 1 | 4 | –\$1 | 10 | –\$6 |
| 2 | 8 | –\$2 | 0 | \$16 |
| 3 | 4 | –\$1 | 12 | –\$8. |

Here, $\sum_{j=1}^{I} p_j^t (C_j^t - q_j^t) = \$2$, and $\sum_{j=1}^{I} p_j^t \,|\, C_j^t - q_j^t \,| = \$30$. The market manager can therefore require additional payments of \$12/30, \$32/30, and \$16/30, from users 1, 2, and 3, respectively, thus achieving revenue neutrality. This method charges all participants, including those with no initial quota. It does not specify a way to scale non-participants.

The extension to fixed charge payments corresponds to a fixed transaction cost on trades: charge each user $\sum_{j=1}^{I} \sum_{t=1}^{T} p_j^t (C_j^t - q_j^t)/I$. Other extensions include the option to charge only buyers, or only sellers, or to adjust each user's initial quota $C_i^t$ to imply the tax $\tau_i^t$, e.g., 4 – 12/30, 8 – 16/30, 4 – 16/30 for each respective user in our example. Indeed, the market manager could use any of many possible methods to achieve the revenue target, but not every method would relate users' rights to the available resource. Unless the initial quota is adjusted, users do not receive direct signals about how their quota will be adjusted in the future. So this method seems appropriate to raise revenue to cover the auction costs, not a way to match rights to the available water. We shall see next a method which relates the users' rights much more closely to the resource than any thus discussed.

**Constraint quota**

To now, I have described rights in terms of the user and his or her location: the quantity of water that can be taken from the user's own well. Rights can instead be defined in terms of the user's impact on the control point.

Montgomery (1972), in the context of air pollution, defined "ambient" permits, perhaps an unfortunate name, where the regulator specified separate rights for each control point. A user wishing to discharge at his or her own location had to obtain sufficient separate rights for each control point that his or her discharge would impact. Montgomery's method allowed for multiple control points, in theory.

Systems based on "trading ratios" (Horan, Shortle, and Abler 2002; Kerr, Rutherford, and Lock 2007; many others) have few control points, ideally one, and sometimes using zonal trading ratios, where all users in a give zone have the same ratio. These ratios correspond to oversimplified $F^t_{ik}$ coefficients in Model GWMarket. In a way, the rights adjustment methods that I proposed to now are similar to these trading ratio systems, in that the right is usually defined in terms of the user's behavior, not the user's impact. Montgomery's ambient permit system defined rights in terms of the user's impact, not their behavior.

Of course, even with only one control point, these systems suffer from high transaction costs. Market designers oversimplified the physics in an attempt to reduce the transaction costs, understanding that by doing so, they were creating fewer commodities, which should allow easier trade. But the transaction cost was not nearly reduced enough (Stavins 1995).

The smart market suffers from none of these limitations. Linear programming can easily manage a rich set of control points and impact factors, which affect a given control point dynamically over time. The auction format provides a venue for the market. The market clearing process assures the regulator that the commons is respected. And the model formulation lets us choose whether to specify rights based on the user's behavior or the user's impact. So let us now explore the latter.

*Quantity constraint quota*

Denote the dual price of constraint ($k$, $t$) of set 4 as $\lambda_k^t$. From linear programming duality, $\sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{b=1}^{B} P^t_{i,b}q^t_{i,b} = \sum_{t=1}^{T}\sum_{k=1}^{K}\lambda^t_k U^t_k$. Thus, the value of the right to take water corresponds to the value of the resource. This insight changes the notion of a right to take water into a right to reduce head at a set of control points. This latter interpretation corresponds to a claim of $F^t_{ik}C^t_i$ against the right hand side $U_k^t$.

I will use the term *constraint consent* to mean a long-term right to reduce head at a control point, and *constraint quota* $C^t_{ik}$ to mean the right of user $i$ to reduce head by $C^t_{ik}$ at control point $k$ in period $t$. A constraint quota is for a single period and a single control point. Abstraction of water from a given well will likely affect many control points over many time periods. To take a unit of water, the user at well $i$ must have quota to every control point where $F^t_{i,k} > 0$. Rather than holding $T$ quota, one for each period, the user would be required to hold $KT$ separate constraint quota, a right for every control point in every period. The market manager now has:

$$\text{net revenue} = -\sum_{k=1}^{K}\sum_{t=1}^{T}\lambda^t_k\left(\sum_{i=1}^{I}C^t_{i,k} - \sum_{i=1}^{I}\sum_{u=1}^{t}F^{t-u+1}_{i,k}q^u_i\right). \tag{25}$$

Scaling quota, then, is necessarily different. To achieve revenue neutrality, the market manager may wish to scale users' constraint quota $C^t_{ik}$ by $\alpha^t_{ik}$, in its most general form. It seems appropriate to scale the quota equally for all users at a given control point and time period, so I drop the subscript $i$ and use $\alpha_k^t$. For revenue neutrality,

$$\lambda_k^t\left(\sum_{i=1}^{I}\alpha_k^t C_{i,k}^t-\sum_{i=1}^{I}\sum_{u=1}^{t}F_{i,k}^{t-u+1}q_i^u\right)=0, \text{ for each control point } k \text{ and time period } t. \qquad (26)$$

The scale factor $\alpha_k^t$ that we seek is easily calculated as

$$\alpha_k^t=\sum_{i=1}^{I}\sum_{u=1}^{t}F_{i,k}^{t-u+1}q_i^u\Big/\sum_{i=1}^{I}C_{i,k}^t, \text{ for each } k \text{ and } t. \qquad (27)$$

Interestingly, $\alpha_k^t$ does not depend directly on the price $\lambda_k^t$. It depends on $\lambda_k^t$ indirectly through the price impact on the solution $q_i^t$. This is different to the well quota method, which depended directly on the price $p_i^t$.

Following market clearing, each user $i$ would be given firm rights to induce drawdown of $F^1_{ik}q_i^1$ at every control point $k$ in the current period, and constraint quota of $C_{i,k}^t=\sum_{u=1}^{t}F_{i,k}^{t-u+1}q_i^t$ for each control point $k$ and each period $t$=2,…,$T$. A non-participating user $i$ would be restricted to the tightest constraint: $q_i^1=\min_{k,t}\{\alpha_k^t C^t_{ik}/F^1_{ik}\}$ for each time period $t$, which would incentivise participation.

Let's look at our continuing example. Recall equations 7 and 8 as

Control point 1: $q_1 + q_2 \leq 10$
Control point 2: $q_2 + q_3 \leq 12$

Initial quota were 4, 8, and 4, respectively. Suppose instead that the rights were recorded by control point. Thus, user 1 has rights of 4 to control point 1, and no rights to control point 2. User 2 has rights of 8 to control point 1 and 8 to control point 2. User 3 has rights of 4 to control point 2 only.

Maximizing $q_1 + q_2 + q_3$ as before, the primal solution is (10, 0, 12) as before, and $\lambda_1 = \lambda_2 = \$1$, but those prices will not be required until we calculate the final user payments. We seek a scale factor for each constraint:

For constraint 1, $\alpha_1$ = (1*10 + 1*0 + 0*12)/(1*4 + 1*8 + 0*12) = 5/6.

For constraint 2, $\alpha_2$ = (0*10 + 1*0 + 1*12)/(0*4 + 1*8 + 1*4) = 1.

Because only control point 1 was over-allocated, we need not adjust the rights for user 3. We set $\alpha_1$ = 5/6 and $\alpha_2$ = 1. Table 1 compares the earlier user trades method to this constraint quota method.

**Table 1. Comparison of well quota to constraint quota.**

| | | | Well quota method | | | Constraint quota method | | |
|---|---|---|---|---|---|---|---|---|
| User | $C_i$ | $q_i$ | $p_i$ | $\alpha C_i$ | User gain | $\lambda_1, \lambda_2$ | $\alpha_1 C_{i,1}, \alpha_2 C_{i,2}$ | User gain |
| 1 | 4 | 10 | –\$1 | 3.67 | –\$6.33 | \$1, \$1 | 3.33, 0 | –\$6.67 |
| 2 | 8 | 0 | –\$2 | 7.33 | \$14.67 | \$1, \$1 | 6.67, 8 | \$14.67 |
| 3 | 4 | 12 | –\$1 | 3.67 | –\$8.33 | \$1, \$1 | 0, 4 | –\$8.00 |

Compared to the well quota method, user 1 is slightly worse off and user 3 is slightly better off. In the well quota method, user 3 was restricted due to the over-allocated control point 1, then bought back the quota; that money offset user 1's restriction. So user 1 obtained a benefit (better than a free ride) at user 3's expense. The constraint quota method is likely to be viewed as fairer than the well quota method, as it assigns the full cost of the environmental impact to the correct users. If environmental standards were changed, then $U_k^t$ changes, and the constraint quota method would specify which users would be affected.

The constraint quota method could be used with a quantity reduction, reducing first those users with the largest impact, $\max_i C^t_{ik}$ for each $k$ and $t$. In our example, user 2's constraint quota for control point 1 would be reduced from 8 to 6. The solution (10, 0, 12) is unchanged, but user 1 pays \$6, user 2 gains \$14, and user \$3 pays \$8. Thus, in comparison to the proportional method, user 1 gains at the expense of user 2.

Each user requires constraint quota, and the solution $q_i^t$ applies, for every relevant control point and time period. We therefore might expect that the constraint quota system would converge over some long run to be identical to the well quota system. This would be true if $U_k^t$ (which reflect natural flows) move together, and in the same fashion that rights are adjusted, which is unlikely. Further, if environmental standards change, the market manager will wish to know exactly which users are affected, and by how much, in order to determine any compensation or rights reductions.

To clarify how constraint quota $C^t_{ik}$ works over time, consider the following expanded example, in which each user's abstraction in period 1 also has some impact in period 2.

Control point 1, period 1: $$q_1^1 + q_2^1 \leq 10. \tag{28}$$

Control point 1, period 2: $$0.5\, q_1^1 + 0.5\, q_2^1 + q_1^2 + q_2^2 \leq 10. \tag{29}$$

Control point 2, period 1: $$q_2^1 + q_3^1 \leq 12. \tag{30}$$

Control point 2, period 2: $$0.5\, q_2^1 + 0.5\, q_3^1 + q_2^2 + q_3^2 \leq 12. \tag{31}$$

Translating the users' original well quota of 4, 8, and 4, respectively, in both periods, one might think that the constraint quota is ambiguous in constraint 29 for user 1 in period 2. Is the user's right for 0.5*4 corresponding to $q_1^1$ in constraint 29, or for 1*4 corresponding to $q_1^2$ in constraint 29? The answer is both. User 1 has quota of 0.5*4 + 1*4 = 6 at control point 1 for period 2. Whether to use that quota in period 1 or period 2 is user 1's choice. Table 2 shows the initial constraint quota.

**Table 2. Initial constraint quota for the extended example.**

| User | 1 | 2 | 3 | **1** | **2** | **3** | 1 | 2 | 3 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Control point | 1 | 1 | 1 | **1** | **1** | **1** | 2 | 2 | 2 | 2 | 2 | 2 |
| Period | 1 | 1 | 1 | **2** | **2** | **2** | 1 | 1 | 1 | 2 | 2 | 2 |
| $C^t_{ik}$ | 4 | 8 | 0 | **6** | **12** | **0** | 0 | 8 | 4 | 0 | 12 | 6 |

Constraint 29 is over-allocated by 6+ 12 – 10 = 8 units (bold in Table 2). Similarly, constraint 31 is over-allocated by 12 + 6 – 12 = 6. Calculating $\alpha_k^t$ is straightforward.

Some users may find that managing *KT* different rights is too complicated. Those users could be told their rights in the simpler well quota terms, with the understanding that they would be charged based on the constraint quota method. The constraint quota would be some of the fine print in the contract with the market manager, though savvier users would want to contest their impact coefficients.

*Per cent constraint quota*

Earlier, it was argued that recording quota as a per cent of available water made little sense, because available water depends on where and when it is taken. But if we specify quota in a way that *does* depend on where and when it is taken, then a percentage quota can work. In this method, we view the constraint quota $C^t_{ik}$ as a per cent of the available drawdown $U_k^t$, easily calculated from the initial well quota $C_i^t$.

$$C^t_{i,k} = \sum\nolimits_{u=1}^{t} F^{t-u+1}_{i,k} C^t_i \Big/ \sum\nolimits_{j=1}^{I} \sum\nolimits_{u=1}^{t} F^{t-u+1}_{j,k} C^t_j \,. \tag{32}$$

Note that $\sum_i C^t_{ik} = 1$. We could have define $C^t_{ik}$ as $C^t_{i,k} = \sum\nolimits_{u=1}^{t} F^{t-u+1}_{i,k} C^t_i \Big/ U^t_k$, but then this would change when $U_k^t$ changed, and we would also have to rescale $C^t_{ik}$ so that $\sum_i C^t_{ik} = 1$.

Following market clearing, each user is given firm rights for the current period to draw down each control point *k* by the fraction $C^1_{i,k} = F^1_{i,k} q^t_i \Big/ \sum\nolimits_{j=1}^{I} F^1_{j,k} q^t_j$, and each user is given quota for periods 2,…,*T* to draw down control point *k* by the fraction $C^t_{i,k} = \sum\nolimits_{u=1}^{t} F^{t-u+1}_{i,k} q^t_i \Big/ \sum\nolimits_{j=1}^{I} \sum\nolimits_{u=1}^{t} F^{t-u+1}_{j,k} q^t_j$ .

Do we need to rewrite constraint 4? No, because all the rights calculations are done after the model is solved. The market manager has

$$\text{net revenue} = -\sum_{k=1}^{K}\sum_{t=1}^{T}\lambda_k^t U_k^t\left(\sum_{i=1}^{I} C_{i,k}^t - \sum_{i=1}^{I}\sum_{u=1}^{t} F_{i,k}^{t-u+1} q_i^u \Big/ \sum_{i=1}^{I}\sum_{u=1}^{t} F_{i,k}^{t-u+1} q_i^u\right). \tag{33}$$

The terms in parentheses cancel as (1 – 1), so the net revenue is guaranteed to be zero. Thus, this method needs no scaling to achieve revenue neutrality. If the market manager wants $V \neq 0$, a range of methods are readily available. This percentage approach simplifies the adjustment process considerably, and would be handy if $U_k^t$ often changes contrary to forecast.

How might users be compensated for tighter environmental standards? In a user pays system, the policy maker could wait for the beginning of the hydrological year to adjust $U_k^t$. For systems with long-term consents, the policy maker could wait for the consent renewal to adjust $U_k^t$ fractionally, as users' renewals are probably not simultaneous. For systems with permanent consents, the market manager may be tempted to tell users that their rights were defined as a percentage, and they still have the same fraction, just of a smaller pie, and therefore no compensation will be forthcoming. But once society has made a precise agreement with commerce regarding the rights to the commons, changes to the agreement must follow due process. The policy maker and users, and presumably the courts, would need to give careful attention to the wording of the rights documents.

# 3 Conclusion

Water rights should be adjusted to match the sustainability of the resource, for the sake of future generations. Within the limits of sustainability, the available water can be allocated many ways, and this article has demonstrated several of them. The problem of revenue neutrality is identical to that of matching quota to the available resource.

A revenue-positive market will be disliked by users. With a revenue-neutral market, users would be incentivised to participate voluntarily. A strongly revenue-negative market is almost certainly unsustainable from the market manager's business point of view. With appropriative rights, a given user may get less water than a more junior user, due to local conditions and availability, and sometimes water may be left unallocated. Recording well quota as a per cent of "total water in the catchment" is not a good idea, as the water available depends on where and when it is taken. Users will probably view constraint quota to be the fairest method, as free riding is eliminated. In addition, constraint quota could be recorded as a percentage.

Because the methods proposed in this article depend mainly on linear programming duality, results are not restricted to ground water, and should apply to many smart markets which are cleared by linear programs, and theoretical market analyses that rely on optimization. Extension, for example, to a smart market for the combination of surface and water rights, should be entirely straight forward, based on the appropriate hydrology model, linear programming duality, and definition of rights related to surface water conveyances.

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