Risk and Precautionary Saving in Two-Person Households

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The existing literature on precautionary saving is based almost entirely on the assumption that the household acts as if it consisted of a single individual decision-taker. In reality saving decisions are typically taken by two-person households. This paper examines the implications of this observation for the existence of precautionary saving, and shows that the assumption that the individual utility functions satisfy the conditions for precautionary saving to exist can imply that the household exhibits precautionary saving, but only under strong assumptions on the type of risk change being considered.

I. Introduction

Almost without exception, the papers in the large literature on saving decisions under risk1 take the decision unit to be a single individual and base the analysis on a model of individual preferences, most usually that of expected utility theory. This therefore ignores the fact that most saving is done by households in which

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1For good surveys see Browning and Lusardi (1996), Eeckhoudt, Gollier and Schlesinger (2005) and Gollier (2001).
typically there are two adults, and so actually or potentially two income earners. When making predictions or analysing data relating to this class of households, the implicit assumption must be that in some sense the two-person characteristics of the household do not matter. We do not have any precise idea however of the conditions under which such an assumption would be justified. The aim of this paper is to clarify these conditions. Again almost without exception, the models in the large literature on the economics of family-based households\(^2\), though dealing extensively with the two-earner case, ignore the existence of risk. There is a need to bring these two literatures together, in an analysis of two-person household decision taking under risk.

This paper takes a step in that direction by considering the extension of the theory of precautionary saving to the case of a two-person household.\(^3\) We show, in the context of a specific household model, that for risk increases of the first and second order,\(^4\) with which the savings literature has primarily been concerned, household precautionary saving will take place whenever the individuals’ preferences satisfy the standard conditions of strictly concave utility (risk aversion) and strictly convex marginal utilities (prudence). However, when we extend the sense in which the income distribution becomes "more risky" to risk increases of order higher than the second we show that this result no longer holds. We also make clear that the standard assumption in the risk sharing literature, that

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\(^2\)For a recent survey of this literature see Apps and Rees (2009), Chs 2-5..

\(^3\)A related paper, Mazzocco (2004), considers the question of whether, when two individuals pool their incomes and exchange risk efficiently, holding the probability distributions of their incomes fixed, their saving will fall relative to the sum of their saving as independent individuals. He shows that this will hold only under very strict conditions. Here, on the other hand, we consider the comparative statics of the saving decision of a couple when the distribution of their joint income varies.

\(^4\)See Ekern (1980). A cumulative distribution function (cdf) \(F(y)\), where \(y \in (y_0, y_1) \subset \mathbb{R}_+\), represents an \(n\)’th order increase in risk over a distribution \(G(y)\) if \(G\) dominates \(F\) by \(n\)’th order stochastic dominance and the first \(n - 1\) moments of the two distributions are equal.
individuals fully commit to implement their agreement in whichever state of the world is realised even in the absence of a formal contract,\(^5\) plays an important role in this analysis.

II. Household precautionary saving under first and second order risk increases

The model which forms the basis of our analysis is a straightforward extension of the "collective model" of the household\(^6\) to a two-period economy with income uncertainty in the second period. Let \(c_i, \tilde{c}_i\) denote the consumption of individual \(i = 1, 2\) in the first and second periods respectively, with the latter a random variable. Their exogenously given (labour) incomes are likewise \(y_i, \tilde{y}_i\) with their sums given by \(y, \tilde{y}\). No insurance or asset markets exist that allow trade in state-contingent incomes, there is only a bond market with certain interest rate \(r \geq 0\) which allows trade in incomes between periods. Individual utility functions \(u_i(c_i)\) are neither time- nor state-dependent, though future utilities may be discounted by a "felicity discount factor" \(\rho \in (0, 1]\).

The couple finds its optimal saving by solving the problem

\[
\text{(1)} \quad \max_{c_i,s,\tilde{c}_i} \sum_{i=1}^{2} u_i(c_i) + \rho E[u_i(\tilde{c}_i)]
\]

\[
\text{(2)} \quad \text{s.t.} \sum_{i=1}^{2} c_i \leq y - s
\]

\(^5\)As for example in Townsend (1994) and Mazzocco (2004).

\(^6\)See Apps and Rees (1988), Chiappori (1988) and Browning and Chiappori (1998). It was proposed as a generalisation of the Nash bargaining model of the household, which is the other widely adopted model in the literature.
\begin{equation}
\sum_{i=1}^{2} \tilde{c}_i \leq \tilde{y} + (1 + r)s
\end{equation}

The parameters $\mu_i \in (0, 1)$, which we are free to normalise by setting $\sum_i \mu_i = 1$, are exogenously given weights reflecting in some sense the "bargaining power" of individual $i$, or more generally, the weight the household gives to her well-being in its collective decision process.\(^7\) The expectation $E[u_i(\tilde{c}_i)]$ embodies the assumption that the individuals have identical probability beliefs. We also assume that the household at the initial decision point can commit to implement its chosen allocations of future consumption in whatever state of the world is realised, and that the weights $\mu_i$ are constant over time.\(^8\)

In the precautionary saving literature, the problem is usually taken to be\(^9\) that of deriving the conditions under which saving increases when there is a mean preserving spread in the probability distribution of income, for example when a certain income $y$ is replaced by the distribution of incomes $y + \tilde{\epsilon} = \tilde{y}$, where $\tilde{\epsilon}$ is a random variable with $E\tilde{\epsilon} = 0$. More generally, $\tilde{y} \in (y^0, y^1)$ is an uncertain income with cdf $G(\tilde{y})$, which is replaced by a new cdf $F(\tilde{y})$, where both have equal means. $F$ is taken to be more risky than $G$ in the sense of second order stochastic dominance.\(^10\) Replacing $G$ by $F$ is then a second order increase in risk.

\(^7\)These are assumed to be functions of what are usually referred to as "distributional variables" strictly exogenous to the household, such as for example individual holdings of exogenous income or wealth, divorce laws, and the ratio of males to females of marriageable age.

\(^8\)These closely related assumptions are not innocuous. For example, in a bargaining model, variations in one individual’s exogenous income would change that individual’s threat point and therefore her bargaining power. The same could be true of a change in the riskiness of an individual’s future income. Moreover, as Basu (2006) shows, future bargaining weights might be influenced by current choices of endogenous variables, in this case saving. It is nevertheless useful to see what results can be derived when the standard assumption of full commitment is maintained.


\(^10\)Implying that $G(z) = \int_{y^0}^{z} G(t)dt \leq F(z) = \int_{y^0}^{z} F(t)dt$ for all $z \in (y^0, y^1)$, with strict inequality for some $z$.\)
The underlying idea stems from the proposition that every risk averse decision taker strictly prefers $G$ to $F$.

Turning now to the household’s saving decision, as modeled in (1)-(3) above, the optimal allocations within any one period/state can be shown to be independent of probabilities and the discount factor and are found for any given available total income $z$ by solving the problem

$$\max_{c_i} \sum_{i=1}^{2} \mu_i u_i(c_i) \text{ s.t. } \sum_{i=1}^{2} c_i \leq z$$

yielding as solution the two functions $c_1(z)$ and $c_2(z) \equiv z - c_1(z)$. Following Paul Samuelson (1956), we call this solution the "sharing rule", and $c_1(z)$ the "share functions". Obviously the properties of these functions are determined by those of the $u_i(\cdot)$, but are more complex than those of any one of these functions because of the effects of the implicit exchange of state-contingent incomes between the individuals. Note in particular that, given the first order condition for an interior solution

$$\mu_1 u_1'(c_1(z)) = \mu_2 u_2'(z - c_1(z))$$

we have from the Implicit Function Theorem:

$$c_1'(z) = \frac{\mu_2 u_2''(c_2(z))}{\mu_1 u_1''(c_1(z)) + \mu_2 u_2''(c_2(z))} > 0$$

with of course $\sum_{i=1}^{2} c_i'(z) = 1$. It is also worth noting at this point that $\sum_{i=1}^{2} c_i''(z) = \sum_{i=1}^{2} c_i'''(z) = \cdots = 0$.

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12 In a slight abuse of notation, we simplify by allowing $z$ to denote either $y - s$ or $\tilde{y} + (1 + r)s$, as the case may be, and $c_i$ to denote consumption in any period or state.
Define the indirect household welfare function \( H(W) \) as the value function of the problem in (4). Clearly, choice of \( s \) is equivalent to choice of \( z \). Then the household chooses its optimal saving and at the same time allocates total incomes to individual consumptions at each time/state by solving the problem \( \max_s H(z(s)) + \rho E[H(z(s))] \).

A single individual maximises the expected present value of utility \( u_i(c_i) + \rho E[u_i(\tilde{c}_i)] \) subject to the appropriate budget constraints. Strict concavity of the utility function \( u_i(.) \) is sufficient to satisfy the second order condition for the problem to yield a unique global saving optimum, and also for a first order increase in risk \(^{13}\) to result in an increase in saving. Furthermore, as already noted, prudence, or \( u''_i > 0 \), is sufficient for a second order risk increase to increase saving. \(^{14}\) By comparing the single’s maximand with that in the couple’s problem, it is clear that \( H(.) \) plays exactly the same role as \( u_i(.) \), and so extending the results on precautionary saving in the former case simply requires us to examine the derivatives of \( H(.) \). The important difference is that changes in saving affect individual utilities via the sharing rule, and, given our assumptions so far, this is the only source of additional complexity created by a two-person household.

We characterize the optimal household saving \( s^* \) by the first order condition

\[
(7) \quad H'(z(s^*)) = \rho(1 + r)E[H'(z(s^*))]
\]

We establish the result on a first order increase in risk simply by showing that \( H \) is strictly concave in \( s \). Although risk aversion would guarantee this in the case of single individuals, the presence of the sharing rule must now also be taken into

\(^{13}\)Under which the distribution \( F \) is more risky than \( G \) in the sense of first order stochastic dominance: \( F(y) \geq G(y) \) for all \( y \in (y^0, y^1) \), with strict inequality for some \( y \).

\(^{14}\)See Kimball (1990).
Proposition 1: A first order increase in risk at the household equilibrium will cause an increase in saving.

Proof: Since \(z(.)\) is linear in \(s\), it suffices to show that

\[
H''(z) = \sum_{i=1}^{2} \mu_i \left[ u''_i(c_i(z))(c'_i)^2 + u'_i(c_i(z))c''_i \right] < 0
\]

This follows immediately by noting that from the first order condition (5) and the fact that \(\sum_i c''_i = 0\) we have \(\sum_{i=1}^{2} \mu_i u'_i(.)c''_i = 0\), and so

\[
H''(z) = \sum_{i=1}^{2} \mu_i u''_i(c_i(z))(c'_i)^2 < 0
\]

as a result of risk aversion.

This simple proposition has interesting economic applications. Consider for example a young couple planning to start a family. Since this will very likely be associated with a fall in income of at least one individual as time is diverted from market work to child care, the couple will anticipate a first order increase in risk in future income - in every future state of the world household income will be lower, the cdf shifts to the left. Therefore they will increase their current saving. However their saving on average after the arrival of the child will fall, since their average income will be lower.\(^{15}\) This "humped" shape of saving in younger households is strongly confirmed by the data.\(^{16}\)

Turning now to the second order risk increase, we can establish:

\(^{15}\)As Eeckhoudt and Schlesinger (2008) also point out, it implies that "prudence" is not necessary for precautionary saving under first order risk increases, and, as they also show, it is not sufficient for third order risk increases. Their general point is that we should always specify the order of risk increase when deriving necessary and sufficient conditions under which precautionary saving is supposed to hold.

\(^{16}\)See for example Apps and Rees (2009), Ch. 5.
Proposition 2: For a second order risk increase, the condition \( u_i''(.) \geq 0 \) for all \( i \) with strict inequality for at least one \( i \) is sufficient for joint precautionary saving.

Proof: Using standard arguments and the result in (9) we can show that for precautionary saving to result from a second order risk increase it is necessary and sufficient that

\[
H'' = \sum_{i=1}^{2} \mu_i [u''(c_i(z))(c'_i)^3 + 2u''(c_i(z))c'_i c''_i] > 0
\]

Since \( c'_i > 0 \) the first term in (10) is positive under the condition of the proposition. Substituting for \( c'_i \) from (6) into the second term and rearranging gives

\[
\sum_{i=1}^{2} \mu_i u''(c_i(z))c'_i c''_i = \frac{\mu_1 u''(c_1(z)) \mu_2 u''(c_2(z))}{\mu_1 u''(c_1(z)) + \mu_2 u''(c_2(z))} \sum_i c''_i(z) = 0
\]

which gives the result.

Although not hard to establish, this result is certainly not trivial. We should expect in general that the existence of precautionary saving in the two-person household must depend on some conditions on the household sharing rule, since this is the element that the household model adds to the individual model. The key point about the second order risk increase case is that the comparative statics depend only on the first order derivatives of the share functions \( c_i(z) \). The positivity of these derivatives and the fact that they must sum to 1 suffices for the result.\(^{17}\)

The results do not however extend to higher orders of risk increase than the second, since then the counterparts of condition (10) involve higher order deriva-

\(^{17}\)In the first order case not even the signs of the \( c'_i \) matter.
tives of the share functions, and conditions only on the signs of the derivatives of the utility functions are no longer sufficient. This is most easily seen in the case of a third order risk increase, where \( G \) is taken to be less risky than \( F \) in the sense of third order stochastic dominance. This is the case of increasing downside risk, that is, increased negative skewness in the income distribution with mean and variance held constant.\(^{18}\) In that case it can be shown that the counterpart of the condition in (10) is

\[
H''' = \sum_{i=1}^{2} \mu_i [u''''(c_i(z))(c'_i)^4 + 3u''''(c_i(z))c'''_i(c'_i)^2] < 0
\]

In this case however the term \( \sum_{i=1}^{2} \mu_i u''''(.)c'''_i(c'_i)^2 \) cannot be eliminated and, since \( c'_1 = -c'_2 \), its sign is in general indeterminate.\(^{19}\) Only in the case of a linear sharing rule, where then of course all derivatives higher than the first vanish, can we guarantee that the satisfaction of the condition for precautionary saving in the individual case also implies the existence of household precautionary saving for all orders of risk increase. But this is a very special case, requiring that the individuals have linear absolute risk tolerance with identical slope coefficients.\(^{20}\)

### III. Conclusion

This paper poses the question: Given a two-person household faced with a risky joint income, under what conditions can its saving be guaranteed to increase when it experiences a risk increase of any given order? This is the standard question of the comparative statics analysis of precautionary saving that until

\(^{18}\)Menezes, Geiss and Tressler (1980) present the theory of third order risk increases together with a number of economic examples.

\(^{19}\)Apps, Andrienko and Rees (2012) also provide a counterexample to the proposition that the standard conditions ensure precautionary saving in this case.

\(^{20}\)See Gollier (2001), Ch 21.
now has been considered only for households consisting of single individuals, and predominantly for second order risk increases.

For a first order risk increase the condition that answers this question is very mild: we simply require strict concavity of the joint maximand in total income, i.e. that the first order necessary condition for optimum saving also be globally sufficient. This is guaranteed by individual risk aversion. For second order risk increases, the conditions now appear to be more complex, depending as they do on the signs of the derivatives of the household share functions. However, because only the first order derivatives of these functions turn out to matter, the condition of a positive third derivative of individual utility - prudence - is still sufficient for the existence of precautionary saving. For higher order risk increases, unless the share functions are linear, conditions on the signs of the corresponding derivatives of the individual utility functions are no longer sufficient, because the higher order derivatives of the share functions do in general come into play. Since the linearity condition is very special, we should conclude that for higher orders of risk increase households may be observed to reduce their saving in the face of increasing riskiness of their joint income even when their individual utility functions satisfy the conditions for precautionary saving to exist. The fact that individuals in households pool their incomes and share consumption risk matters.

Some comfort may be derived from the fact that the results from individual choice models can under some circumstances carry over to the two-person case. However, many interesting problems will involve risk changes that do not cor-

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21 Note that even if one of the individuals has zero income in all states, so that we have a single-earner household, as long as both weights are positive and utility functions differ the household will in general behave differently to one consisting of a single individual.

22 Eeckhoudt and Schlesinger (2008) provide the generalisation to all orders of risk increase for the single individual household.

23 For example Davies and Hoy (2002) show that a change from proportional to progressive taxation can represent a third order risk increase. Eeckhoudt and Schlesinger (2006) provide
respond to a mean-preserving spread, and in such cases we cannot assume in
general that the comparative statics predictions drawn from individual choice
models will apply to two-person households: an explicit analysis with an appro-
priate household model will in general be necessary.

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interesting insights into the intuitive interpretation of higher order risk increases.


