Proper Scoring Rules: Incentives, Stakes and Hedging*

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Abstract

Proper scoring rules (PSR) are among the most popular incentivized belief elicitation mechanisms. A well known result is that risk averters facing PSR misreport their beliefs by stating more uniform probabilities. We show that this result does not generalize when i) the PSR payments are increased, ii) the agent has a financial stake in the event she is predicting, and iii) the agent can hedge her prediction by taking an additional action. Instead, combining theory and experiment, we find that agents distort their reported probabilities in complex, yet mostly predictable manners. We argue that our results have implications for the elicitation of beliefs in most environments of interest to economists, both in academia and in practice.

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1. Introduction

Proper Scoring Rules (PSR hereafter) are belief elicitation techniques designed to provide an agent the incentives to report her subjective beliefs thoughtfully and truthfully. Although it is well known that PSR are only incentive compatible under expected payoff maximization, the biases (i.e. the systematic differences between an agent's subjective and reported beliefs) a PSR may generate have not been characterized in general economic environments. This paper combines theory and experiment in an attempt at partially filling this void. More precisely, we characterize the possible PSR biases for all risk averse agents and all PSR under three effects: i) an increase in the PSR payments, ii) the introduction of a financial stake in the event the agent is predicting, and iii) the possibility for the agent to hedge her prediction by taking an additional action. The empirical significance of the biases identified are then tested in an experiment.

Accurate measurements of probabilistic beliefs are important both in practice and in academia. In practice, numerous websites now offer public opinions (about e.g. consumer products, movies or restaurants) and predictions (about e.g. sporting or political events).[1] To be meaningful these opinions and predictions must be informative. When soliciting advice from experts (about e.g. health, environmental or financial issues) individuals typically expect unbiased recommendations. Recent suspicions of conflict of interest suggest that this may not always be the case.[2] Finally, in an effort to better manage risk, firms are increasingly turning to their employees to forecast (e.g.) sales, completion dates, or industry trends.[3] Precise beliefs assessments are also important in academia. In particular, Manski (2002, 2004) argues that separate measures of choices and beliefs are needed in order to properly estimate decision models. Modern Macroeconomic theory considers that monetary policy consists in large part in managing expectations (about e.g. inflation).[4] In practice, such management of expectations requires

[1] Such opinion websites include *ePinion, Ebay, Zagat,* or Amazon. Prediction websites include the *Iowa Electronic Market*, the *Hollywood Stock Exchange*, or *Intrade*.

[2] See e.g. the New York Times article "Questions Grow About a Top CNBC Anchor" (February 12, 2007) about TV anchors for the financial news channel CNBC. Likewise, the pharmaceutical industry has long been suspected to influence doctors' prescription behavior through various "marketing" campaigns.

[3] Such firms include *Yahoo!, Microsoft, Google, Chevron, General Electric,* and *General Motors*. The economic value of accurate forecasts may be illustrated with the case of the *Dreamliner*'s delays which are expected to cost the *Boeing Corporation* up to $10 billion.

[4] See e.g. Woodford (2005), or Blinder et al. (2008) for a review of this literature.

correct measures of probabilistic beliefs.[5] Finally, experimental economists are increasingly eliciting their subjects' beliefs in order to better understand observed behavior.[6]

Because they are incentive compatible under expected payoff maximization, PSR have been one of the most popular belief elicitation technique, with applications to numerous fields such as meteorology, business, education, psychology, finance, and economics.[7] Recently, with the rapid development of prediction markets, there has been an upsurge of interest in PSR. In particular, Market Scoring Rules have been proposed as a way to overcome the liquidity problems that have affected some prediction markets.[8] In short, Market Scoring Rules may be described as follows. A group of agents is sequentially asked to make a prediction about a particular event. Each agent is paid for her prediction according to a PSR, but she also agrees to pay the previous agent for his prediction according to the same PSR. Because of their attractive properties, Market Scoring Rules have been rapidly adopted by several firms operating prediction markets.[9]

Some economists, however, have been concerned about the possible biases generated by PSR when agents are not risk neutral (Winkler and Murphy 1970). The nature of these biases has typically been analyzed in specific contexts. Namely, a particular PSR and a given utility function are selected, and the agent's wealth is generally assumed to vary only with the PSR payments. To the best of our knowledge, however, no general result exists. Some mechanisms aimed at overcoming risk aversion when eliciting subjective probabilities with PSR have recently been proposed (Offerman et al. 2009, Andersen et al. 2009). The general effectiveness of these approaches still needs to be confirmed, and their ability to be easily implemented outside the lab remains to be demonstrated. It therefore can be useful to characterize the direction of the biases PSR may generate in order in particular i) to interpret reported beliefs as either lower or upper bonds on subjective beliefs, ii) to understand whether biases may be reduced with appropriate incentives, or iii) to help design better belief elicitation techniques.

When first developed by statisticians in the 1950's, PSR were studied in sim-

[5] Efforts are currently underway at the Federal Reserve Bank of New York to develop better instruments to measure individuals' inflation expectations (Bruine de Bruin et al. 2009).

[6] Wagner (2009) identifies more than forty economic experiments using incentivized beliefs elicitation techniques.

[7] See Camerer (1995), Offerman et al. (2009), or Palfrey and Wang (2009) for references.

[8] See, e.g., Hanson (2003), Ledyard (2006) or Abramowicz (2007).

[9] E.g. *Inkling Markets*, *Consensus Point*, *Yoopick*, *Crowdcast*, and *Microsoft Corporation*.

ple environments.[10] In this paper, we generalize the analysis of PSR to richer economic environments, by characterizing the possible biases PSR generate in response to three effects. First, we consider how varying the PSR payments affect reported probabilities. Although experimental economists have long debated how incentives affects choices (Camerer and Hogarth 1999, Hertwig and Ortmann 2001), to the best of our knowledge, the problem has not been explicitly addressed for PSR.[11] Second, we consider an environment in which the agent has a financial stake in the event. Such a situation is common in practice. For instance, an agent may be asked to make a prediction about an economic indicator (e.g. the stock market, the inflation level) or an event (e.g. a flood, a favorable jury verdict). Similarly, an agent facing a Market Scoring Rule always has a stake in the event she predicts, as her payment to the previous predictor depends on the outcome of the event. Finally, subjects in (e.g.) public good experiments are often asked to predict the contributions of others.[12] In all of those cases, independently of the PSR payments, the agent's wealth varies with the outcome of the random variable she is predicting. As we shall see, such violations of the "no stake" condition (Kadane and Winkler 1988) may induce further biases. Third, we offer the agent the possibility to hedge her prediction by taking an additional action whose payoff also depends on the event. For instance, in the previous examples, the agent predicting an economic indicator might also have to choose how to diversify her portfolio, while the agent predicting a catastrophic event may also have to decide on her insurance coverage. Likewise, subjects in public good experiments have to choose their own contributions.[13] As we shall see, the prediction and the additional action are in general not independent, which may create an additional source of bias.

[10] Early references include Brier (1950), Good (1952) and McCarthy (1956). It seems that De Finetti informally invented PSR (De Finetti 1981). A broad and very influential paper in the statistical literature is Savage (1971). Early references in economics include Friedman (1979) and Holt (1979).

[11] Some have compared financially versus non-financially incentivized belief elicitation techniques (Beach and Philips 1967, Rustrom and Wilcox 2009). We extend this analysis by considering variations of strictly positive financial incentives.

[12] See e.g. Croson (2000), Gachter and Renner (2006), or Fischbacher and Gachter (2008).

[13] Although experimental economists have recently been aware of stakes and hedging opportunities (Fehr et al. 2009, Palfrey and Wang 2009, Andersen et al. 2009), these issues have typically been ignored (Costa-Gomez and Weizsacker 2008, Fischbacher and Gachter 2008). Concerns have also been raised that eliciting beliefs of subjects engaged in a game, even absent of any stakes and hedging considerations, may lead them to think more strategically, therefore affecting their behavior (Croson 2000, Gachter and Renner 2006, Rustrom and Wilcox 2009).

Our analysis is divided in two parts. In the theory part, we assume expected utility, and consider the class of all PSR (Gneiting and Raftery 2007). We first show that there is a monotone relationship between reported and subjective probabilities. We then generalize previous results (Winkler and Murphy 1970, Offerman, et al. 2009, Andersen et al. 2009) by showing that risk averters report more uniform probabilities, i.e. probabilities skewed toward 1/2 in the case of binary events. In contrast with popular beliefs, we find that changing the rewards of the PSR has an ambiguous impact on reported probabilities. In particular, smaller PSR payments can either reduce or reinforce the PSR biases depending on whether the utility function displays increasing or decreasing relative risk aversion. We then show that the presence of a bonus (i.e. a positive stake) when the event occurs lowers reported probabilities under risk aversion. Finally, we show that the possibility of hedging by betting on the event alter predictions. Interestingly, we identify a region where the reported probabilities remain unchanged, and are therefore completely independent of the agent's subjective probabilities.

In the second part, we report on an experiment aimed at testing whether the PSR biases are empirically relevant. The basic design is similar to Offerman et al. (2009) (OSKW hereafter). Subjects are presented with a list of events describing the possible outcome of the roll of two 10-sided dice. Without making reference to "probability" or "belief", the probabilities are elicited with a quadratic scoring rule. In addition to a control treatment, a total of six treatments are conducted by varying i) the PSR payments (the "High Incentives" and "Hypothetical Incentives" treatments), ii) the stake in the event (the "Low Stakes" and "High Stakes" treatments), and iii) the returns on the amount bet on the event (the "Low Hedging" and "High Hedging" treatments). Although not perfectly consistent with the theory, the experimental results are generally in line with the predictions made under risk aversion. In particular, we find significantly larger biases when the PSR pays higher amounts which suggests that subjects exhibit increasing relative risk aversion. In contrast, the absence of incentives produces less biased, yet noisier elicited probabilities. Consistent with the theory, the presence of a stake leads subjects to report significantly lower probabilities. Furthermore, we find a positive correlation between the amount bet on an event and the bias in the reported probability for that event, thereby providing evidence of hedging. Finally, we observe larger biases when the underlying objective probabilities are compounded or complex, which, although inconsistent with expected utility, can be rationalized with a model of ambiguity aversion.

2. Theory

In this section, we study the properties of the "response function", that is the function that gives the optimal reported probabilities of an agent who is rewarded according to a PSR. The properties derived hold for all PSR, and are therefore not restricted to the quadratic scoring rule (QSR hereafter) we use in the experiment.

2.1. Preliminary Assumptions

We consider a binary random variable, that is an event and its complement. We assume probabilistic coherence, and thus restrict our attention to the subjective probability of this event p held by the agent. We assume that p is exogenous: it is fixed and cannot be affected by the agent (i.e. there is no learning and no moral hazard).[14]

Let $q \in [0, 1]$ be the agent's reported probability. A scoring rule gives the agent a monetary reward $S_1(q)$ if the event occurs, and $S_0(q)$ if the event does not occur. We assume that the scoring rule is differentiable and real-valued, except possibly that $S_1(0) = -\infty$ and $S_0(1) = -\infty$.

We assume that the agent is an expected utility maximizer with a thrice differentiable and strictly increasing state-independent von Neumann Morgenstern utility function denoted $u(.)$. We start by assuming that the agent's wealth does not depend on whether the event occurs or not. We will relax this "no-stake" condition in sections 2.5 and 2.6. Furthermore, we relax in Appendix B the assumption of expected utility, where we study the effect of ambiguity aversion.

2.2. Proper Scoring Rules

A scoring rule is said to be proper if and only if a risk neutral agent reveals truthfully her subjective probability.

Definition 2.1. Proper scoring rule. *A scoring rule $S = (S_1(q), S_0(q))$ is proper if and only if:*

$$p = \arg\max_{q\in[0,1]} pS_1(q) + (1-p)S_0(q) \tag{2.1}$$

[14] See Osband (1989), Ottaviani and Sorensen (2007) and Wagner (2009) for somewhat related theoretical results obtained when p is endogenous.

In the remainder of this section, we will illustrate some of our results using the popular QSR, defined by

$$\begin{aligned} S_1(q) &= 1-(1-q)^2 \\ S_0(q) &= 1-q^2 \end{aligned} \tag{2.2}$$

The QSR is represented on Figure 1a. It is straightforward to show that a QSR satisfies (2.1) and is therefore proper.

We now provide a simple characterization of all PSR for binary random variables. This characterization has been recently proposed in the statistics literature by Gneiting and Raftery (2007) in the multi-event situation, building on the pioneering work of McCarthy (1956), Savage (1971), and Schervish (1989).[15]

Proposition 2.1. *A scoring rule is proper if and only if there exists a function* $g(.)$ *with* $g'' > 0$ *such that*

$$\begin{aligned} S_1(q) &= g(q)+(1-q)g'(q) \\ S_0(q) &= g(q)-qg'(q) \end{aligned} \tag{2.3}$$

The sufficiency of Proposition 2.1 is easy to prove. Indeed, under (2.3), the agent's expected payoff equals

$$\pi(q) \equiv pS_1(q)+(1-p)S_0(q) = g(q)+(p-q)g'(q)$$

which reaches a maximum at $q=p$ since g' is strictly increasing.

Proposition 2.1 indicates that a PSR can be fully characterized by a single function $g(.)$, and by a simple property on this function, namely its convexity. Note also that any positive affine transformation of a PSR remains proper. In particular, it is easy to show that the QSR can be obtained from $g(q) = q^2+(1-q)^2-1$ together with the application of a positive affine transformation.

Observe finally that

$$\begin{aligned} S_1'(q) &= (1-q)g''(q) > 0 \\ S_0'(q) &= -qg''(q) < 0 \end{aligned}$$

Hence when a scoring rule is proper, the convexity of $g(.)$ implies the intuitive property that $S_1(q)$ must be increasing, and that $S_0(q)$ must be decreasing (see, again, Figure 1a). This implies that $S_1(q)$ and $S_0(q)$ can cross at most once, an important property for what follows.

[15] The proofs of the Propositions are relegated to Appendix A.

2.3. Risk Aversion

From now on, we relax the assumption of risk neutrality, and allow for risk aversion. Winkler and Murphy (1970), Kadane and Winkler (1988) and Offerman et al. (2009) have examined the response function of a risk averse agent facing a QSR. They show that risk aversion leads the agent to report probabilities skewed towards 1/2 in the case of binary events. This makes a risk-averter better off since this reduces the difference across terminal payoffs. We now generalize this result to the class all PSR, and to the notion of comparative risk aversion.

We define the response function $R(p)$ as follows

$$R(p) = \arg\max_{q\in[0,1]} pu(S_1(q)) + (1-p)u(S_0(q)) \tag{2.4}$$

where $S = (S_1, S_0)$ is a PSR as defined in (2.3). Our objective in the remainder of this section is to analyze the properties of this response function. In particular, we want to characterize the response function's "bias", $R(p) - p$.

The first order condition of the program above can be written as follows

$$f(p,q) \equiv p(1-q)u'(S_1(q)) - (1-p)qu'(S_0(q)) = 0. \tag{2.5}$$

It is easy to see that $\frac{\partial f(p,q)}{\partial q} < 0$, so that the program is concave and $R(p)$ is unique. It is also easy to check that it is optimal to report 0 when $p = 0$ and to report 1 when $p = 1$. Observe moreover that $\frac{\partial f(p,q)}{\partial p} > 0$, so that the response function $R(p)$ is increasing in p, as stated in the following Lemma.

Lemma 2.1 *$R'(p) > 0$ together with $R(0) = 0$ and $R(1) = 1$.*

Notice that this Lemma only requires the utility function to be increasing. Next, we show that truthful revelation of subjective probability is in general not optimal under risk aversion. Moreover, we show that the deviation from truthtelling is systematic, and depends on p. To see this, observe that the left hand side of (2.5) evaluated at $q = p$ has the sign of $u'(S_1(p)) - u'(S_0(p))$, which captures the marginal benefit of increasing q at $q = p$. This means that under risk aversion the response function $R(p)$ is larger (respectively lower) than p when $S_1(p)$ is lower (respectively larger) than $S_0(p)$. Yet, using (2.3), we have $S_1(p) \leq S_0(p)$ if and only if $p \leq 1/2$. This implies that the agent reports more uniform probabilities in the following sense: the response function is higher than p

when $p < 1/2$ and lower than p when $p > 1/2$, as stated in the following corollary.[16]

Corollary 2.1 *$R(p) \geq p$ if and only if $p \leq 1/2$.*

The response function is therefore "regressive" (i.e., it crosses the diagonal from above), with a fixed point equal to 1/2. Figure 1b displays such a regressive response function for the QSR in (2.2) together with a quadratic utility function $u(x) = -(2-x)^2$ with $x \leq 2$.

Note that Corollary 2.1 implies $R(1/2) = 1/2$, so that the agent truthfully reveals his subjective probability at $p = 1/2$. This is because we use the definition of a PSR in (2.3), which imposes the restrictive property $S_1(1/2) = S_0(1/2)$. However, if one applies a positive affine transformation to the PSR, $S_1(1/2)$ and $S_0(1/2)$ can differ. In that case, the result that risk aversion leads to report more uniform probabilities does not hold anymore. We study this effect in section 2.5.

Finally, we derive a Proposition that generalizes Corollary 2.1. This Proposition states that more risk averse agents (in the classical sense of Pratt 1964) always report more uniform probabilities. We denote $R_u(p)$ and $R_v(p)$ the response functions associated with the utility functions $u(.)$ and $v(.)$.

Proposition 2.2. *Let $v(w) = \Phi(u(w))$ with $\Phi' > 0$ and $\Phi'' \leq 0$. Then*

$$R_v(p) \geq R_u(p) \text{ if and only if } p \leq 1/2$$

The intuition is that a more risk averse agent is willing to sacrifice more in terms of expected payoff in order to reduce the difference across terminal payoffs. This can be achieved by reporting probabilities closer to 1/2.

2.4. Incentives

We now study the effect of changing the incentives provided by the PSR. More precisely, we study the effect of changing $a > 0$ on the response function

$$R(p, a) = \arg\max_{q \in [0,1]} pu(aS_1(q)) + (1-p)u(aS_0(q)) \tag{2.6}$$

We show that this effect depends on the relative risk aversion coefficient $\gamma(x) = \frac{-xu''(x)}{u'(x)}$.

[16] In contrast, risk lovers facing a PSR always report more extreme probabilities. In particular, when an interior solution exists, it is easy to show that the response function is lower than p when $p < 1/2$ and higher than p when $p > 1/2$.

Proposition 2.3. *Under* $\gamma'(x) \geq (\leq) 0$,

$$\frac{\partial R(p,a)}{\partial a} \geq (\leq) 0 \text{ if and only if } p \leq 1/2.$$

In words, under increasing (decreasing) relative risk aversion, an increase in the rewards of the PSR leads to report more (less) uniform probabilities. One can present the intuition as follows. There are two effects when the PSR payments increase: i) a wealth effect, as the agent gets a higher reward for any given reported probability, and ii) a risk effect, as the difference between the rewards in the two states becomes more important. The derivative of the relative risk aversion $\gamma(x)$ ensures that one effect always dominates the other. In particular, when relative risk aversion is increasing, the risk effect dominates the wealth effect so that the agent reports more uniform probabilities in order to reduce the variability of her payoff.[17]

An implication of this result is that, under constant relative risk aversion, changing the PSR incentives has no effect on the response function. This indicates that it may not be possible to eliminate the response function's bias by adjusting the incentives of the PSR. In particular, letting a tend to 0 does not guarantee that the response function $R(p,a)$ tends to p for all concave utility functions. Hence, the common belief that paying agents small amounts induces risk-neutral behavior leading to truthful revelation is not correct in general.[18]

The result of Proposition 2.3 is illustrated in Figure 1c. The added response function compared to Figure 1b is calculated for $a = 2$. Observe that both response functions are regressive with a fixed point at $1/2$. Yet, the increased incentives lead to report more uniform probabilities. This is because the quadratic utility function used for the numerical example displays increasing relative risk aversion.

2.5. Stakes

We now relax the "no-stake" condition by adding a constant to the reward S_1 of the PSR. We know that the PSR remains proper in this case since it is a simple positive affine transformation of the initial PSR. Moreover, observe that adding this constant is formally identical to assuming that the agent's wealth changes when the event obtains. We consider the latter interpretation in the remainder.

[17] Note that Proposition 2.3 relies on the assumption that the initial wealth is equal to zero.

[18] See, e.g., Kadane and Winkler (1988: 359) or Karni (1999: 480).

The response function is defined by

$$R(p, \triangle) = \arg\max_{q \in [0,1]} pu(\triangle + S_1(q)) + (1-p)u(S_0(q)) \tag{2.7}$$

in which the added reward $\triangle$ is a finite (positive or negative) "stake". The first order condition can be written

$$f(\triangle, q) \equiv p(1-q)u'(\triangle + S_1(q)) - (1-p)qu'(S_0(q)) = 0 \tag{2.8}$$

As before $\frac{\partial f(\triangle,q)}{\partial q} < 0$ under risk aversion, so that the program is concave. In addition, it is immediate to see that the properties of Lemma 2.1 are still satisfied with a stake. Finally, observe that $\frac{\partial f(\triangle,q)}{\partial \triangle} < 0$ so that the response function is decreasing in $\triangle$ under risk aversion, as stated in the following Lemma.

Lemma 2.2 $\frac{\partial R(p,\triangle)}{\partial \triangle} \leq 0$

The intuition for this result is straightforward. Under risk aversion, an increase in $\triangle$ reduces the marginal utility when the event occurs. Therefore, in order to compensate for the difference in marginal utility across states, the agent increases the rewards of S_0 compared to S_1. This can be done by reducing the reported probability. We can now state a Proposition that characterizes the response function when the agent has a stake.

Proposition 2.4. *The response function $R(p, \triangle)$ is characterized as follows:*
i) if there exists a $\widehat{p}$ such that $\triangle + S_1(\widehat{p}) = S_0(\widehat{p})$, then we have

$$R(p, \triangle) \geq p \text{ if and only if } p \leq \widehat{p}$$

ii) if $\triangle + S_1(p) \geq (\leq) S_0(p)$ for all p, then we have

$$R(p, \triangle) \leq (\geq) p$$

Proposition 2.4 is illustrated in Figure 1d. Compared to Figure 1b, there are two additional response functions on Figure 1d, one for $\triangle = 1/2$ and the other for $\triangle = 1$. Both response functions are regressive, but the first has a fixed point at $1/4$, and the second is below the diagonal everywhere. Note finally that Corollary 1 is a particular case of Proposition 2.4 for $\triangle = 0$.

2.6. Hedging

We now allow the agent to make another decision in addition to her prediction. Suppose the agent can invest an amount α in $[0, \overline{\alpha}]$ in a lottery which returns $(k+1)\alpha$ if the event occurs, and zero if the event does not occur. We assume that k and $\overline{\alpha}$ are strictly positive and finite. The problem becomes

$$\max_{q \in [0,1], \alpha \in [0, \overline{\alpha}]} pu(S_1(q) + k\alpha) + (1-p)u(S_0(q) - \alpha) \tag{2.9}$$

It is easy to see that this program is well behaved under strict risk aversion. The following Proposition presents properties of the response function when this particular form of hedging is available.

Proposition 2.5. *The solutions $R(p)$ and $\alpha(p)$ to program (2.9) satisfy the following properties:*

i) for $p \leq \underline{p}(k)$, we have $\alpha(p) = 0$ and $R(p) \in [0, \frac{1}{1+k}]$ with $R'(p) > 0$,
ii) for p in $[\underline{p}(k), \overline{p}(k)]$, we have $\alpha(p) \in [0, \overline{\alpha}]$ with $\alpha'(p) > 0$ and $R(p) = \frac{1}{1+k}$,
iii) for $p \geq \overline{p}(k)$, we have $\alpha(p) = \overline{\alpha}$ and $R(p) \in [\frac{1}{1+k}, 1]$ with $R'(p) > 0$,

together with

$$\underline{p}(k) = \frac{u'(S_0(\frac{1}{1+k}))}{ku'(S_1(\frac{1}{1+k})) + u'(S_0(\frac{1}{1+k}))} \text{ and}$$

$$\overline{p}(k) = \frac{u'(S_0(\frac{1}{1+k}) - \overline{\alpha})}{ku'(S_1(\frac{1}{1+k}) + k\overline{\alpha}) + u'(S_0(\frac{1}{1+k}) - \overline{\alpha})}$$

This Proposition tells us that there are two critical threshold values for subjective probabilities that shape the optimal investment rule: when $p \leq \underline{p}(k)$, the agent does not invest at all, and when $p \geq \overline{p}(k)$ the agent invests the maximum amount $\overline{\alpha}$. That is intuitive since the investment opportunity is not attractive when p is low, and becomes more attractive as p increases. In particular, when p belongs to $]\underline{p}(k), \overline{p}(k)[$, the agent invests some amount in $]0, \overline{\alpha}[$. Perhaps most interestingly, in this intermediate range of subjective probabilities, the agent report probabilities that are constant, and are therefore independent from p. The intuition is that the PSR is used as a transfer scheme across states, while the investment opportunity is used to adjust risk exposure and therefore changes as p changes.[19] As an illustration, we plot in Figures 1e and 1f the optimal investment share $\alpha(p)/\overline{\alpha}$ and the response function under $k = 1$ and $\overline{\alpha} = 0.75$.

[19] More formally, observe that the critical optimality condition yielding a constant reported probability when the investment is interior is $\frac{1-q^*}{q^*} = k$. This condition equalizes the ratio of

3. Experimental Treatments

The main features of the experimental design are similar to OSKW's (2009) calibration experiment without explicit reference to beliefs or probability. In short, subjects are presented with a list of events and asked to select a "choice number" for every event. To each of the choice numbers corresponds two payments generated with a QSR. The first is the payment to the subject when the event occurs, while the second is the payment to the subject when the event does not occur. Note that, like in OSKW's, each event has an objective probability (i.e. it can be calculated using standard probability theory), and the instructions are neutral in the sense that they never refer to "probabilities", "predictions" or "beliefs".

3.1. The Control Treatment

We now provide additional information about the control treatment (T_0), and point out the differences with OSKW's design. The events presented to subjects concern the outcome of the roll of two 10-sided dice. Unlike OSKW, the events are not homogenous, and consist in three different series of 10 events. A precise description of the events and series is postponed to subsection 3.5.

The subjects' payments are generated with a QSR of the form $S_1(q) = a \cdot [1 - (1-q)^2]$ and $S_0(q) = a \cdot [1 - q^2]$, where $a = 4,000FCFA$ in our experiment.[20] A subject's possible choice numbers, as well as their corresponding payments were presented in the form of a "choice table" (see Appendix C).[21] The table consists of 149 possible choice numbers, instead of 100 in OSKW.[22] Each entry in the choice

the marginal return across states of the PSR $\frac{1-q^*}{q^*}$ to the ratio of the marginal returns across states of the investment opportunity k. If $\frac{1-q^*}{q^*}$ is larger than k, the PSR is more efficient than the investment opportunity to transfer wealth from state 0 to state 1, which can be done by increasing q^*. Conversely, if $\frac{1-q^*}{q^*}$ is lower than k, the PSR is more efficient than the investment opportunity to transfer wealth from state 1 to state 0, which can be done by decreasing q^*. At the limit, the agent must therefore set $q^* = 1/(1+k)$. This intuition also suggests that the result that q^* may be independent from p is specific to the decision-making environment we have considered.

[20] The Franc CFA is the currency used in Burkina Faso where the experiment was conducted (see Section 3.6 for details). The conversion rate at the time was roughly \$1 for 455 FCFA.

[21] How to best present PSR to subjects remains an open question. Tables, although not ideal, have been often adopted in part because they are simple to implement (see e.g. McKelvey and Page 1990, Sonnemans and Offerman 2004, Rustrom and Wilcox 2009, Blanco et al. 2009).

[22] We adopted this approach so that the choice number is different from the event's objective probability. For instance, the choice number corresponding to the objective probability 12% is

table, and in particular the link between choices and payments, were explained in details, and illustrated through several examples (see Appendix C). After reading the instructions, the subjects' understanding of the table was submitted to a test, which was then solved by the experimenter. The subjects were then presented with the list of 30 events, one series at a time. No time limit was imposed, and the subjects could modify any of their previous choices at anytime.

Once every subject completed his task, the experimenter randomly drew one of the thirty events, and rolled the two dice once to determine whether this event occurred or not. Every subject in a session was then paid according to the outcome of the roll and the choice number she selected for the event randomly drawn. For instance, if a subject selects the choice number 30 for the event randomly drawn, she receives either 1,440 FCFA if the event obtains, or 3,840 FCFA if the event does not obtain (see Appendix C). This amount constitutes the entirety of a subject's payments, as no show-up fee was provided in the control treatment.

Based on the theoretic analysis conducted in the previous section, we can frame the experimental hypotheses for each treatment in terms of properties of the response function $R(p)$. In particular, assuming subjects in our experiment are risk averse, we can use Corollary 2.1, to formulate our first hypothesis.

$\mathbf{H_0}$: *The response function in T_0 is i) regressive (i.e. it crosses the diagonal from above) and ii) has a fixed point at* $1/2$.

3.2. The "High Incentives" and "Hypothetical Incentives" treatments

Two treatments were conducted to study the effect of incentives. As indicated in Table 1 where the difference between treatments are summarized, the "High Incentives" treatment (T_1) is identical to the control treatment except that every payment in the choice table is now multiplied by $a = 10$. For instance, if a subject chooses row 30, she received either 14,400 FCFA (instead of 1,440 FCFA in T_0) if the event obtains, or 38,400 FCFA (instead of 3,840 FCFA in T_0) if the event does not obtain. The "Hypothetical Incentives" treatment (T_2) is identical to the "High Incentives" treatment except that payments are now hypothetical. More specifically, subjects in T_2 were told they would not be paid the amount in the choice table. Instead, they received a flat fee of 3,000 FCFA for completing the task, regardless of their choices.

18, while it is 12 in OSKW. To simplify the analysis we normalize the subjects' responses by multiplying the choice number they selected by $2/3$. The resulting measure $q \in [0, 100]$ can then be interpreted as the probability inferred from the subject's choice.

Proposition 2.3 shows that incentives affect the response function only when the relative risk aversion is non-constant. To derive an hypothesis, we follow most of the economic literature (including recent papers on scoring rules such as OSKW or Andersen et al. 2009) and assume constant relative risk aversion.

$\mathbf{H_1}$: *The response function in* T_1 *is identical to the response function in* T_0.

Since subjects' choices are not incentivized in T_2, no theoretical prediction can be derived. However, recent experimental results by Holt and Laury (2002) suggest that treatments with high hypothetical payments and treatment with low real payoffs yield similar results. This leads to our next hypothesis.

$\mathbf{H_2}$: *The response function in* T_2 *is identical to the response function in* T_0.

3.3. The "Low Stake" and "High Stake" Treatments

Two treatments were conducted to study the effect of stakes. These treatments are identical to the control treatment except that subjects receive a bonus when the event occurs. The bonus is 2,000 FCFA in the "Low Stake" treatment (T_3), and 8,000 FCFA in the "High Stake" treatment (T_4).

Lemma 2.2 demonstrates that adding a positive stake when the event occurs lowers the response function. Therefore the response functions in T_3 and T_4 are predicted to be less elevated everywhere than the response function in T_0. Moreover, Proposition 2.4 identifies two special cases, one in which the response function is regressive with an interior fixed point, and one in which the response function is below the diagonal everywhere (and therefore has no interior fixed point). It is easy to show, given the size of the stakes and the specific QSR we used, that T_3 corresponds to the first case with a fixed point at 1/4, while T_4 corresponds to the second case. This leads to the following hypotheses.

$\mathbf{H_3}$: *The response function in* T_3 *i) is lower than in* T_0 *and ii) has a fixed point at* 1/4.

$\mathbf{H_4}$: *The response function in* T_4 *i) is lower than in* T_3 *and ii)* is *lower than* p.

3.4. The "Low Hedging" and "High Hedging" Treatments

Two treatments were conducted to study the effect of hedging. These treatments are identical to the control treatment except that subjects are asked to make an additional decision for each event. Namely, subjects were offered the opportunity to bet an amount between 0 and 2,000 FCFA. If the event does not occur the bet is lost. If the event occurs, the bet multiplied by 2 (respectively, 4) is paid to the subjects in the "Low Hedging" treatment T_5 (respectively, the "High Hedging" treatment T_6). Finally, in both states of the world, the subjects retain the part of the $2,000$ FCFA they did not bet. In other words, in addition to selecting a choice number, subjects in the hedging treatments are asked to make a simple portfolio decision with two assets, a riskless and a risky asset.

Proposition 2.5 shows that, with the possibility for hedging, the response function is regressive, but constant over an interval located to the right of the fixed point. Observe also that T_5 corresponds to the case $k = 1$, and T_6 to the case $k = 3$ in section 2.6. It is then immediate to show that the fixed points in T_5 and T_6 are respectively $1/2$ and $1/4$. Moreover, Proposition 2.5 provides some information about what share of her initial endowment a subject should invest in the risky asset. This leads to the following hypotheses.

$\mathbf{H_5}$: *The response function in T_5 is i) regressive with a fixed point at $1/2$, ii) equal to $1/2$ when the share invested is in]0,1[, and iii) lower than in T_0 when p is close to 1; Moreover, the share invested is iv) 0 if and only if $p \leq 1/2$ and v) 1 when p is close to 1.*

$\mathbf{H_6}$: *The response function in T_6 is i) regressive with a fixed point at $1/4$, ii) equal to $1/4$ when the share invested is in]0,1[, and iii) lower than in T_5 when p is close to 1; Moreover, the share invested is iv) 0 if and only if $p \leq 1/4$, v) 1 when p is close to 1, and vi) no less than in T_5 for any p.*

3.5. Comparison of the Three Series

As mentioned previously, the 30 events presented to subjects has been split into 3 series of 10 events. In each series, the 10 events describe the possible outcome resulting from the roll of two 10-sided dice (one black, the other red). To better compare choices across series, the 10 events in each series have the same objective probabilities 3%, 5%, 15%, 25%, 35%, 45%, 61%, 70%, 80%, and 90%. The events in Series 1 are similar to those in OSKW's calibration exercise. Namely, we told subjects that the red die determines the first digit, and the black die determines the second digit of a number between 1 and 100. For instance, the event with objective probability 25% was described as "the number drawn is between 1 (included) and 25 (included)". A complete description of the events, and the order in which they were presented to subjects may be found in Appendix C.

In Series 2, we consider events whose probabilities, although still objective, are arguably more difficult to calculate than in Series 1. Namely, we told subjects that the two dice would be added to form a number between 0 and 18. For instance, the event with 25% probability was described as "the sum obtained is between 2 (included) and 6 (included)".

The object of Series 3 is to test how subjects respond when faced with (objective) compounded probabilities. To do so, we asked subjects to select a single choice number not for one, but for two possible events. The subjects were told that the experimenter would throw a fair coin to determine which of the two possible events would be taken into consideration for payments. The events used in Series 3 are similar to those in Series 1, i.e. the roll of the red and black dice produces a number between 1 and 100 and the events give a possible range for that number. For instance, the event with probability 25% was described as "If the coin falls on the **Head** side, then the event is : "the number drawn is between 82 (included) and 89 (included)" ; otherwise, if the coin falls on the **Tail** side, then the event is : "the number drawn is between 25 (included) and 66 (included)"".

Since the objective probabilities are the same in each series, we expect no difference across the three series if subjects behave in a way consistent with expected utility based on von Neumann Morgenstern axioms. Nevertheless, experimental evidence suggests that subjects' responses may be affected by the complexity in calculating objective probabilities. In particular Halevy (2007) finds that most subjects in his experiments do not reduce compound probabilities. From a theoretical perspective, there are different ways to relax the standard reduction axiom. We opted for a simple model of ambiguity recently introduced by Klibanoff, Marinacci and Mukerji (2005) (KMM hereafter).[23] Under this model, we show in Proposition 6.1 (see Appendix B) that, in the context of our experiment, ambiguity aversion reinforces the effect of risk aversion. As a result, the response functions obtained with Series 2 and 3 may more biased than the response function obtained with Series 1. To derive an hypothesis, however, we consider the standard expected utility approach.

$\mathbf{H_7}$: *The response functions obtained in Series 1, 2 and 3 are identical.*

3.6. Implementation of the Experiment

The experiment took place in Ouagadougou (Burkina Faso) in June 2009.[24] The choice of the location was motivated by two factors. First, we wanted to take

[23] One reason for using this model is that KMM preferences enable one to distinguish ambiguity aversion from risk aversion. Also, Halevy (2007) finds some support for these preferences.

[24] Burkina Faso is a Francophone country in West Africa with over 13 million inhabitants, among which around 1.4 million live in the capital city Ouagadougou.

advantage of a favorable exchange rate i) to create salient financial differences between treatments (e.g. between the reference and the "Hypothetical Incentives" treatments), and ii) to provide subjects with substantial incentives so that risk aversion had a fair chance to play a role.[25] Second, one of the authors had conducted several experiments in Ouagadougou over the past three years (see e.g. Armantier and Boly 2008, 2009). Building on our experience, we followed a well established protocol to hire subjects and rent a lab to conduct the experiment.

More specifically, we used a local recruiting firm (Opty-RH) to place flyers around Ouagadougou stating that we were looking for subjects for a paid economic experiment. The subjects had to be at least 18 years old, and be current or former university students. People interested had to come to the recruiting firm location with a proof of identification, and either a valid university diploma or a proof of university enrollment. After validating their credentials, subjects were randomly assigned to a session and told when and where to show-up for the experiment.

The sessions were conducted in a centrally located high school we had already rented in the past to conduct other experiments. Upon arrival, the subjects were gathered in a large room. The instructions were read aloud, followed by questions, and the comprehension test. The subjects were then presented with the 30 events and asked to make their choices using pen and paper. Finally, the subjects filled a short questionnaire, after which they were paid in cash. Two sessions were conducted for each treatment, with each session taking on average 90 minutes to complete.

As indicated in Table 2, a total of 301 subjects participated in the experiment, with a minimum of 41 subjects per treatment. The subjects are mostly composed of men (74%) and students currently enrolled at the university (68%), ranging in age between 19 and 38 (with a median at 25 years old). In a post experiment survey, slightly more than half of the subjects reported having taken a probability or a statistics class at the university. Finally, most of the subjects (86%) reported not having participated in a similar economic or psychology experiment. Excluding the "Hypothetical Incentives" treatment (where earnings were fixed at $3,000$ FCFA), the average earnings of a subject were $8,861$ FCFA. As indicated in Table 2, however, earnings vary greatly across subjects and treatments (the smallest amount paid was 100 FCFA and the maximum was $40,000$ FCFA).

[25] The maximum payment of $40,000$ FCFA in the "High Incentives" treatment exceeds the monthly average entry salary (pre-tax) for a university graduate.

4. Experimental Results

4.1. The control Treatment (T_0)

Figure 2 shows for each of the 7 treatments conducted the subjects' average responses to the events in the three series. According with hypothesis H_0, the three response functions in the control treatment are regressive. Moreover, they exhibit the traditional inverse S-shape with a fixed point around 1/2. This observation is consistent with the literature, as similar shapes have been previously identified when eliciting beliefs (Huck and Weizsaker 2002, Sonnemans and Offerman 2004, Hurley and Shogren 2005, as well as OSKW). Figure 2 also reveals that the three series can be ordered with respect to their respective biases. Indeed, it appears that Series 1 (the simple probabilities) yields the smallest biases for virtually all objective probabilities (i.e. its corresponding response function is consistently the closest to the diagonal), while Series 2 (the complex probabilities) generates the largest biases. This result appears to contradict hypothesis H_7 stating that under expected utility there should be no systematic differences across the three series.

To test our hypotheses more formally, we compare statistically the subjects' choices across series and treatments with a parametric model of the form:

$$\widehat{P}_{it} = \varphi\left(P_t\right) + \eta_i + u_{it} \tag{4.1}$$

where $\widehat{P}_{it}$ is the reported probability corresponding to the choice number N_{it} selected by subject i for event $t = 1, ..., 30$ (i.e. $\widehat{P}_{it} = 2/3 \cdot N_{it}$), P_t is the objective probability of occurrence of event t, η_i is a mean zero normally distributed individual specific error term, u_{it} follows a normal distribution truncated such that $\widehat{P}_{it} \in [0, 1]$, and $\varphi\left(.\right)$ is a continuous function satisfying $\varphi\left(0\right) = 0$, $\varphi\left(1\right) = 1$ and $\varphi'\left(.\right) > 0$. Consistent with previous literature, we consider a function that may exhibit an inverse S-shape:

$$\varphi\left(P_t\right) = \exp\left(\left[\ln P_t\right]^b \cdot \left[\ln(a)\right]^{1-b}\right) \tag{4.2}$$

where $a \in [0, 1]$ and $b > 0$.

Observe that under the reparametrization $\left\{b = \alpha; a = \exp\left(-\beta^{\frac{1}{1-\alpha}}\right)\right\}$, (4.2) is in fact the probability weighting function $w\left(P_t\right) = \exp\left(-\beta\left[-\ln P_t\right]^{\alpha}\right)$ proposed in a different context by Prelec (1998). The specification in (4.2) was preferred to Prelec's because the parameters are easier to interpret in our context. Indeed, observe that $\varphi\left(a\right) = a$ and $\varphi'\left(a\right) = b$. In other words, a captures where the function φ crosses the diagonal, while b captures the slope of φ at this fixed point. Finally, we control for treatment and series effects by modelling the parameters in (4.2) as follows:

$$a = a_0 + a_1 \cdot (S_2 + S_3) + a_2 \cdot S_3 + a_3 \cdot T_0 + a_4 \cdot T_0 \cdot (S_2 + S_3) + a_5 \cdot T_0 \cdot S_3$$

where T_0 is a dummy variable equal to 1 when the observation was collected in the control treatment, while S_2 and S_3 are dummy variables equal to 1 when the event belongs respectively to Series 2 and Series 3. The parameter b is modelled in an analog fashion. To estimate the model with the data collected only in the control treatment, the parameters a_3 to a_5, as well as b_3 to b_5 are all set equal to zero. The parameters, estimated by Maximum Simulated Likelihood, are reported in Table 4.

First, observe that in the control treatment a_0 is not significantly different from 1/2, while b_0 is significantly smaller than 1. This therefore confirms that the subjects' response function for the events in Series 1 exhibits an inverse S-shape and crosses the diagonal near 1/2. Note also that a_1 and a_2 are not significant in the control treatment. In other words, the fixed points do not appear to vary significantly for the three series in the control treatment. In contrast, b_1 is significantly smaller than 0, while b_2 is positive and significant. This result confirms that the inverse S-Shape is the most pronounced for the events in Series 2, and the least pronounced for the events in Series 1. Observe also in Table 4 that the sign and significance of (a_1, a_2), as well as (b_1, b_2), are generally consistent across treatments. This therefore implies that the ranking of the three series in terms of the biases they generate is generally preserved regardless of treatments.[26]

To gain a different perspective on the data, we calculate in Table 3 four statistics. The first is the average number of "extreme predictions", that is the average number of time a subject selects a choice number below 10 (which corresponds to a reported probability below 6.66%) or above 140 (which corresponds to a reported probability above 93.33%). We also calculate two measures of the errors made by subjects when ranking the objective probabilities. The first one (Error 1) consists of the average number of time a subject incorrectly ranks two consecutive choice numbers with respect to their objective probabilities (e.g. the choice

[26] Similar conclusions can be reached nonparametrically by using Friedman tests for each objective probabilities and series. Table 6 shows that in most treatments Series 1 (Series 2) generally has the lowest (highest) ranking of reported probabilities for objective probabilities below 50%, and the lowest (highest) ranking of reported probabilities for objective probabilities above 50%.

number selected by a subject for the objective probability 5% is higher than the choice number he selected for the objective probability 15%). The second measure captures the number of reported probabilities on the incorrect side of 1/2. More specifically, "Error 2" consists of the average number of times a subject selects a choice number above (below) 75 for an objective probability below (above) 50%. Finally, the choice numbers a subject selects for a given objective probability are ordered across the three series from least to most biased. We can see in Table 3 that these four criteria paint a consistent picture: Series 1 (respectively, Series 2) has the most (least) extreme predictions, the fewest (most) errors, and the best (worst) ranking in terms of bias. These results therefore provide further support against the hypothesis that subjects respond similarly to the events in the three series.

To summarize, we find statistical evidence that the response functions in the control treatment exhibit the traditional inverse S-shape with a fixed point around 1/2. This result is consistent with subjects being risk averse expected utility maximizers, and therefore supports hypothesis H_0. Moreover, we find that the responses to the events in Series 1 (Series 2) are statistically the most (least) biased. This result cannot be explained under expected utility, and therefore it contradicts hypothesis H_7. Instead, the systematic differences between the three series can be rationalized by ambiguity aversion under the additional assumption that Series 1 (the simple probabilities), 3 (the compounded probabilities) and 2 (the complex probabilities) are ranked in increasing order of ambiguity. Arguably, this assumption could find support in the fact that, although all objective, these three types of probabilities (simple, compounded and complex) require different levels of computational sophistication to calculate. In a recent paper, Halevy (2007) concludes that attitudes toward ambiguity and compound objective lotteries are tightly associated. Our experimental results not only support Halevy's conclusion, but they also extend it by suggesting that complex objective probabilities may also be perceived as ambiguous.[27]

[27]Our experiment also points out a potential problem with OSKW approach to correct for risk aversion: For the same agent, different correction functions could emerge in their calibration exercise depending on the type of objective probabilities considered. An argument could be made for using the "simplest" objective probabilities possible (such as those in Series 1), but our experience suggests that simplicity is a relative concept: while obvious for anyone with basic knowledge of probability, the events in Series 1 turned out to be surprisingly challenging for some of our subjects.

4.2. The Incentives Treatments (T_1 and T_2)

Figure 2 indicates that the response functions for the three series in the "High Incentives" treatment are flatter than in the control treatment, although they still cut the diagonal around 1/2. This observation is confirmed statistically in Table 4. Indeed, a_3 to a_5 are not significantly different from 0 in T_1, thereby suggesting that the fixed points of the different response functions cannot be distinguished statistically across the two treatments. In contrast, we find the parameter b_3 to be positive and significant in T_1. This confirms that, compared to the control treatment, the response functions are generally flatter in the "High Incentive" treatment. Similar conclusions are reached nonparametrically by using Mann-Whitney tests for each objective probability and each series. Indeed, Table 5 shows that for most series and objective probabilities (except some objective probabilities around 1/2) the distributions of responses in T_1 are closer to the diagonal than in T_0. Note also that the the signs and magnitudes of b_4 and b_5 in Table 4 suggest that subjects' choices are more homogenous across the three series in T_1. This observation finds additional support in the criteria reported in Table 3. Indeed, responses to the events in Series 2 (Series 1) remain the most (least) biased and the most (least) prone to errors, but the differences across series are not as severe as in T_0.

To sum up, responses in T_1 are significantly different than in T_0, which refutes hypothesis H_1 derived under the assumption of constant relative risk aversion. Instead, as explained in Section 2.4, choices in T_1 are consistent with subjects exhibiting increasing relative risk aversion. In a recent paper, Andersen et al. (2009) also conclude that their subjects' behavior in a similar belief elicitation experiment may be best described under increasing relative risk aversion. It is also interesting to note that our results imply that paying more do not necessarily yield "better" answers. Instead, we find that, because of our subjects' specific form of relative risk aversion, using a PSR that provides higher incentives generate more biases.

As for the "Hypothetical Incentives" treatment T_2, Figure 2 reveals that subjects' responses, although still exhibiting the inverse S-shape, are on average closer to the diagonal than in the control treatment. This observation is confirmed statistically in Table 4 as b_3 is found to be negative and significant in T_2. Note also that σ_u, the standard deviation of the error term u_{it} in (4.1), is significantly larger for T_2 than for T_0. In addition, observe in Table 3 that the number of extreme predictions and errors (of both forms) are systematically greater in T_2 compared to T_0. In other words, it appears that, although not as biased, subjects' responses are noisier in the "Hypothetical Incentives" treatment. These results therefore do not support hypothesis H_2. Instead, we find that, when eliciting beliefs with a QSR, subjects behave differently if they are provided with real or hypothetical incentives. Our conclusions are only partially consistent with the literature. Like Gachter and Renner (2006), we find that financial incentives reduce the noise in the beliefs elicited. In contrast with our experiment however, Sonnemans and Offerman (2004) find no difference between rewarding predictors with a QSR or with a flat fee.

4.3. The Treatments with Stakes (T_3 and T_4)

For the smallest objective probabilities, no obvious difference is visible in Figure 2 between the response functions in the control treatment and those in the low and high stakes treatments. In contrast, Figure 2 clearly shows that, compared to T_0, responses for the highest objective probabilities are lower in T_3, and the lowest in T_4. These observations are confirmed by the nonparametric tests in Table 5. There, we can see that the samples of responses for each of the three series are stochastically lower in T_0 for most objective probabilities above 25%. In addition, the comparison of the low and high stakes treatments in Table 5 indicates that in general there exist a significant difference between the two treatments, whereby the probabilities stated by subjects are generally lower in T_4 than in T_3. The parametric estimations in Table 4 confirm these results. Indeed, a_3 and b_3 are positive and significant for both treatments T_3 and T_4, but significantly larger for treatment T_4. This implies that, compared to T_0, the response functions become lower and flatter in the "Low Stake" treatment, and that the magnitude of this effect is stronger in the "High Stake" treatment. In other words, part i) of predictions H_3 and H_4 (i.e. the response functions are less elevated in T_3 and T_4) are verified. Behavior in the experiment, however, is not fully consistent with our predictions. Indeed, observe in Table 4 that the parameter a_3 is significantly different from $1/4$ in T_3, and from 0 in T_4, thereby contradicting part ii) of H_3 and H_4.

To sum up, we find that when they have a stake in the event, subjects do tend to smooth their payoffs across the two states, especially when the event is likely to occur. This treatment effect is only partially in agreement with the theory: the direction is correct, but the magnitude is insufficient.

4.4. The Treatments with Hedging (T_5 and T_6)

The response functions for the two hedging treatments plotted in Figure 2 reveal several differences compared to those in the control treatment. First, for the highest objective probabilities, the response functions become lower in T_5, and the lowest in T_6. Second, the response functions have a fixed point around 1/2 in T_5 and slightly above 40% in T_6. Third, the response functions appear slightly flatter (but not perfectly flat) around the diagonal in both hedging treatments. Most of these observations are confirmed statistically by the parametric and nonparametric tests in Tables 4 and 5. In particular, observe in Table 4 that the estimate of a_3 is insignificant in T_5, while it is positive and significant in T_6. The former is consistent with part i) of hypothesis H_5, as we cannot exclude that, as in the control treatment, the response functions in the "Low Hedging" treatment cut the diagonal at 1/2. In the "High Hedging" treatment, however, the parameter a_0 is found to be significantly greater than 1/4 which contradicts part i) of hypothesis H_6. Observe also that b_3 is significant and positive in both T_5 and T_6, thereby indicating flatter responses around the diagonal in the two hedging treatments compared to T_0. The nonparametric tests in Table 5 also confirm that, compared to T_0, responses for most objective probabilities above 60% may be considered statistically lower in T_5, and the lowest in T_6. This result therefore supports part iii) of hypotheses H_5 and H_6.

Turning now to the subjects' betting behavior in the two hedging treatments, we can see in Figure 2 that, according with part vi) of hypothesis H_6, subjects in the "High Hedging" treatment invest more in the risky asset for any objective probability than in the "Low Hedging" treatment. This observation is confirmed statistically by the nonparametric tests reported in the last column of Table 5. The subjects betting behavior, however, is not fully consistent with the theory. In particular we can see in Figure 2 that, in contrast with part iv) and v) of hypotheses H_5 and H_6, bets are too low for the high objective probabilities in T_5 (where they should be near or at 100%), and too high for the low probability in T_6 (where they should be near or at 0%).

To summarize, although not fully consistent with the theory, subjects in our experiment appear to take partial advantage of their hedging opportunities. In particular, we find that subjects tend to bet high on the most likely events, while simultaneously making lower predictions than in the control treatment. In other words, it seems that subjects are willing to take some risk on the bet, while using the scoring rule as an insurance in case the event does not occur.

5. Conclusion and Discussion

Introduced in the 1950's by statisticians, Proper Scoring Rules (PSR) have arguably become the most popular incentivized belief elicitation mechanism. In the simplest environment, a well known result is that risk averters are better off misreporting their beliefs by stating more uniform probabilities (i.e. closer to 1/2 in the case of a binary event). Combining theory and experiment, we find that this result does not generalize to richer environments of particular interest to economists. Instead, we have shown that higher incentives, stakes and hedging lead agents to distort their reported probabilities in complex, yet mostly predictable manners. We believe our results may have several important implications.

First, as argued in the introduction, belief elicitation in non-experimental settings typically involves some form of a stake or the possibility to hedge one's prediction. In particular, agents who participate in Prediction Markets based on PSR (the so called Market Scoring Rule) always have a stake in the event they predict. In addition, like in our experiment, the stakes in these field environments are likely to be far larger than the prediction's reward.[28] Our results therefore suggest that eliciting beliefs in the field with a PSR can generate severe biases. Nevertheless, the beliefs elicited may still be considered informative, as we find they tend to be consistent with our theoretic predictions.

Second, our results may be relevant for traditional lab experiments. In particular, stakes and hedging opportunities may explain why i) some subjects fail to best-respond to their stated beliefs (e.g. Costa-Gomez and Weizsaker 2008), and ii) observers make different predictions about the play of a game than the subjects actually playing the game (e.g. Palfrey and Wang 2009). Observe however, that our subjects are paid more than in most lab experiments.[29] Consequently, our results may not generalize to traditional lab settings where risk aversion may play a lesser role. In fact, Blanco et al. (2009) find that lab subjects do not seem to take advantage of hedging opportunities while eliciting beliefs about one's opponent in a prisoner's dilemma game.[30]

Third, the effects of stakes and hedging we identify in this paper are not spe-

[28] For instance, think about an agent who is asked to make a prediction about the stock market, a natural catastrophe, or the future of her industry. Most prediction payments are likely to pale in comparison of the stakes the agent could have in these events.

[29] A notable exception is Andersen et al. (2009) whose subjects were paid up to \$100 for a similar belief elicitation task.

[30] Although stakes and hedging may not affect average outcomes in the lab, a few sophisticated or highly risk averse subjects may suffice to pollute the data collected.

cific to PSR. It is easy to show that other belief elicitation techniques, although they may offer some protection against risk aversion in the simplest environments, are not immune to stakes and hedging. This is the case for instance for the standard lottery mechanism (Kadane and Winkler, 1988) and for the direct revelation mechanism recently proposed by Karni (2009). In fact, we are not aware of any incentivized belief elicitation method that would directly address these issues.[31] In the remainder, we briefly discuss possible remedial measures that may be taken in an effort to elicit unbiased beliefs.

The method the most frequently implemented in order to mitigate the effect of risk aversion consists in using a PSR that pays small amounts (e.g. Nyarko and Schotter 2002, Rustrom and Wilcox 2009). Contrary to a widespread belief however, this approach may not be effective as there is no guarantee it leads agents to report their beliefs more truthfully. Instead, we have shown theoretically that smaller payments can either accentuate or attenuate the PSR biases depending on the agent's relative risk aversion.

The second remedial approach recognizes that if an agent's primitives (e.g. utility, wealth, stakes) are known, then his optimal reported probability can be calculated for any subjective probability. This function could then be inverted in order to recover the agent's unobserved beliefs from his stated probabilities. One could therefore imagine an approach akin to OSKW or Andersen et al. (2009), where relevant data are first collected in a calibration exercise in order to estimate this correction function.[32]

Another well known remedial measure is to induce risk neutrality by paying agents in lottery tickets that give them a chance to win a prize (Roth and Malouf 1979, Allen 1987, Schlag and van der Weele 2009). In theory, it is easy to show that this approach is effective under expected utility to elicit truthful beliefs as long as all payments (including the stakes and hedging revenues) are made in lottery tickets. In practice however, doubts have been expressed about the ability of this approach to control for risk attitude (Davis and Holt 1993, Selten, Sadrieh and Abbink 1999).

A fourth possible approach when eliciting beliefs while playing a game is to make the prediction and the game decision independent. For instance, Blanco et

[31] Importantly, Karni and Safra (1995) show that unbiased belief elicitation is impossible when stakes are not observed by the experimenter. This impossibility result holds even if the utility function is observable, and even if several experiments can be implemented.

[32] This approach, however, tends to make belief elicitation with a PSR (an already intrusive method) even more cumbersome.

al. (2009) propose to randomly pay subjects either their game or their prediction payoffs. Likewise, a subject in Armantier and Treich (2009) is randomly matched with two partners. The subject plays the game against the first partner, and her prediction is scored against the play of the second partner.

The last, and arguably simplest approach, consists in eliciting beliefs without offering any financial reward for accuracy. Such non-incentivized methods are common in statistics, psychology and in field surveys. In his review of the surveys literature in economics, Manski (2004) concludes that the beliefs elicited in such a way are informative. Our results also suggest that hypothetical payoffs may be preferred if one is willing to trade noise for unbiaseness.

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Table 1 : Financial Differences between Treatments (in FCFA)							
	T0 Control	T1 High Incentives	T2 Hypothetical Incentives	T3 Low Stakes	T4 High Stakes	T5 Low Hedging	T6 High Hedging
Show-up-fee	0	0	3,000	0	0	0	0
Maximum Scoring Rule Payment	4,000	40,000	0	4,000	4,000	4,000	4,000
Stakes	0	0	0	2,000	8,000	0	0
Maximum Return on Investment	0	0	0	0	0	4,000	8,000

Table 2 : Characteristics of the Subject Pool							
	T0 Control	T1 High Incentives	T2 Hypothetical Incentives	T3 Low Stakes	T4 High Stakes	T5 Low Hedging	T6 High Hedging
Number of Subjects	43	43	48	41	44	41	41
Age	24.581 (2.373)	25.119 (2.350)	24.208 (2.042)	24.350 (2.315)	25.605 (2.977)	24.850 (3.286)	25.475 (2.792)
% of Female	27.9%	23.8%	25.0%	29.3%	23.3%	25.0%	29.3%
% Currently Enrolled at the University	67.2%	64.5%	71.6%	70.5%	60.8%	68.0%	61.5%
% with University Course in Probability	60.0%	52.8%	54.2%	61.0%	57.5%	50.0%	53.7%
% with Previous Participation in Experiment	14.6%	15.0%	16.7%	9.8%	20.9%	15.4%	12.2%
Subjects' average Earnings	3,055.2 (1,565.5)	28,341.8 (8,121.7)	3,000 (__)	5,277.3 (1,173.2)	7,864.9 (4,189.5)	4,992.8 (1,226.3)	6,155.8 (1,583.9)

Table 3 : Features of Subjects' Responses								
	Series	T0 Control	T1 High Incentives	T2 Hypothetical Incentives	T3 Low Stakes	T4 High Stakes	T5 Low Hedging	T6 High Hedging
Extreme Prediction[1]	S1	1.070 (0.985)	0.023 (0.152)	2.116 (1.531)	1.148 (1.236)	0.977 (1.000)	1.122 (1.166)	1.244 (1.985)
	S2	0.558 (0.796)	0.093 (0.366)	1.465 (1.120)	0.415 (0.774)	0.864 (1.268)	0.439 (0.808)	0.780 (2.019)
	S3	0.721 (0.959)	0.070 (0.258)	1.488 (1.470)	0.902 (0.944)	0.932 (1.208)	0.683 (1.059)	1.000 (1.987)
Error 1[2]	S1	1.093 (0.947)	0.953 (1.045)	1.698 (1.301)	1.049 (0.973)	0.955 (0.861)	1.171 (1.093)	1.049 (0.973)
	S2	1.744 (1.157)	1.465 (1.437)	2.349 (1.066)	1.683 (0.934)	1.727 (1.169)	1.878 (1.364)	1.439 (1.285)
	S3	1.395 (1.158)	1.209 (1.186)	2,372 (1.047)	1.537 (1.247)	1.409 (1.085)	1.488 (0.952)	1.293 (1.123)
Error 2[3]	S1	0.419 (0.626)	0.372 (0.655)	0.605 (0.760)	0.634 (0.733)	0.932 (1.087)	0.537 (0.636)	0.878 (1.345)
	S2	0.767 (0.947)	0.674 (0.919)	1.163 (0.998)	0.878 (0.872)	1.523 (1.191)	0.683 (0.960)	1.220 (1.475)
	S3	0.674 (0.680)	0.372 (0.725)	1.163 (0.924)	0.805 (0.782)	1.227 (1.309)	0.390 (0.737)	1.073 (1.403)
Series Ranking[4]	S1	1.515 (0.669)	1.828 (0.629)	1.795 (0.721)	1.732 (0.254)	1.748 (0.694)	1.793 (0.251)	1.741 (0.263)
	S2	2.398 (0.724)	2.112 (0.712)	2.091 (0.792)	2.318 (0.250)	2.255 (0.731)	2.220 (0.242)	2.310 (0.241)
	S3	2.087 (0.708)	2.060 (0.676)	2.114 (0.783)	1.950 (0.292)	1.998 (0.717)	1.988 (0.257)	1.949 (0.228)

In each cell, the first number is the average per subject, while the number in parenthesis is the standard deviation.

[1] For each subject and each Series, "Extreme Prediction" captures the number of time his choice number is below 10 or above 140.

[2] For each subject and each Series, "Error 1" captures the number of time his choice numbers are incorrectly ordered (e.g. In Series 1, the choice number associated with the 5% probability event is greater than the choice number associated with the 15% probability event).

[3] For each subject and each Series, "Error 2" captures the number of time a choice number above 75 is selected for an event with probability below 50%, plus the number of time a choice number below 75 is selected for an event with probability above 50%.

[4] For each subject and each of the 10 objective probabilities, the three series are ranked from least to most biased. "Series Ranking" therefore equals to 1, 2 or 3, depending on that ranking.

Table 4 : Estimation of the Response Function

	T0 Control	T1 High Incentives	T2 Hypothetical Incentives	T3 Low Stakes	T4 High Stakes	T5 Low Hedging	T6 High Hedging
a_0	0.480^{***} (0.024)	0.501^{***} (0.006)	0.499^{***} (0.156)	0.425^{***} (0.016)	0.393^{***} (0.032)	0.493^{***} (0.012)	0.402^{***} (0.029)
a_1 $(S_2 + S_3)$	0.026 (0.021)	0.004 (0.005)	-0.018 (0.016)	0.019 (0.013)	0.006 (0.014)	-0.024 (0.016)	0.013 (0.013)
a_2 (S_3)	0.004 (0.011)	-0.002 (0.006)	-0.020 (0.043)	2.851E-4 (0.014)	-0.007 (0.012)	0.009 (0.013)	-0.010 (0.008)
a_3 (T_0)	—	-0.034 (0.023)	-0.038 (0.059)	0.048^{**} (0.023)	0.109^{***} (0.040)	-0.031 (0.029)	0.101^{***} (0.035)
a_4 $(T_0.[S_2 + S_3])$	—	0.020 (0.020)	0.047 (0.048)	0.005 (0.024)	0.003 (0.031)	0.052^{*} (0.028)	-0.009 (0.023)
a_5 $(T_0.S_3)$	—	0.008 (0.013)	0.024 (0.045)	0.004 (0.017)	0.015 (0.016)	-0.005 (0.017)	0.025^{*} (0.014)
b_0	0.736^{***} (0.054)	0.289^{***} (0.033)	0.901^{***} (0.060)	0.622^{***} (0.037)	0.518^{***} (0.038)	0.543^{***} (0.057)	0.499^{***} (0.040)
b_1 $(S_2 + S_3)$	-0.242^{***} (0.029)	-0.053^{***} (0.015)	-0.195^{***} (0.043)	-0.189^{***} (0.025)	-0.119^{***} (0.024)	-0.210^{***} (0.032)	-0.157^{***} (0.017)
b_2 (S_3)	0.067^{***} (0.021)	0.009 (0.013)	0.015 (0.035)	0.129^{***} (0.024)	0.056^{**} (0.026)	0.086^{***} (0.024)	0.072^{***} (0.013)
b_3 (T_0)	—	0.434^{***} (0.061)	-0.147^{**} (0.068)	0.107^{**} (0.051)	0.222^{***} (0.071)	0.207^{**} (0.083)	0.197^{***} (0.062)
b_4 $(T_0.[S_2 + S_3])$	—	-0.179^{***} (0.032)	-0.058 (0.055)	-0.048 (0.039)	-0.103^{**} (0.042)	-0.042 (0.046)	-0.049 (0.031)
b_5 $(T_0.S_3)$	—	0.055 (0.034)	0.052 (0.042)	-0.063 (0.042)	0.012 (0.036)	-0.017 (0.035)	-0.010 (0.024)
σ_η	0.020^{***} (0.004)	0.024^{***} (0.003)	0.029^{***} (0.003)	0.026^{***} (0.005)	0.093^{***} (0.017)	0.025^{***} (0.003)	0.114^{***} (0.015)
σ_u	0.107^{***} (0.006)	0.097^{***} (0.003)	0.133^{***} (0.004)	0.101^{***} (0.003)	0.106^{***} (0.004)	0.117^{***} (0.004)	0.103^{***} (0.004)
$\ln(L)$	-2602.37	-5362.18	-5532.78	-5165.09	-5220.24	-4832.89	-4937.02

In each cell, the first number corresponds to the point estimate, while the number in parenthesis is the estimated standard deviation of the parameter. ***, **, and * respectively indicate parameters significant at the 1%, 5% and 10% levels.

Table 5 : Non-Parametric Comparison of Treatments

Objective Probability	Series	T_0 vs. T_1	T_0 vs. T_2	T_0 vs. T_3	T_0 vs. T_4	T_3 vs. T_4	T_0 vs. T_5	T_0 vs. T_6	T_5 vs. T_6	
									Prediction	Bet
3%	S1	2.60E+02 8.64E-09	1.20E+03 1.74E-01	8.29E+02 6.37E-01	9.48E+02 9.90E-01	9.60E+02 6.08E-01	7.02E+02 1.07E-01	8.78E+02 9.71E-01	1.02E+03 9.10E-02	5.92E+02 1.60E-02
	S2	3.98E+02 5.19E-06	1.37E+03 7.00E-03	9.65E+02 4.55E-01	1.07E+03 2.90E-01	9.33E+02 7.88E-01	5.98E+02 1.10E-02	1.01E+03 2.40E-01	1.19E+03 1.00E-03	5.00E+02 1.00E-03
	S3	3.50E+02 6.55E-07	1.22E+03 1.37E-01	9.88E+02 3.42E-01	1.08E+03 2.39E-01	9.42E+02 7.28E-01	7.90E+02 4.09E-01	1.05E+03 1.23E-01	1.06E+03 3.90E-02	5.63E+02 6.00E-03
5%	S1	2.23E+02 1.30E-09	1.26E+03 7.10E-02	8.97E+02 8.93E-01	8.49E+02 4.09E-01	7.97E+02 3.52E-01	7.18E+02 1.43E-01	8.38E+02 6.93E-01	9.55E+02 2.87E-01	5.05E+02 1.00E-03
	S2	3.83E+02 2.70E-06	1.28E+03 4.50E-02	8.22E+02 5.94E-01	1.13E+03 1.26E-01	1.13E+03 4.80E-02	7.15E+02 1.34E-01	9.12E+02 7.85E-01	1.04E+03 6.50E-02	4.41E+02 6.44E-05
	S3	3.60E+02 1.04E-06	1.35E+03 1.10E-02	1.16E+03 1.20E-02	1.19E+03 4.10E-02	8.56E+02 6.82E-01	7.78E+02 3.54E-01	1.07E+03 9.20E-02	1.08E+03 2.40E-02	3.96E+02 1.89E-05
15%	S1	2.29E+02 1.73E-09	1.14E+03 3.75E-01	6.91E+02 8.80E-02	7.38E+02 7.70E-02	8.98E+02 9.72E-01	5.36E+02 2.00E-03	9.34E+02 6.41E-01	1.17E+03 2.00E-03	3.37E+02 1.96E-06
	S2	3.31E+02 2.71E-07	1.36E+03 9.00E-03	7.65E+02 2.95E-01	1.09E+03 2.26E-01	1.15E+03 3.20E-02	6.38E+02 2.90E-02	9.68E+02 4.38E-01	1.13E+03 7.00E-03	3.38E+02 1.80E-06
	S3	3.77E+02 2.04E-06	1.16E+03 2.92E-01	8.09E+02 5.13E-01	1.06E+03 3.46E-01	1.07E+03 1.35E-01	5.22E+02 1.00E-03	8.79E+02 9.79E-01	1.15E+03 4.00E-03	2.94E+02 1.89E-07
25%	S1	3.54E+02 7.59E-07	1.33E+03 1.80E-02	8.77E+02 9.68E-01	1.14E+03 9.90E-02	1.09E+03 9.50E-02	7.44E+02 2.18E-01	9.06E+02 8.26E-01	9.76E+02 2.09E-01	2.58E+02 4.34E-08
	S2	4.84E+02 1.24E-04	1.52E+03 9.87E-05	1.11E+03 4.50E-02	1.25E+03 1.00E-02	9.83E+02 4.76E-01	8.65E+02 8.82E-01	9.60E+02 4.84E-01	9.39E+02 3.60E-01	2.84E+02 1.29E-07
	S3	4.48E+02 3.36E-05	1.35E+03 1.10E-02	1.10E+03 4.80E-02	1.20E+03 3.00E-02	9.16E+02 9.02E-01	9.51E+02 5.32E-01	1.22E+03 3.00E-03	1.08E+03 2.90E-02	2.51E+02 1.83E-08
35%	S1	3.54E+02 7.02E-07	9.24E+02 3.88E-01	7.29E+02 1.72E-01	9.38E+02 9.42E-01	1.02E+03 2.88E-01	3.99E+02 1.46E-05	8.02E+02 4.76E-01	1.22E+03 3.98E-04	2.25E+02 5.29E-09
	S2	7.46E+02 1.13E-01	1.16E+03 3.13E-01	1.10E+03 4.50E-02	1.36E+03 3.70E-04	1.09E+03 9.80E-02	7.73E+02 3.18E-01	1.04E+03 1.42E-01	1.08E+03 2.10E-02	2.14E+02 3.14E-09
	S3	4.95E+02 1.36E-04	1.16E+03 3.13E-01	9.35E+02 6.34E-01	1.25E+03 1.00E-02	1.16E+03 2.40E-02	7.11E+02 1.22E-01	9.29E+02 6.73E-01	1.01E+03 1.22E-01	1.47E+02 5.35E-11
45%	S1	7.81E+02 1.95E-01	1.03E+03 9.90E-01	9.90E+02 3.26E-01	1.25E+03 1.00E-02	1.09E+03 8.70E-02	8.59E+02 8.34E-01	1.11E+03 4.20E-02	1.08E+03 2.50E-02	1.63E+02 1.21E-10
	S2	9.98E+02 5.08E-01	1.11E+03 5.37E-01	1.12E+03 2.80E-02	1.24E+03 1.10E-02	9.73E+02 5.28E-01	1.00E+03 2.47E-01	1.17E+03 7.00E-03	1.02E+03 7.00E-02	1.46E+02 3.90E-11
	S3	1.12E+03 7.50E-02	1.21E+03 1.61E-01	1.07E+03 8.40E-02	1.41E+03 7.18E-05	1.16E+03 2.30E-02	1.02E+03 2.05E-01	1.38E+03 5.68E-06	1.22E+03 2.48E-04	2.40E+02 1.01E-08
61%	S1	1.28E+03 2.00E-03	9.79E+02 6.73E-01	1.18E+03 7.00E-03	1.33E+03 1.00E-03	1.03E+03 2.43E-01	1.11E+03 3.80E-02	1.14E+03 2.10E-02	8.86E+02 6.69E-01	1.43E+02 1.48E-11
	S2	1.27E+03 3.00E-03	1.13E+03 4.31E-01	1.08E+03 7.80E-02	1.44E+03 2.63E-05	1.16E+03 2.10E-02	1.29E+03 2.52E-04	1.35E+03 1.91E-05	1.05E+03 4.50E-02	1.42E+02 2.68E-11
	S3	1.11E+03 1.04E-01	1.08E+03 7.04E-01	8.59E+02 8.40E-01	1.22E+03 2.00E-02	1.18E+03 1.40E-02	9.82E+02 3.51E-01	1.16E+03 1.30E-02	1.02E+03 8.20E-02	1.59E+02 7.82E-11
70%	S1	1.48E+03 1.60E-06	1.11E+03 5.21E-01	1.23E+03 2.00E-03	1.38E+03 2.45E-04	9.51E+02 6.69E-01	1.07E+03 9.10E-02	1.29E+03 2.71E-04	1.05E+03 4.50E-02	2.51E+02 6.00E-09
	S2	1.23E+03 7.00E-03	7.81E+02 4.50E-02	1.10E+03 4.80E-02	1.45E+03 1.47E-05	1.20E+03 9.00E-03	1.10E+03 4.50E-02	1.22E+03 3.00E-03	9.78E+02 1.98E-01	1.63E+02 8.45E-11
	S3	1.32E+03 1.00E-03	1.08E+03 7.26E-01	9.48E+02 5.51E-01	1.25E+03 9.00E-03	1.14E+03 4.00E-02	1.07E+03 9.70E-02	1.27E+03 1.00E-03	1.05E+03 4.60E-02	2.65E+02 1.65E-08
80%	S1	1.36E+03 1.62E-04	7.57E+02 2.90E-02	9.89E+02 3.38E-01	1.38E+03 2.57E-04	1.22E+03 5.00E-03	9.35E+02 6.34E-01	1.24E+03 1.00E-03	1.10E+03 1.40E-02	3.30E+02 8.78E-08
	S2	1.07E+03 2.19E-01	6.33E+02 1.00E-03	1.10E+03 4.60E-02	1.16E+03 7.50E-02	9.05E+02 9.79E-01	1.07E+03 9.50E-02	1.09E+03 5.90E-02	8.73E+02 7.65E-01	2.16E+02 1.91E-09
	S3	1.31E+03 1.00E-03	8.81E+02 2.28E-01	9.66E+02 4.51E-01	1.38E+03 2.34E-04	1.25E+03 2.00E-03	9.92E+02 3.21E-01	1.18E+03 7.00E-03	1.03E+03 8.40E-02	2.35E+02 1.91E-09
90%	S1	1.50E+03 5.68E-07	8.06E+02 7.10E-02	1.17E+03 9.00E-03	1.53E+03 6.37E-07	1.24E+03 3.00E-03	1.10E+03 5.20E-02	1.27E+03 4.60E-04	9.82E+02 1.88E-01	3.15E+02 2.66E-08
	S2	1.42E+03 1.92E-05	9.48E+02 5.04E-01	1.12E+03 3.50E-02	1.51E+03 1.87E-06	1.24E+03 3.00E-03	1.18E+03 7.00E-03	1.39E+03 5.68E-06	1.05E+03 4.70E-02	2.55E+02 5.56E-09
	S3	1.43E+03 1.45E-05	8.10E+02 7.70E-02	1.20E+03 5.00E-03	1.50E+03 2.69E-06	1.24E+03 3.00E-03	1.24E+03 1.00E-03	1.36E+03 1.67E-05	9.47E+02 3.20E-01	3.26E+02 9.87E-08

In each cell, the first number is the Mann-Whitney U Test Statistic, while the second number is the p-value.
Cells shaded in dark (light) gray indicate treatment effect significant at the 5% (10% level).

Table 6 : Non-Parametric Comparison of Series

Objective Probability		Series	T_0	T_1	T_2	T_3	T_4	T_5		T_6	
								Prediction	Bet	Prediction	Bet
3%	Rank Sum	S1	53.5	76	78.5	68.5	77.5	67.5	88.5	71.5	82.5
		S2	108	92.5	102.5	100	103.5	104	75.5	97.5	83
		S3	96.5	89.5	107	77.5	83	74.5	82	77	80.5
	Friedman Statistic		40.256	5.024	11.313	14.327	10.652	23.638	5.93	11.827	0.230
	p-value		1.81E-9	0.081	0.003	0.001	0.005	7.36E-6	0.052	0.003	0.892
5%	Rank Sum	S1	58	76.5	78.5	65	79	68.5	86	67	84.5
		S2	103	90	108.5	111	101	95	75	100.5	76
		S3	97	91.5	101	70	84	82.5	85	78.5	85.5
	Friedman Statistic		28.429	4.299	11.538	32.253	7.189	11.071	4.625	16.797	2.627
	p-value		6.71E-7	0.117	0.003	9.92E-5	0.027	0.004	0.099	2.25E-4	0.269
15%	Rank Sum	S1	60	78.5	82	65	88	67.5	90.5	65	84
		S2	102.5	90	104	103	93	86.5	76.5	96	75
		S3	95.5	89.5	102	78	83	92	79	85	87
	Friedman Statistic		25.024	2.793	6.265	19.128	1.307	10.836	6.559	16.197	4.727
	p-value		3.68E-6	0.247	0.044	7.02E-5	0.520	0.004	0.038	3.04E-4	0.094
25%	Rank Sum	S1	67	74.5	89	76	75	78.5	92	74.5	96.5
		S2	97.5	89.5	91	91	92	92	76.5	98	69.5
		S3	93.5	94	108	79	97	75.5	77.5	73.5	80
	Friedman Statistic		13.485	7.068	4.589	3.294	6.734	4.828	7.167	12.607	17.435
	p-value		0.001	0.029	0.101	0.193	0.034	0.089	0.028	0.002	1.64E-4
35%	Rank Sum	S1	59	79.5	88.5	73.5	84.5	80	90.5	72.5	90
		S2	112.5	89	106.5	88.5	97.5	89.5	80.5	92	69.5
		S3	86.5	89.5	93	84	82	76.5	75	81.5	86.5
	Friedman Statistic		35.565	2.209	3.795	3.058	3.693	2.919	6.024	5.953	12.998
	p-value		1.89E-8	0.331	0.150	0.217	0.158	0.232	0.049	0.051	0.002
45%	Rank Sum	S1	69.5	86.5	93.5	80.5	81.5	76.5	88	80.5	88.5
		S2	95	91	97.5	82	96.5	85.5	72.5	88.5	77.5
		S3	93.5	80.5	97	83.5	86	84	82.5	77	80
	Friedman Statistic		10.920	2.056	0.22	0.123	3.038	1.706	5.957	2.482	3.800
	p-value		0.004	0.358	0.896	0.940	0.219	0.426	0.051	0.289	0.150
61%	Rank Sum	S1	102	92	106	87.5	96.5	86	94	98	89
		S2	77.5	81.5	95	74.5	81.5	72.5	69	70	73.5
		S3	78.5	84.5	87	84	86	87.5	83	78	83.5
	Friedman Statistic		9.859	1.814	3.978	2.277	3.203	4.707	13.362	14.726	8.157
	p-value		0.007	0.404	0.137	0.320	0.202	0.095	0.001	0.001	0.014
70%	Rank Sum	S1	109	95	105	86	108.5	97	91.5	94	88.5
		S2	65.5	77	96	63.5	63	65.5	67	72	73.5
		S3	83.5	86	87	96.5	92.5	83.5	87.5	80	84
	Friedman Statistic		23.448	5.184	3.447	14.392	27.856	16.65	14.702	8.198	11.023
	p-value		8.10E-6	0.075	0.178	0.001	8.93E-7	2.42E-4	0.001	0.017	0.004
80%	Rank Sum	S1	109.5	89.5	110	96.5	98.5	93.5	98.5	92	88
		S2	59.5	86	92.5	61	88.5	68	66	71.5	73.5
		S3	89	82.5	85.5	88.5	77	84.5	81.5	82.5	84.5
	Friedman Statistic		32.191	0.778	6.777	17.337	6.431	11.34	19.757	7.322	13.086
	p-value		1.02E-7	0.678	0.034	1.72E-4	0.040	0.003	5.13E-5	0.026	0.001
90%	Rank Sum	S1	105	95	103	96	101	98	86.5	103	85.5
		S2	69.5	77	83	71.5	75.5	73.5	73	64.5	78.5
		S3	83.5	86	102	78.5	87.5	74.5	86.5	78.5	82
	Friedman Statistic		15.988	5.184	5.676	8.327	8.857	12.924	6.000	28.13	6.125
	p-value		1.74E-3	0.075	0.059	0.016	0.012	0.002	0.050	7.79E-7	0.047

Friedman tests are conducted to compare a subject's individual responses across Series. Under the null hypothesis, the distributions of a subject's responses are the same across series. Cells shaded in dark (light) gray indicate differences across Series significant at the 5% (10% level)

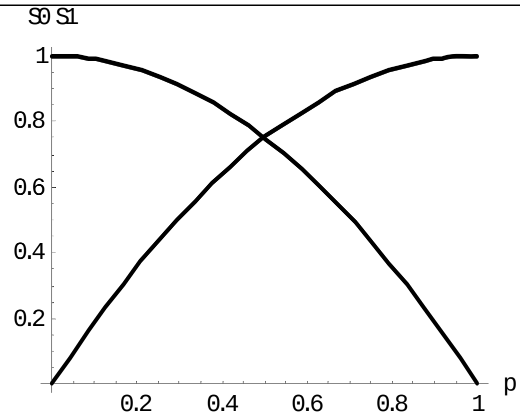

Figure 1a: Quadratic scoring rule

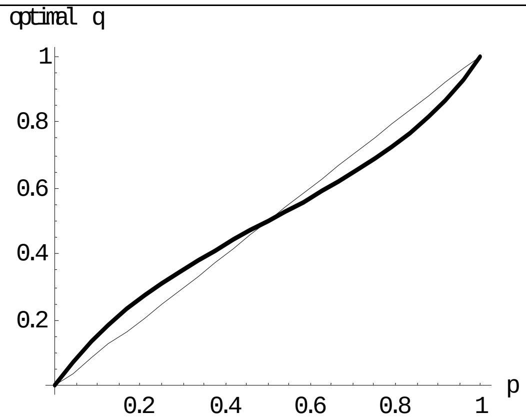

Figure 1b: Risk-aversion – response function

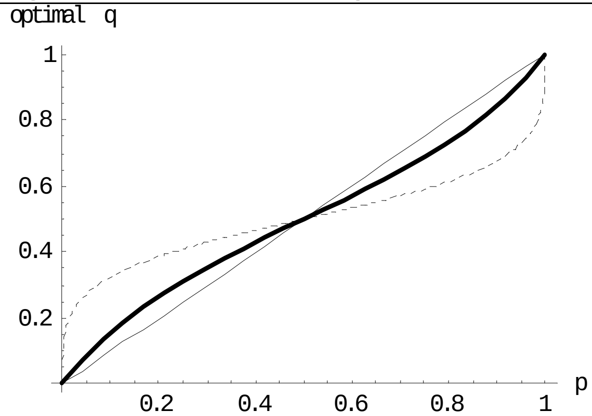

Figure 1c: Incentives – response functions

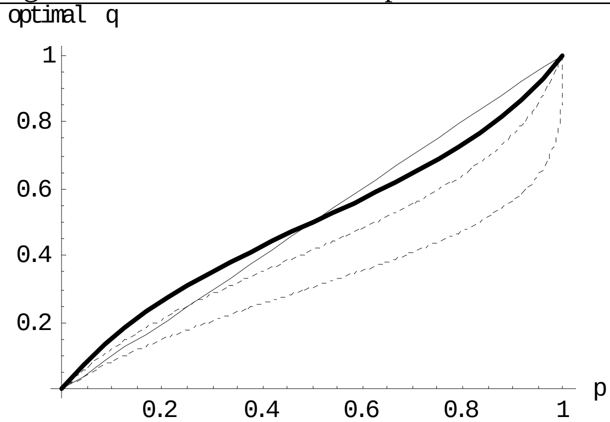

Figure 1d: Stakes – response functions

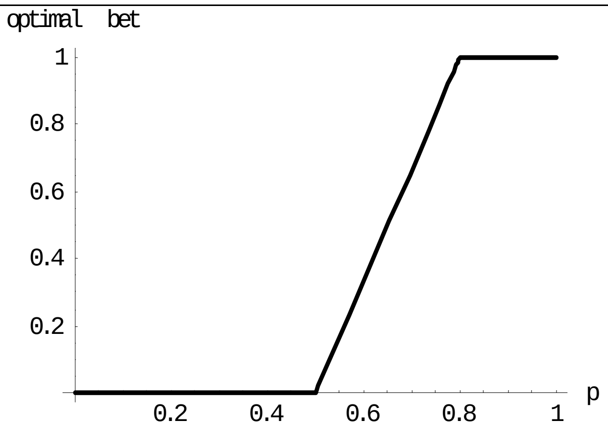

Figure 1e: Hedging – share invested (bet)

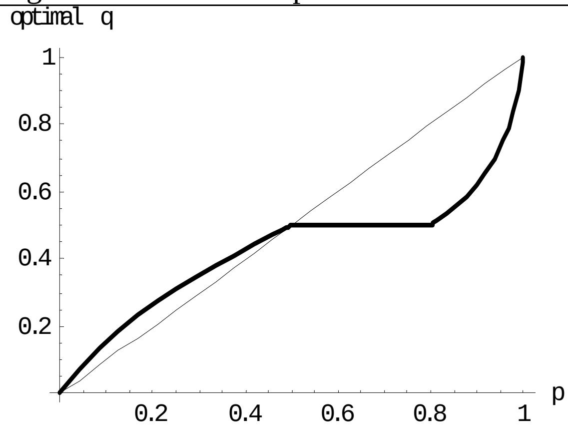

Figure 1f: Hedging – response function

Simulations were done with Mathematica.
Figure 1a represents a quadratic scoring rule (QSR): $S_1(q)=1-(1-q)^2$ and $S_0(q)=1-q^2$.
Figures 1b, 1c and 1d represent the response functions under a QSR and a quadratic utility function: $u(x)=-(2-x)^2$ with $x\leq 2$. Figure 1c represents the response functions for respective incentives $a=1$(plain curve) and $a=2$ (dashed curve). Figure 1d represents the response function for respective stakes $\Delta=0$ (plain curve), $\Delta=½$ (dashed curve) and $\Delta=1$ (dashed curve, below the diagonal).
Figures 1e and 1f represent respectively the optimal share invested (investment divided by maximal possible amount to invest) and the response function under a QSR, a quadratic utility function, a double-or-nothing investment opportunity ($k=1$) and a maximal possible amount to invest equal to 0.75.

Figure 1

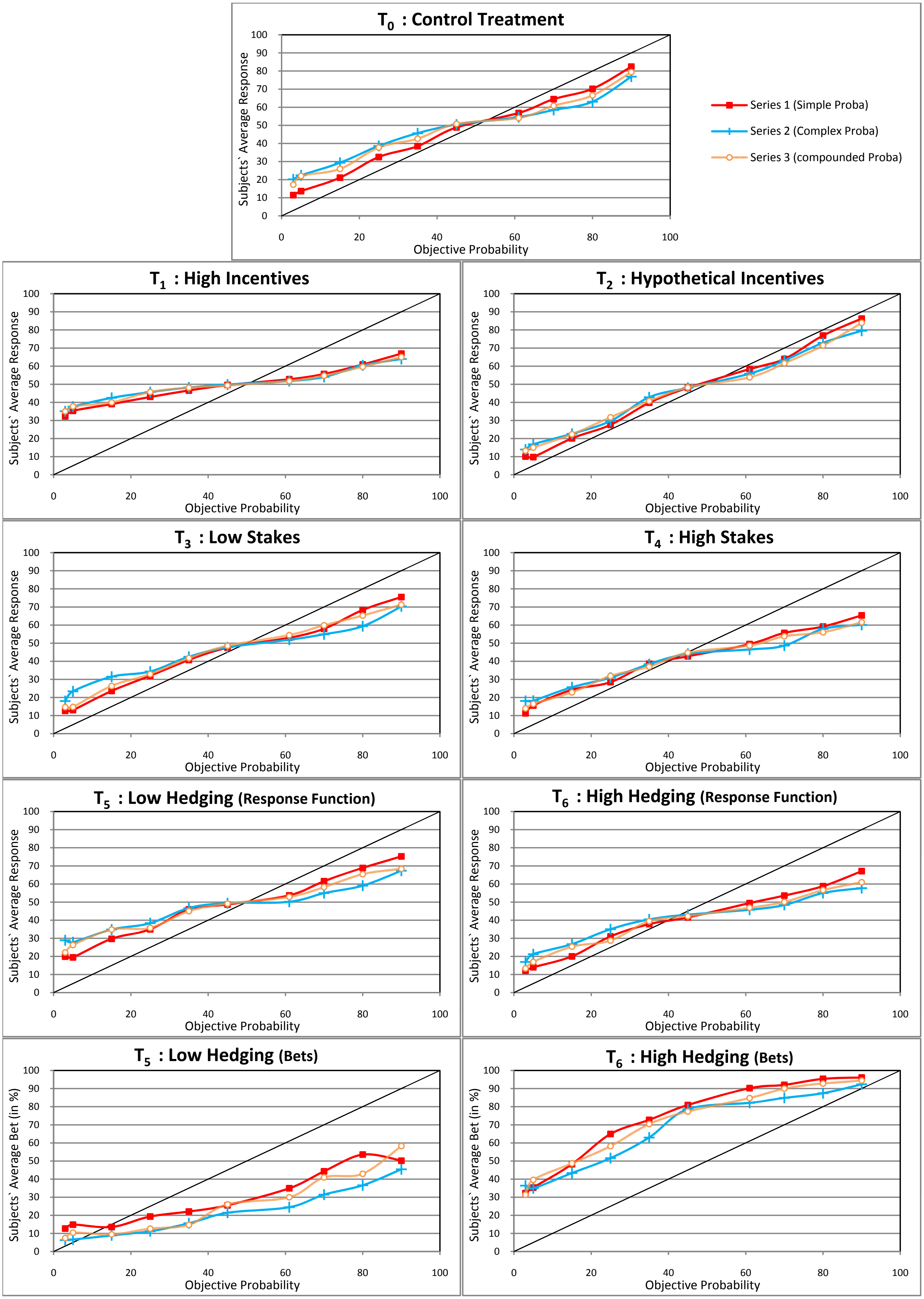
T0 : Control Treatment
T1 : High Incentives
T2 : Hypothetical Incentives
T3 : Low Stakes
T4 : High Stakes
T5 : Low Hedging (Response Function)
T6 : High Hedging (Response Function)
T5 : Low Hedging (Bets)
T6 : High Hedging (Bets)
Series 1 (Simple Proba)
Series 2 (Complex Proba)
Series 3 (compounded Proba)
Subjects` Average Response
Subjects` Average Bet (in %)
Objective Probability

Figure 2

Appendix A: Demonstration of the Propositions in section 2

Proposition 2.1 *A scoring rule is proper if and only if there exists a function $g(.)$ with $g'' > 0$ such that*

$$\begin{aligned} S_1(q) &= g(q) + (1-q)g'(q) \\ S_0(q) &= g(q) - qg'(q) \end{aligned}$$

Proof: We proved the sufficiency in the text. We now prove the necessity. Define

$$g(p) \equiv \max_q pS_1(q) + (1-p)S_0(q) = pS_1(p) + (1-p)S_0(p) \tag{5.1}$$

by the definition of a proper scoring rule. By the envelope theorem, we have $g'(p) = S_1(p) - S_0(p)$. Replacing $S_1(p)$ by $g'(p) + S_0(p)$ and $S_0(p)$ by $S_1(p) - g'(p)$ in (5.1) directly gives the result.■

Proposition 2.2 *Let $v(w) = \Phi(u(w))$ with $\Phi' > 0$ and $\Phi'' \leq 0$. Then*

$$R_v(p) \geq R_u(p) \text{ if and only if } p \leq 1/2$$

Proof: The function $R_u(p) \equiv R_u$ is defined by the first order condition:

$$p(1-R_u)u'(S_1(R_u)) - (1-p)R_u u'(S_0(R_u)) = 0 \tag{5.2}$$

We want to examine the sign of the similar first order condition for $v(.)$ evaluated at $R_v(p) = R_u$:

$$\begin{aligned} L(p) &\equiv p(1-R_u)v'(S_1(R_u)) - (1-p)R_u v'(S_0(R_u)) \\ &= p(1-R_u)u'(S_1(R_u))\Phi'(u(S_1(R_u))) - (1-p)R_u u'(S_0(R_u))\Phi'(u(S_0(R_u))) \\ &= (1-p)R_u u'(S_0(R_u))[\Phi'(u(S_1(R_u))) - \Phi'(u(S_0(R_u)))] \end{aligned}$$

where the second inequality uses $v(w) = \Phi(u(w))$, and the last inequality uses (5.2). Observe that $L(p)$ has the sign of the term into bracket, and thus of $[S_0(R_u))) - S_1(R_u))]$ since Φ' is decreasing. Consequently, $L(p)$ is positive if and only if $R_u \leq 1/2$. By the properties of R_u exhibited in Corollary 2.1, this holds true if and only if $p \leq 1/2$.■

Proposition 2.3 *Under* $\gamma'(x) \geq (\leq) 0$,

$$\frac{\partial R(p,a)}{\partial a} \geq (\leq) 0 \text{ if and only if } p \leq 1/2.$$

Proof: The response function $R(p,a) \equiv R$ is defined by the first order condition

$$M(a) = p(1-R)au'(aS_1(R)) - (1-p)Rau'(aS_0(R))) = 0 \tag{5.3}$$

Since the program is concave, the sign of $\frac{\partial R(p,a)}{\partial a}$ is the same as that of $M'(a)$. We obtain

$$\begin{aligned} M'(a) &= p(1-R)aS_1(R)u''(aS_1(R)) - (1-p)RaS_0(R)u''(aS_0(R)) \\ &= p(1-R)u'(aS_1(R))\frac{aS_1(R)u''(aS_1(R))}{u'(aS_1(R))} - \\ &\quad (1-p)Ru'(aS_0(R))\frac{aS_0(R)u''(aS_0(R))}{u'(aS_0(R))} \\ &= p(1-R)u'(aS_1(R))[\gamma(aS_0(R)) - \gamma(aS_1(R))] \end{aligned}$$

where the last equality uses (5.3) and the definition of $\gamma(x)$. Notice that $M'(a)$ has the sign of the term into bracket. Using the properties of S_0 and S_1 in (2.3) and those of R, we conclude that, under $\gamma(x)$ increasing (respectively decreasing), $M'(a)$ is positive if and only if p is lower (respectively larger) than $1/2$. ■

Proposition 2.4 *The response function $R(p, \triangle)$ is characterized as follows:*
i) if there exists a $\widehat{p}$ such that $\triangle + S_1(\widehat{p}) = S_0(\widehat{p})$, then we have

$$R(p, \triangle) \geq p \text{ if and only if } p \leq \widehat{p}$$

ii) if $\triangle + S_1(p) \geq (\leq) S_0(p)$ for all p, then we have

$$R(p, \triangle) \leq (\geq) p$$

Proof: The response function $R(p,a) \equiv R$ is characterized by the following first order condition

$$g(\triangle, R) \equiv p(1-R)u'(\triangle + S_1(R)) - (1-p)Ru'(S_0(R)) = 0$$

We compute the marginal benefit of increasing R at p :

$$g(\triangle, p) = p(1-p)[u'(\triangle + S_1(p)) - u'(S_0(p))]$$

which is positive if and only if

$$N(p) \equiv \triangle + S_1(p) - S_0(p) \leq 0$$

We thus have $R \geq p$ if and only if $N(p) \leq 0$. Since, by our assumptions on the PSR, $N(p)$ is strictly increasing, there is at most one $\widehat{p}$ satisfying $N(\widehat{p}) = 0$. Therefore either $\widehat{p}$ exists and we are in case i), or $\widehat{p}$ does not exist and we are in case ii). It is then direct to conclude the proof for each case i) and ii).■

Proposition 2.5 *The solutions* $R(p)$ and $\alpha(p)$ *to program (2.9) satisfy the following properties:*

i) *for* $p \leq \underline{p}(k)$, *we have* $\alpha(p) = 0$ *and* $R(p) \in [0, \frac{1}{1+k}]$ *with* $R'(p) > 0$,
ii) *for* p *in* $[\underline{p}(k), \overline{p}(k)]$, *we have* $\alpha(p) \in [0, \overline{\alpha}]$ *with* $\alpha'(p) > 0$ *and* $R(p) = \frac{1}{1+k}$,
iii) *for* $p \geq \overline{p}(k)$, *we have* $\alpha(p) = \overline{\alpha}$ *and* $R(p) \in [\frac{1}{1+k}, 1]$ *with* $R'(p) > 0$,

together with

$$\begin{aligned} \underline{p}(k) &= \frac{u'(S_0(\frac{1}{1+k}))}{ku'(S_1(\frac{1}{1+k})) + u'(S_0(\frac{1}{1+k}))} \text{ and} \\ \overline{p}(k) &= \frac{u'(S_0(\frac{1}{1+k}) - \overline{\alpha})}{ku'(S_1(\frac{1}{1+k}) + k\overline{\alpha}) + u'(S_0(\frac{1}{1+k}) - \overline{\alpha})} \end{aligned}$$

Proof: The conditions which characterize the interior solutions (α^*, q^*) of the program (*2.9*) are

$$pku'(S_1(q^*) + k\alpha^*) - (1-p)u'(S_0(q^*) - \alpha^*) = 0 \tag{5.4}$$

$$p(1-q^*)u'(S_1(q^*) + k\alpha^*) - (1-p)q^*u'(S_0(q^*) - \alpha^*) = 0 \tag{5.5}$$

which imply $q^* = \frac{1}{1+k}$.

Therefore the condition (5.4) writes

$$pku'(S_1(\frac{1}{1+k}) + k\alpha^*) - (1-p)u'(S_0(\frac{1}{1+k}) - \alpha^*) = 0 \tag{5.6}$$

Differentiating with respect to p and rearranging yields

$$\frac{\partial \alpha^*}{\partial p} = -\frac{ku'(S_1(\frac{1}{1+k}) + k\alpha^*) + u'(S_0(\frac{1}{1+k}) - \alpha^*)}{pk^2u''(S_1(\frac{1}{1+k}) + k\alpha^*) + (1-p)u''(S_0(\frac{1}{1+k}) - \alpha^*)} > 0$$

Therefore α^* can only increase in p; moreover, the condition (5.4) cannot be satisfied at $p = 0$ or at $p = 1$. Indeed it is strictly negative at $p = 0$ and strictly positive at $p = 1$. Consequently, $\alpha(p)$ is first equal to zero, then equal to $\alpha^* > 0$ and strictly increasing in p, and finally constant and equal to $\overline{\alpha}$. There are thus two critical values of subjective probability denoted $\underline{p}(k)$ and $\overline{p}(k)$ with $0 < \underline{p}(k) < \overline{p}(k) < 1$ such that the optimal α is equal to zero for $p \leq \underline{p}(k)$ and is equal to $\overline{\alpha}$ for $p \geq \overline{p}(k)$. Moreover the response function $R(p) = \frac{1}{1+k}$ when p is in $[\underline{p}(k), \overline{p}(k)]$.

We now study more specifically the response function when $p \leq \underline{p}(k)$. Since $\alpha^* = 0$, the response function $R(p)$ is equal to the reported probability without hedging effects, as characterized by q solving

$$p(1-q)u'(S_1(q)) - (1-p)qu'(S_0(q)) = 0$$

The threshold probability $\underline{p}(k)$ is defined by the p solving

$$pku'(S_1(\frac{1}{1+k})) - (1-p)u'(S_0(\frac{1}{1+k})) = 0$$

We thus have

$$\underline{p}(k) = \frac{u'(S_0(\frac{1}{1+k}))}{ku'(S_1(\frac{1}{1+k})) + u'(S_0(\frac{1}{1+k}))}$$

This directly gives the condition presented in the Proposition. Moreover, this implies $\underline{p}(k) > \frac{1}{1+k}$ if and only if $k < 1$. Notice also that an increase in k decreases $\underline{p}(k)$. Finally, it is easy to check that $R(\underline{p}(k) = \frac{1}{1+k}$.

We finally study the response function when $p \geq \overline{p}(k)$. Since $\alpha^* = \overline{\alpha}$, the optimal reported probability q is defined by

$$p(1-q)u'(S_1(q) + k\overline{\alpha}) - (1-p)qu'(S_0(q) - \overline{\alpha}) = 0$$

The threshold probability $\overline{p}(k)$ is defined by the p solving

$$pku'(S_1(\frac{1}{1+k}) + k\overline{\alpha}) - (1-p)u'(S_0(\frac{1}{1+k}) - \overline{\alpha}) = 0,$$

that is by

$$\overline{p}(k) = \frac{1}{k\frac{u'(S_1(\frac{1}{1+k})+k\overline{\alpha})}{u'(S_0(\frac{1}{1+k})-\overline{\alpha})} + 1}$$

■

Appendix B: The effect of ambiguity aversion

We consider the theory of ambiguity recently introduced by KMM (2005). We assume that the agent's subjective probabilities are represented by a random variable $\widetilde{p}$, of which the realizations represent each possible subjective probability which belongs to $[0, 1]$.

Consistent with KMM preferences, when faced with a PSR, the ambiguity averse agent's response function solves

$$\max_q (E\phi(\widetilde{p}u(S_1(q)) + (1-\widetilde{p})u(S_0(q))))$$

in which ϕ is assumed to be continuously differentiable, strictly increasing and concave. Under KMM preferences, the concavity of ϕ corresponds to ambiguity aversion. Under ϕ linear, the agent is ambiguity neutral and essentially behaves as an expected utility maximizer. The first order condition is

$$K(q) \equiv E\{(\widetilde{p}(1-q)u'(S_1(q)) - (1-\widetilde{p})qu(S_0(q)))\phi'(\widetilde{p}u(S_1(q)) + (1-\widetilde{p})u(S_0(q)))\} = 0 \tag{5.7}$$

It can be easily checked that the second order condition is satisfied under u and ϕ concave.

Our objective is to study the effect of ambiguity aversion on the response function. We therefore compare the solution to the previous equation assuming a concave ϕ to the solution q^* of the same equation assuming ϕ linear (i.e., under expected utility):

$$E(\widetilde{p}(1-q^*)u'(S_1(q^*)) - (1-\widetilde{p})q^*u(S_0(q^*))) = 0$$

Observe that this last condition is equivalent to (2.5) with $p = E\widetilde{p}$.

We are done if we can compute the sign $K(q^*)$, which expresses the marginal benefit under ambiguity aversion of increasing q at the optimal reported probability q^* under ambiguity neutrality. Denoting as before $f(p,q) \equiv p(1-q)u'(S_1(q)) - (1-p)qu'(S_0(q))$, and using the previous equality, we have

$$K(q^*) = Cov_{\widetilde{p}}[f(\widetilde{p}, q^*), \phi'(\widetilde{p}u(S_1(q^*)) + (1-\widetilde{p})u(S_0(q^*)))]$$

We now use the following Lemma stating the well-known covariance rule (e.g., Kimball 1951).

Lemma 6.1 *If $X(p)$ is increasing in p, then $Cov_{\widetilde{p}}(X(\widetilde{p}), Y(\widetilde{p})) \leq 0$ if and only if $Y(p)$ is decreasing in p.*

Observe that $f(p, q)$ is increasing in p. Moreover observe that the derivative of $\phi'(pu(S_1(q^*)) + (1-p)u(S_0(q^*)))$ with respect to p has the sign of $S_0(q^*) - S_1(q^*)$ under $\phi'' < 0$. Consequently $K(q^*)$ is positive if and only if q^* is lower than 1/2. We then use the Corollary 2.1 that characterizes q^* and the properties of the PSR in (2.3) to obtain the next Proposition. In this Proposition, we denote $R_a(\widetilde{p})$ the optimal reported probability under ambiguity aversion (ϕ concave) and $R(\widetilde{p})$ the optimal reported probability under expected utility (ϕ linear) with a subjective probability $E\widetilde{p}$.

Proposition 6.1 *$R_a(\widetilde{p}) \geq R(\widetilde{p})$ if and only if $E\widetilde{p} \leq 1/2$.*

Essentially, this result indicates that ambiguity aversion leads to report more uniform probabilities compared to ambiguity neutrality (i.e., expected utility). In other words, ambiguity aversion reinforces the bias induced by risk aversion. It is also possible to show that ambiguity aversion reinforces the effect of risk aversion when there is a stake. That is, ambiguity aversion leads to increase the response function before the fixed point and to decrease the response function after the fixed point.

A particular case is $u(x) = x$. In that case, the agent is ambiguity averse, but risk neutral. This implies $q^* = E\widetilde{p}$: when the agent is both ambiguity neutral and risk neutral, she always reports the mean of her subjective beliefs $E\widetilde{p}$ by the definition of a PSR. Proposition 6.1 then reduces to the following Corollary.

Corollary 6.1 *$R_a(\widetilde{p}) \geq E\widetilde{p}$ if and only if $E\widetilde{p} \leq 1/2$.*

This Corollary, similar to Corollary 2.1, thus further suggests that the effect of ambiguity aversion is similar to that of risk aversion.

Code d’identification :

Appendix C

INSTRUCTIONS POUR L’EXPERIENCE

Vous êtes sur le point de participer à une expérience visant à mieux comprendre les décisions face à l’incertitude. Au cours de cette expérience, vous aurez la possibilité de gagner une somme d’argent. Cette somme vous sera payée dès la fin de l’expérience, à l'extérieur du laboratoire, en privé, et en argent comptant. La somme d’argent que vous allez être en mesure de gagner pourra être plus importante si :

1. Vous lisez avec attention les instructions qui vont vous être données.
2. Vous suivez ces instructions scrupuleusement.
3. Vous réfléchissez bien aux décisions que vous allez prendre au cours de l’expérience.

Si vous avez des questions à poser pendant la lecture des instructions ou pendant le déroulement de l’expérience, n’hésitez surtout pas à nous appeler **en levant la main.** Par contre, **toute communication entre participants est absolument interdite pendant l'expérience. Si vous ne respectez pas ces consignes, nous serons obligés de vous exclure de l'expérience, sans aucun paiement.**

Tache

Nous allons vous présenter 30 «événements» différents, divisés en 3 séries de 10. Chacun de ces événements sera déterminé par le lancer de 2 dés. L’un des dés est rouge, l’autre dé est noir. Chacun des 2 dés a 10 faces numérotées de 0 à 9. Lors du lancer de l’un ou l’autre des 2 dés, chacune des 10 faces a exactement la même chance de sortir. Nous vous donnons maintenant 2 exemples d’événements que nous pourrions vous proposer :

- Événement 1: ***«le dé rouge est égal à 5 et le dé noir est égal à 3»***.
- Événement 2: ***«le dé rouge est strictement plus grand que le dé noir»***.

Comme nous vous l’expliquerons dans un instant, 1 des 30 événements sera sélectionné au hasard à la fin de l’expérience. Nous effectuerons alors un (1) unique lancer de chacun des 2 dés afin de déterminer si l’événement se produit ou ne se produit pas. Par exemple, si l’événement 1 ci-dessus a été sélectionné au hasard, alors nous dirons que l’événement se produit si à la suite du lancer des 2 dés, le dé rouge est égal à 5 et le dé noir est égal à 3. Pour toute autre valeur du dé rouge ou du dé noir, nous dirons que l’événement ne se produit pas. Pareillement, si c’est l’événement 2 ci-dessus qui a été sélectionné au hasard, alors nous dirons que l’événement se produit si à la suite du lancer des 2 dés, le dé rouge est strictement plus grand que le dé noir. Dans le cas contraire, nous dirons que l’événement ne se produit pas.

Vos Choix :

Pour chacun des 30 événements, nous vous demanderons de faire un choix. Ce choix déterminera à la fois votre paiement dans le cas ou l’événement se produit, et votre paiement dans le cas ou l’événement ne se produit pas. Votre choix consistera à sélectionner un numéro entre 1 et 149 dans la table qui vous a été remise. Nous allons maintenant vous expliquer comment votre choix va affecter votre paiement.

En vous reportant à la table, vous pouvez voir que chacun des 149 choix possibles est accompagné de deux autres chiffres dans les colonnes adjacentes. Le premier chiffre correspond au paiement que vous recevrez si l’événement se produit. Le second chiffre correspond au paiement que vous recevrez si l’événement ne se produit pas. Par exemple, vous pouvez voir que les chiffres associés au choix numéro 1, sont 53 et 4000. Cela signifie que vous serez payé 53FCFA si l’événement se produit ou 4000FCFA si l’événement ne se produit pas. Comme vous pouvez le constater, quand le numéro de choix augmente de 1 à 149, les montants dans la 1ere colonne augmentent, alors que les montants dans la 2eme colonne diminuent. Par exemple, les montants associés au choix numéro 90 sont 3360FCA et 2560FCFA. Autrement dit, quand vous choisissez le numéro 90 plutôt que le numéro 1 vous gagnez plus si l’événement se produit (3360FCFA au lieu de 53FCFA), mais vous gagnez moins si l’événement ne se produit pas (2560FCFA au lieu de 4000FCFA). De la même manière, notez que les numéros de choix les plus élevés (c'est-à-dire, se rapprochant de 149) vous donnent les plus grands montants dans le cas ou l’événement se produit, mais les plus petits montants dans le cas ou l’événement ne se produit pas. Par exemple, le choix numéro 140 vous rapporte 3982FCFA si l’événement se produit, contre seulement 516FCFA si l’événement ne se produit pas.

Pour chacun des 30 événements, vous êtes totalement libre de choisir le numéro que voulez. Notez aussi qu’il n’y a pas de bon ou de mauvais choix. Les choix peuvent tout à fait être différents d’un individu à l’autre. De façon générale cependant, notez qu’il pourra être plus profitable de choisir un plus grand numéro si vous pensez que l’événement a de plus grandes chances de se produire. En effet, comme nous venons de l’expliquer, un tel choix vous donnera un montant d’argent plus élevé dans le cas ou l’événement se produit. A l’inverse, il pourra être plus profitable de choisir un plus petit numéro si vous pensez que l’événement à moins de chance de se produire.

Détermination du Paiement

Nous déterminerons votre paiement en 3 temps. Dans un premier temps nous sélectionnerons au hasard un événement. Dans un deuxième temps nous effectuerons un (1) unique lancer de chacun des 2 dés afin de déterminer si l’événement sélectionné se produit ou ne se produit pas. Enfin, dans un troisième temps nous regarderons le choix que vous avez effectué pour cet événement afin de déterminer le montant que nous vous paierons.

Nous procéderons de la façon suivante pour sélectionner l’un des 30 événements. Au tout début de l’expérience nous vous demanderons d’inscrire votre code d’identification sur un bout de papier que vous plierez ensuite. Vous trouverez votre code d’identification dans le coin, en haut a droite de cette page. A la fin de l’expérience, la personne en charge de l’expérience tirera au hasard un des papiers. La personne dont le code d’identification a été tiré au hasard choisira au hasard dans un sac 1 jeton parmi 30 jetons numérotés de 1 à 30. Le numéro du jeton sélectionné indiquera l’événement qui sera considéré pour le paiement. Le même événement sera utilisé pour déterminer le paiement de toutes les personnes dans la salle.

La personne en charge de l’expérience tirera ensuite au hasard un autre papier. La personne dont le code d’identification a été tiré au hasard effectuera alors un (1) unique lancer de chacun des 2 dés à 10 faces afin de déterminer si l’événement sélectionné se produit ou ne se produit pas. Cet unique lancer sera utilisé pour déterminer le paiement de toutes les personnes dans la salle.

Code d'identification :

Si vous ne souhaitez pas être tiré au hasard pour effectuer le lancer de dés ou la sélection de l'événement utilisé pour le paiement vous pouvez nous remettre une feuille blanche, sans inscrire votre code d'identification.

Test de Bonne Compréhension :

Une bonne compréhension des instructions et de la table est fondamentale si vous souhaitez avoir les meilleures chances d'obtenir un bon paiement au cours de l'expérience. Afin de vous assurer que c'est bien le cas, nous allons effectuer un test rapide sans enjeu. Considérez tout d'abord que l'événement 1: ***«le dé rouge est égal à 5 et le dé noir est égal à 3»*** ait été sélectionné, et imaginez qu'un individu ait sélectionné le numéro de choix **98** pour cet événement, alors qu'un autre individu a sélectionné le numéro de choix **139**. Pouvez-vous écrire dans la table ci-dessous le paiement que recevrait chacun de ces individus dans le cas ou le lancer de dé produirait les résultats suivants :

Résultat du lancer de dé	Paiement de l'individu avec	
	Numéro de choix égal à **98**	Numéro de choix égal à **139**
Le dé rouge est égal à 6 et le dé noir est égal à 4	________FCFA	________FCFA
Le dé rouge est égal à 5 et le dé noir est égal à 4	________FCFA	________FCFA
Le dé rouge est égal à 5 et le dé noir est égal à 3	________FCFA	________FCFA

Imaginez maintenant c'est l'événement 2: ***«le dé rouge est strictement plus grand que le dé noir»*** qui ait été sélectionné. D'autre part, imaginez qu'un individu ait sélectionné le choix numéro de choix **6** pour cet événement, alors qu'un autre individu a sélectionné le numéro de choix **71**. Pouvez-vous écrire dans la table ci-dessous le paiement que recevrait chacun de ces individus dans le cas ou le lancer de dé produirait les résultats suivants :

Résultat du lancer de dé	Paiement de l'individu avec	
	Numéro de choix égal à **6**	Numéro de choix égal à **71**
Le dé rouge est égal à 3 et le dé noir est égal à 9	________FCFA	________FCFA
Le dé rouge est égal à 5 et le dé noir est égal à 2	________FCFA	________FCFA
Le dé rouge est égal à 0 et le dé noir est égal à 5	________FCFA	________FCFA

Dans le cas ou les instructions que nous venons de vous lire n'étaient pas parfaitement claires, nous vous invitons à nous poser des questions dés maintenant, en levant la main. Une fois que l'expérience aura commencé vous pourrez encore nous poser des questions en levant au préalable la main.

Notez aussi que le paiement que vous recevrez aujourd'hui pourra être plus ou moins élevé en fonction de vos choix et du résultat du lancer des dés. En acceptant de participer à l'expérience vous accepter les conséquences liées à vos choix et au lancer des dés. Dans le cas ou vous ne souhaiteriez pas participer à l'expérience vous êtes libre de partir dés maintenant, auquel cas nous vous remettrons un dédommagement de 500FCFA.

Code d'identification :

Série 1 :

Pour les 10 premiers événements, on considérera que le dé rouge constitue les Dizaines (c'est à dire 0, 10, 20, 30, 40, 50, 60, 70, 80, et 90) et le dé noir les Unités (c'est à dire 0, 1, 2, 3, 4, 5, 6, 7, 8, et 9). Si l'on ajoute les deux dés on obtient alors un nombre entre 1 et 100 (si les dés sont tous les 2 égaux à 0, alors on considère que cela équivaut au nombre 100). Notez que tous les nombres entre 1 et 100 ont tous exactement la même chance d'être obtenu.

Événement	Description	Numéro de Choix
Numéro **1**	***« le nombre obtenu est compris entre 1 (inclus) et 25 (inclus) »***	
Numéro **2**	***« le nombre obtenu est compris entre 62 (inclus) et 66 (inclus) »***	
Numéro **3**	***« le nombre obtenu est compris entre 16 (inclus) et 76 (inclus) »***	
Numéro **4**	***« le nombre obtenu est compris entre 3 (inclus) et 92 (inclus) »***	
Numéro **5**	***« le nombre obtenu est compris entre 52 (inclus) et 96 (inclus) »***	
Numéro **6**	***le nombre obtenu est compris entre 9 (inclus) et 88 (inclus) »***	
Numéro **7**	***« le nombre obtenu est compris entre 44 (inclus) et 58 (inclus) »***	
Numéro **8**	***« le nombre obtenu est compris entre 23 (inclus) et 25 (inclus) »***	
Numéro **9**	***« le nombre obtenu est compris entre 37 (inclus) et 71 (inclus) »***	
Numéro **10**	***« le nombre obtenu est compris entre 28 (inclus) et 97 (inclus) »***	

Série 2 :

Pour les 10 événements suivants, on ajoutera le résultat du dé rouge au résultat du dé noir. Comme les résultats de chaque dé sont entre 0 et 9, on obtiendra alors une somme entre 0 et 18. Notez que certaines sommes ne peuvent être obtenues qu'avec une combinaison unique des 2 dés (par exemple 0), alors que d'autres sommes peuvent être obtenues avec plusieurs combinaisons des 2 dés (par exemple 6). Par conséquent, certaines des 18 sommes ont plus de chances d'être obtenues que d'autres.

Événement	Description	Numéro de Choix
Numéro **11**	***« la somme obtenue est comprise entre 0 (inclus) et 4 (inclus) »***	
Numéro **12**	***« la somme obtenue est comprise entre 2 (inclus) et 10 (inclus) »***	
Numéro **13**	***« la somme obtenue est égale à 16 »***	
Numéro **14**	***« la somme obtenue est comprise entre 4 (inclus) et 14 (inclus) »***	
Numéro **15**	***« la somme obtenue est comprise entre 5 (inclus) et 13 (inclus) »***	
Numéro **16**	***« la somme obtenue est comprise entre 0 (inclus) et 14 (inclus) »***	
Numéro **17**	***« la somme obtenue est comprise entre 10 (inclus) et 18 (inclus) »***	
Numéro **18**	***« la somme obtenue est égale à 4 »***	
Numéro **19**	***« la somme obtenue est comprise entre 11 (inclus) et 17 (inclus) »***	
Numéro **20**	***« la somme obtenue est comprise entre 2 (inclus) et 6 (inclus) »***	

<u>Code d'identification :</u>

<u>Série 3 :</u>

La dernière série de 10 choix ressemble à la première. Le dé rouge constitue à nouveau les Dizaines, le dé noir les Unités, et on ajoute les deux dés pour obtenir un nombre entre 1 et 100. La différence avec la série 1, est qu'au moment où vous sélectionnez un numéro de choix, vous faites maintenant face non plus à 1, mais à 2 événements possibles. Par exemple, le 1er événement pourrait être ***«le nombre obtenu est compris entre 1 (inclus) et 25 (inclus)»*** et le 2ème ***«le nombre obtenu est compris entre 55 (inclus) et 59 (inclus)»***. Vous devez sélectionner un seul numéro de choix sans savoir lequel des ces 2 événements sera utilisé pour calculer votre paiement. Ce n'est qu'à la fin de l'expérience que l'expérimentateur lancera une pièce de monnaie afin d'identifier l'un des 2 événements. Si la pièce tombe du coté **<u>Pile</u>**, alors votre paiement sera calculé en utilisant le 1er événement. Si la pièce tombe du coté **<u>Face</u>**, alors votre paiement sera calculé en utilisant le 2ème événement. Comme pour la série 1, nous lancerons ensuite les deux dés afin de déterminer si l'événement identifié par la pièce de monnaie se produit ou pas. Voici un exemple :

♦ Si la pièce tombe du coté **<u>Pile</u>**, alors l'événement est :

« le nombre obtenu est compris entre 1 (inclus) et 25 (inclus) ».

♦ <u>Ou bien</u>, Si la pièce tombe du coté **<u>Face</u>**, alors l'événement est :

« le dé rouge est compris entre 55 (inclus) et 59 (inclus) ».

Vous devez sélectionner un seul numéro de choix alors que vous ne savez pas encore lequel de ces 2 événement sera utilisé pour calculer votre paiement. Imaginons qu'une personne sélectionne le numéro de choix 70. Pour calculer le paiement de cette personne nous devons distinguer les 2 cas possibles :

♦ Soit la pièce lancée à la fin de l'expérience tombe du coté **<u>Pile</u>**. Dans ce cas, l'événement que nous considérons est : ***« le nombre obtenu est compris entre 1 (inclus) et 25 (inclus) »***. Si ensuite les 2 dés donnent un nombre qui est bien compris entre 1 (inclus) et 25 (inclus), alors l'événement se produit et la personne dans notre exemple reçoit 2862FCFA. Par contre, si les 2 dés donnent un nombre qui n'est pas compris entre 1 (inclus) et 25 (inclus), alors l'événement ne se produit pas et la personne reçoit 3129FCFA.

♦ Soit la pièce lancée à la fin de l'expérience tombe du coté **<u>Face</u>**. Dans ce cas, l'événement que nous considérons est : ***« le nombre obtenu est compris entre 55 (inclus) et 59 (inclus) »***. Si ensuite les 2 dés donnent un nombre qui est bien compris entre 55 (inclus) et 59 (inclus), alors l'événement se produit et la personne dans notre exemple reçoit 2862FCFA. Par contre, si les 2 dés donnent un nombre qui n'est pas compris entre 55 (inclus) et 59 (inclus), alors l'événement ne se produit pas et la personne reçoit 3129FCFA.

Pour résumer, il y a donc 2 scénarios dans lesquels nous dirons que l'événement se produit : 1) la pièce tombe du coté **<u>Pile</u>** et les dés donnent un nombre entre 1 (inclus) et 25 (inclus), <u>ou bien</u> 2) la pièce tombe du coté **<u>Face</u>** et les dés donnent un nombre entre 55 (inclus) et 59 (inclus). Dans tous les autres cas, l'événement ne se produit pas. Quand vous sélectionnez votre numéro de choix, vous devez donc essayer d'imaginer les différents scénarios possibles dans lesquels l'événement se produit ou ne se produit pas.

Si ces explications ne sont pas parfaitement claires, veuillez nous <u>appeler en levant la main</u>. Nous viendrons vous donner des explications supplémentaires. Nous vous rappelons qu'il est très important pour vous de bien comprendre les instructions afin de prendre les décisions qui vous conviennent le mieux.

Code d'identification :

Événement	Description	Numéro de Choix
Numéro **21**	♦ Si la pièce tombe du coté **Pile**, alors l'événement est : ***« le nombre obtenu est compris entre 48 (inclus) et 82 (inclus) »***. ♦ Ou bien, Si la pièce tombe du coté **Face**, alors l'événement est : ***« le nombre obtenu est compris entre 14 (inclus) et 48 (inclus) »***.	
Numéro **22**	♦ Si la pièce tombe du coté **Pile**, alors l'événement est : ***« le nombre obtenu est compris entre 21 (inclus) et 35 (inclus) »***. ♦ Ou bien, Si la pièce tombe du coté **Face**, alors l'événement est : ***« le nombre obtenu est compris entre 30 (inclus) et 44 (inclus) »***.	
Numéro **23**	♦ Si la pièce tombe du coté **Pile**, alors l'événement est : ***« le nombre obtenu est compris entre 25 (inclus) et 89 (inclus) »***. ♦ Ou bien, Si la pièce tombe du coté **Face**, alors l'événement est : ***« le nombre obtenu est compris entre 2 (inclus) et 96 (inclus) »***.	
Numéro **24**	♦ Si la pièce tombe du coté **Pile**, alors l'événement est : ***« le nombre obtenu est compris entre 66 (inclus) et 97 (inclus) »***. ♦ Ou bien, Si la pièce tombe du coté **Face**, alors l'événement est : ***« le nombre obtenu est compris entre 13 (inclus) et 70 (inclus) »***.	
Numéro **25**	♦ Si la pièce tombe du coté **Pile**, alors l'événement est : ***« le nombre obtenu est compris entre 56 (inclus) et 58 (inclus) »***. ♦ Ou bien, Si la pièce tombe du coté **Face**, alors l'événement est : ***« le nombre obtenu est compris entre 78 (inclus) et 80 (inclus) »***.	
Numéro **26**	♦ Si la pièce tombe du coté **Pile**, alors l'événement est : ***« le nombre obtenu est compris entre 82 (inclus) et 89 (inclus) »***. ♦ Ou bien, Si la pièce tombe du coté **Face**, alors l'événement est : ***« le nombre obtenu est compris entre 25 (inclus) et 66 (inclus) »***.	
Numéro **27**	♦ Si la pièce tombe du coté **Pile**, alors l'événement est : ***« le nombre obtenu est compris entre 7 (inclus) et 88 (inclus) »***. ♦ Ou bien, Si la pièce tombe du coté **Face**, alors l'événement est : ***« le nombre obtenu compris entre 3 (inclus) et 100 (inclus) »***.	
Numéro **28**	♦ Si la pièce tombe du coté **Pile**, alors l'événement est : ***« le nombre obtenu est égal à 12 »***. ♦ Ou bien, Si la pièce tombe du coté **Face**, alors l'événement est : ***« le nombre obtenu est compris entre 49 (inclus) et 57 (inclus) »***.	
Numéro **29**	♦ Si la pièce tombe du coté **Pile**, alors l'événement est : ***« le nombre obtenu est compris entre 26 (inclus) et 86 (inclus) »***. ♦ Ou bien, Si la pièce tombe du coté **Face**, alors l'événement est : ***« le nombre obtenu est compris entre 14 (inclus) et 74 (inclus) »***.	
Numéro **30**	♦ Si la pièce tombe du coté **Pile**, alors l'événement est : ***« le nombre obtenu est compris entre 1 (inclus) et 83 (inclus) »***. ♦ Ou bien, Si la pièce tombe du coté **Face**, alors l'événement est : ***« le nombre obtenu est compris entre 36 (inclus) et 91 (inclus) »***.	

Numéro de Choix	Votre Paiement (en FCFA) quand l'Evènement		Numéro de Choix	Votre Paiement (en FCFA) quand l'Evènement		Numéro de Choix	Votre Paiement (en FCFA) quand l'Evènement	
	Se Produit	**Ne se Produit pas**		**Se Produit**	**Ne se Produit pas**		**Se Produit**	**Ne se Produit pas**
1	53	4000	51	2258	3538	101	3573	2186
2	106	3999	52	2293	3519	102	3590	2150
3	158	3998	53	2327	3501	103	3607	2114
4	210	3997	54	2362	3482	104	3624	2077
5	262	3996	55	2396	3462	105	3640	2040
6	314	3994	56	2429	3442	106	3656	2002
7	365	3991	57	2462	3422	107	3671	1965
8	415	3989	58	2495	3402	108	3686	1926
9	466	3986	59	2528	3381	109	3701	1888
10	516	3982	60	2560	3360	110	3716	1849
11	565	3978	61	2592	3338	111	3730	1810
12	614	3974	62	2623	3317	112	3743	1770
13	663	3970	63	2654	3294	113	3757	1730
14	712	3965	64	2685	3272	114	3770	1690
15	760	3960	65	2716	3249	115	3782	1649
16	808	3954	66	2746	3226	116	3794	1608
17	855	3949	67	2775	3202	117	3806	1566
18	902	3942	68	2805	3178	118	3818	1525
19	949	3936	69	2834	3154	119	3829	1482
20	996	3929	70	2862	3129	120	3840	1440
21	1042	3922	71	2890	3104	121	3850	1397
22	1087	3914	72	2918	3078	122	3861	1354
23	1133	3906	73	2946	3053	123	3870	1310
24	1178	3898	74	2973	3026	124	3880	1266
25	1222	3889	75	3000	3000	125	3889	1222
26	1266	3880	76	3026	2973	126	3898	1178
27	1310	3870	77	3053	2946	127	3906	1133
28	1354	3861	78	3078	2918	128	3914	1087
29	1397	3850	79	3104	2890	129	3922	1042
30	1440	3840	80	3129	2862	130	3929	996
31	1482	3829	81	3154	2834	131	3936	949
32	1525	3818	82	3178	2805	132	3942	902
33	1566	3806	83	3202	2775	133	3949	855
34	1608	3794	84	3226	2746	134	3954	808
35	1649	3782	85	3249	2716	135	3960	760
36	1690	3770	86	3272	2685	136	3965	712
37	1730	3757	87	3294	2654	137	3970	663
38	1770	3743	88	3317	2623	138	3974	614
39	1810	3730	89	3338	2592	139	3978	565
40	1849	3716	90	3360	2560	140	3982	516
41	1888	3701	91	3381	2528	141	3986	466
42	1926	3686	92	3402	2495	142	3989	415
43	1965	3671	93	3422	2462	143	3991	365
44	2002	3656	94	3442	2429	144	3994	314
45	2040	3640	95	3462	2396	145	3996	262
46	2077	3624	96	3482	2362	146	3997	210
47	2114	3607	97	3501	2327	147	3998	158
48	2150	3590	98	3519	2293	148	3999	106
49	2186	3573	99	3538	2258	149	4000	53
50	2222	3556	100	3556	2222			